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Lucky Edge Labeling Of Some Special Graphs.

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Abstract- **Let G be a Simple Graph with Vertex set V(G) and Edge set E(G) respectively. Vertex set V(G) is labeled arbitrary by positive integers and let E(e) denote the edge label such that it is the sum of labels of vertices incident with edge e. The labeling is said to be lucky edge labeling if the edge set E(G) is a proper coloring of G, that is, if we have E(e1) ≠ E(e2) whenever e1 and e2 are adjacent edges. The least integer k for which a graph G has a lucky edge labeling from the set {1, 2,….,k} is the lucky number of G denoted by η(G). A graph which admits lucky edge labeling is called Lucky Edge Graph. In this paper, it is proved that $\mathbf{Z-(P_n)}$, Fish Graph $\mathbf{C_n@K_3}$,Butterfly Graph $\mathbf{K_3^2}$, Double Triangular Snake $\mathbf{DT_n}$, Flower Graph $\mathbf{fl_n}$, $\mathbf{P_n^2}$ are Lucky Edge Graphs.**

Keywords: Lucky Edge Graph, Lucky Edge Labeling, Lucky Number, 2010 Mathematics subject classification Number: 05C78.

1. INTRODUCTION

A graph G is a finite non empty set of objects called vertices together with a set of pairs of distinct vertices of G which is called edges. Each e= {uv} of vertices in E is called an edge or a line of G. For Graph Theoretical Terminology, [2].

A vertex labeling of a graph G is an assignment of labels to the vertices of G that induces for each edge *uv* a label depending on the vertex labels of *u* and *v*.

A graph *G* is said to be labeled if the *n* vertices are distinguished from one another by symbols such as v_1, v_2,……,v_n. In this paper, it is proved that $Z-(P_n)$, Fish Graph $C_n@K_3$,Butterfly Graph K_3^2, Double Triangular Snake DT_n, Flower Graph fl_n, P_n^2 are Lucky Edge Graphs.

2. PRELIMINARIES:

Definition: 2.1

Let G be a Simple Graph with Vertex set V(G) and Edge set E(G) respectively. Vertex set V(G) is labeled arbitrary by positive integers and let E(e) denote the edge label such that it is the sum of labels of vertices incident with edge e. The labeling is said to be **Lucky Edge Labeling** if the edge set E(G) is a proper coloring of G, that is, if we have $E(e_1) \neq E(e_2)$ whenever e_1 and e_2 are adjacent edges. The least integer k for which a graph G has a lucky edge labeling from the set {1, 2,….,k} is the **Lucky Number** of G denoted by **η(G)**.

A graph which admits lucky edge labeling is called **Lucky Edge Graph**.

Definition: 2.2

$\boldsymbol{Z-(P_n)}$ is a graph obtained in a pair of path P_n, in which the i^{th} vertex of a path P_1 is joined with $(i+1)^{th}$vertex of a path P_2.

Definition: 2.3

Fish Graph is a graph obtained by attaching one of the vertex of K_3 to any one of the vertex of C_n. It is denoted by $\boldsymbol{C_n@K_3}$.

Definition: 2.4

Butterfly Graph is a planar undirected graph with 5 vertices and 6 edges. It is denoted by $\boldsymbol{K_3^2}$ or $\boldsymbol{C_3@K_3}$ or $\boldsymbol{K_3@K_3}$.

Definition: 2.5

Double Triangular Snake is a graph obtained from a path P_n, by replacing each edge by two triangles C_3. It is denoted by $\boldsymbol{DT_n}$.

Definition: 2.6

Flower Graph is a graph obtained from a Corona of a Wheel in which the end of the pendant vertices are connected to the center of a Wheel. It is denoted by $\boldsymbol{fl_n}$.

Definition: 2.7

$\mathbf{P_n^2}$ is a graph obtained from a path of length $n-1$ by joining a vertex to another vertex which is away from a path of length 2.

3. Main Results

Theorem: 3.1

$Z-(P_n)$ is a Lucky Edge Graph and the Lucky number is 6.

Proof:

Let $G = Z-(P_n)$ be the graph.

Let V(G) = $\{ u_i, v_i : 1 \leq i \leq n\}$

E(G) = $\{(u_i u_{i+1}), (v_i v_{i+1}) : 1 \leq i \leq n\text{-}1\} \cup \{(v_i u_{i+1}) : 1 \leq i \leq n\}$

Let f : V[G] →{1, 2, 3} defined by

$$f(u_i) = \begin{cases} 1 & i \equiv 1, 2 \bmod 6 \\ 3 & i \equiv 3, 4 \bmod 6\,, \quad 1 \leq i \leq n. \\ 2 & i \equiv 0, 5 \bmod 6 \end{cases}$$

$$f(v_i) = \begin{cases} 2 & i \equiv 1, 2 \bmod 6 \\ 1 & i \equiv 3, 4 \bmod 6\,, \quad 1 \leq i \leq n. \\ 3 & i \equiv 0, 5 \bmod 6 \end{cases}$$

Thus the induced edge labeling are

$$f^*(u_i u_{i+1}) = \begin{cases} 2 & i \equiv 1 \bmod 6 \\ 4 & i \equiv 2, 5 \bmod 6 \\ 6 & i \equiv 3 \bmod 6\,, \quad 1 \leq i \leq n-1. \\ 5 & i \equiv 4 \bmod 6 \\ 3 & i \equiv 0 \bmod 6 \end{cases}$$

$$f^*(v_i v_{i+1}) = \begin{cases} 4 & i \equiv 1,4 \bmod 6 \\ 3 & i \equiv 2 \bmod 6 \\ 2 & i \equiv 3 \bmod 6 \\ 6 & i \equiv 5 \bmod 6 \\ 5 & i \equiv 0 \bmod 6 \end{cases}, \quad 1 \le i \le n-1.$$

$$f^*(u_{i+1} v_i) = \begin{cases} 3 & i \equiv 1 \bmod 3 \\ 4 & i \equiv 0 \bmod 3 \\ 5 & i \equiv 2 \bmod 3 \end{cases}, \quad 1 \le i \le n-1.$$

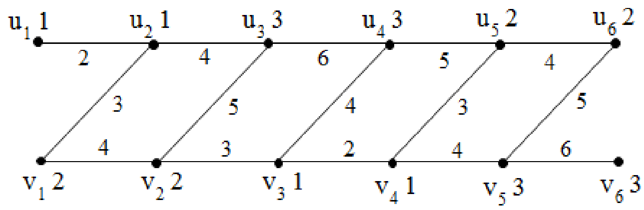


Figure 1

For example, Lucky Edge Labeling of $Z-(P_6)$ is given in figure 1 and η($Z-(P_6)$)= 6.
Thus $Z-(P_n)$ has Lucky Edge labeling and the labeling is {2, 3, 4, 5, 6} and Lucky Number η($Z-(P_n)$)= 6.
Hence $Z-(P_n)$ is a Lucky Edge Graph.

Theorem: 3.2
Fish graph is a Lucky Edge Graph and the Lucky number is 6.

Proof:
Let $G = C_n @ K_3$ be the graph.
Let V(G) = { $u_i : 1 \le i \le n$}, {v_1, v_2}
E(G) = {$(u_i u_{i+1}) : 1 \le i \le n-1$}∪{$(u_1 u_n)$}∪{$(u_1 v_i)$: $1 \le i \le 2$}∪{$(v_1 v_2)$}
Let f : V[G] →{1, 2, 3, 4} defined under 2 cases

Case a:
When $n \equiv 0, 1 (mod\ 4)$
The vertex labeling are

$$f(u_i) = \begin{cases} 1 & i \equiv 1,2 \bmod 4 \\ 2 & i \equiv 0,3 \bmod 4 \end{cases}, \quad 1 \le i \le n-1.$$

$$f(v_i) = \begin{cases} 2 & i = 1 \\ 4 & i = 2 \end{cases}$$

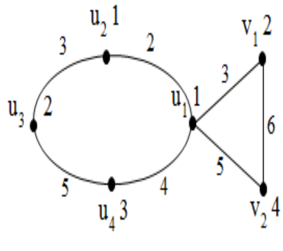
$f(u_n) = 3.$
Thus the induced edge labeling are

$$f^*(u_i u_{i+1}) = \begin{cases} 2 & i \equiv 1 \bmod 4 \\ 3 & i \equiv 0,2 \bmod 4 \\ 4 & i \equiv 3 \bmod 4 \end{cases}, \quad 1 \le i \le n-2.$$

$f^*(u_1 u_n) = 4.$
$f^*(u_n u_{n-1}) = 5.$

$$f^*(u_1 v_i) = \begin{cases} 3 & i = 1 \\ 5 & i = 2 \end{cases}$$

$f^*(v_1 v_2) = 6.$



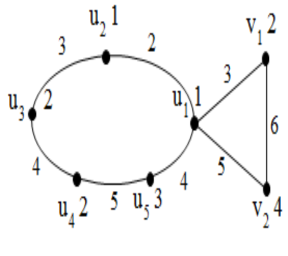


Figure 2 **Figure 3**

For example, Lucky Edge Labeling of $C_4 @ K_3$ and $C_5 @ K_3$ are given in figure 2 and figure 3 respectively.

Case b:
When $n \equiv 2, 3 (mod\ 4)$ and $n \ne 3$
The vertex labeling are

$$f(u_i) = \begin{cases} 1 & i \equiv 1,2 \ (mod\ 4) \\ 2 & i \equiv 0,3 \ (mod\ 4) \end{cases}, \quad 1 \le i \le n-2.$$

$$f(v_i) = \begin{cases} 2 & i = 1 \\ 4 & i = 2 \end{cases}$$

$f(u_i) = 3, i = n, n-1.$
Thus the induced edge labeling are

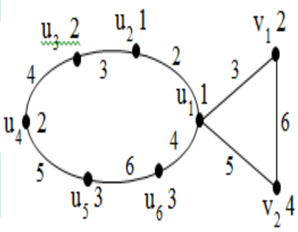
$$f^*(u_i u_{i+1}) = \begin{cases} 2 & i \equiv 1 \ (mod\ 4) \\ 3 & i \equiv 0,2 \ (mod\ 4) \\ 4 & i \equiv 3 \ (mod\ 4) \end{cases}, \quad 1 \le i \le n-3.$$

$f^*(u_1 u_n) = 4.$
$f^*(u_n u_{n-1}) = 6.$

$$f^*(u_{n-1} u_{n-2}) = \begin{cases} 5 & n \equiv 2 \ (mod\ 4) \\ 4 & n \equiv 3 \ (mod\ 4) \end{cases}.$$

$$f^*(u_1 v_i) = \begin{cases} 3 & i = 1 \\ 5 & i = 2 \end{cases}$$

$f^*(v_1 v_2) = 6$



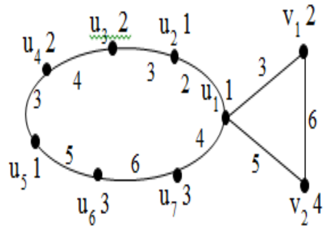


Figure 4 **Figure 5**

For example, Lucky Edge Labeling of $C_6 @ K_3$ and $C_7 @ K_3$ are given in figure 4 and figure 5 respectively.
Thus$C_n @ K_3$ has Lucky Edge labeling and in both the cases the labeling is {2, 3, 4, 5, 6} and Lucky Number η($C_n @ K_3$) is 6. Hence Fish graph is a Lucky Edge Graph.

Remark: 3.3
Butterfly Graph is a Lucky Edge Graph and the Lucky number is 7.

Proof:
From the above theorem, when $n = 3$, the graph we obtain is $C_3 @ K_3$, which is also known as Butterfly graph.
Let $G = C_3 @ K_3$ be the graph.
Let V(G) = { $u_i : 1 \le i \le 5$}
E(G) = {$(u_1 u_i) : 2 \le i \le 5$}∪{$(u_2 u_5)$}∪{$(u_3 v_4)$}
Let f : V[G] →{1, 2, 3, 4, 5} defined by
$f(u_i) = i, 1 \le i \le 5.$
The induced edge labeling are
$f^*(u_1 u_i) = 1 + i,\ 2 \le i \le 5.$
$f^*(u_2 u_5) = f^*(u_3 u_4) = 7.$

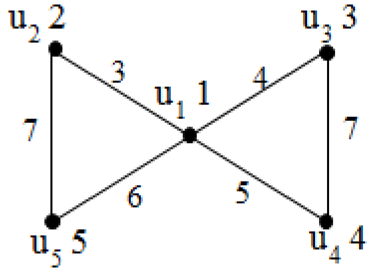


Figure 6

For example, Lucky Edge Labeling of $C_3@K_3$ is given in figure 1 and η($C_3@K_3$)= 7.
Thus Butterfly Graph has Lucky Edge labeling and the labeling is {3, 4, 5, 6, 7} and Lucky Number is 7.
Hence Butterfly Graph is a Lucky Edge Graph

Theorem: 3.4
Double Triangular Snake DT_n is a Lucky Edge Graph and the Lucky number is 10.

Proof:
Let $G = DT_n$ be the graph.
Let V(G) = { $u_i : 1 \leq i \leq n+1$}∪{$v_i : 1 \leq i \leq n$}∪{$w_i : 1 \leq i \leq n$}
E(G) = {$(u_iv_i), (u_iu_{i+1}), (v_iu_{i+1}), (u_iw_i), (w_iu_{i+1}) : 1 \leq i \leq n$}.
Let f : V[G] →{1, 2, 3, 4, 5, 6, 7} defined by

$$f(u_i) = \begin{cases} 1 & i \equiv 1 \bmod 3 \\ 2 & i \equiv 2 \bmod 3 \\ 3 & i \equiv 0 \bmod 3 \end{cases}, \quad 1 \leq i \leq n+1.$$

$$f(v_i) = \begin{cases} 4 & i \equiv 1 \bmod 2 \\ 5 & i \equiv 0 \bmod 2 \end{cases}, \quad 1 \leq i \leq n.$$

$$f(w_i) = \begin{cases} 6 & i \equiv 1 \bmod 2 \\ 7 & i \equiv 0 \bmod 2 \end{cases}, \quad 1 \leq i \leq n.$$

Thus the induced edge labeling are

$$f^*(u_iv_i) = \begin{cases} 5 & i \equiv 1 \bmod 6 \\ 6 & i \equiv 4,5 \bmod 6 \\ 7 & i \equiv 2,3 \bmod 6 \\ 8 & i \equiv 0 \bmod 6 \end{cases}, \quad 1 \leq i \leq n.$$

$$f^*(v_iu_{i+1}) = \begin{cases} 5 & i \equiv 3 \bmod 6 \\ 6 & i \equiv 1,0 \bmod 6 \\ 7 & i \equiv 4,5 \bmod 6 \\ 8 & i \equiv 2 \bmod 6 \end{cases}, \quad 1 \leq i \leq n.$$

$$f^*(u_iu_{i+1}) = \begin{cases} 3 & i \equiv 1 \bmod 3 \\ 4 & i \equiv 0 \bmod 3 \\ 5 & i \equiv 2 \bmod 3 \end{cases}, \quad 1 \leq i \leq n.$$

$$f^*(u_iw_i) = \begin{cases} 7 & i \equiv 1 \bmod 6 \\ 8 & i \equiv 4,5 \bmod 6 \\ 9 & i \equiv 2,3 \bmod 6 \\ 10 & i \equiv 0 \bmod 6 \end{cases}, \quad 1 \leq i \leq n.$$

$$f^*(w_iu_{i+1}) = \begin{cases} 7 & i \equiv 3 \bmod 6 \\ 8 & i \equiv 1,0 \bmod 6 \\ 9 & i \equiv 4,5 \bmod 6 \\ 10 & i \equiv 2 \bmod 6 \end{cases}, \quad 1 \leq i \leq n.$$

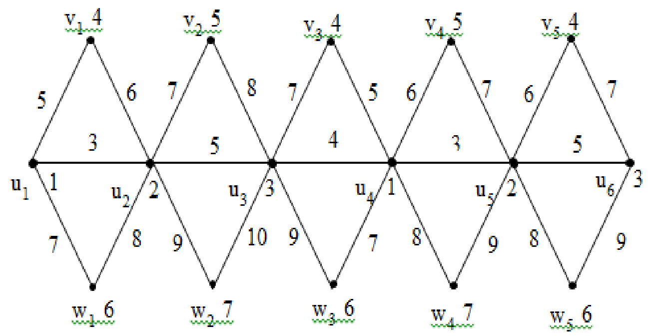


Figure 7

For example, Lucky Edge Labeling of DT_5 is given in figure 7 and η(DT_5)= 10.
Thus DT_n has Lucky Edge labeling and the labeling is {3, 4, 5, 6, 7, 8, 9, 10} and η(DT_n)= 10.
Hence Double Triangular Snake DT_n is a Lucky Edge Graph.

Theorem: 3.5
Flower Graph fl_n is a Lucky Edge Graph and the Lucky number is 2n+3.

Proof:
Let $G = fl_n$ be the graph.
Let V[G] = { $u_i : 1 \leq i \leq 2n+1$}.
E[G] = {$(u_1u_i) : 2 \leq i \leq 2n+1$}∪{$(u_iu_{i+1}) : 2 \leq i \leq n$}∪{$(u_2u_{n+1})$}∪{$(u_iu_j) : 2 \leq i \leq n+1 \,\&\, 2n+1 \geq j \geq n+2$}
Here i increases from 2 to 2n+1 when j decreases from 2n+1 to n+2 (i.e. when $i = 2, j = 2n+1$, when $i = 3, j = 2n$, and so on.)
Let f: V[G] → {1, 2, 3,……, 2n+1} defined by
$f(u_i) = i$ for $1 \leq i \leq 2n+1$
Then the induced edge labeling are
$f^*(u_1u_i) = 1 + i, 2 \leq i \leq 2n+1$
$f^*(u_iu_{i+1}) = 1 + 2i, 2 \leq i \leq n$
$f^*(u_2u_{n+1}) = 3 + n$
$f^*(u_iu_j) = 2n + 3, \; 2 \leq i \leq n+1 \,\&\, 2n+1 \geq j \geq n+2$ such that $i + j = 2n + 3$.

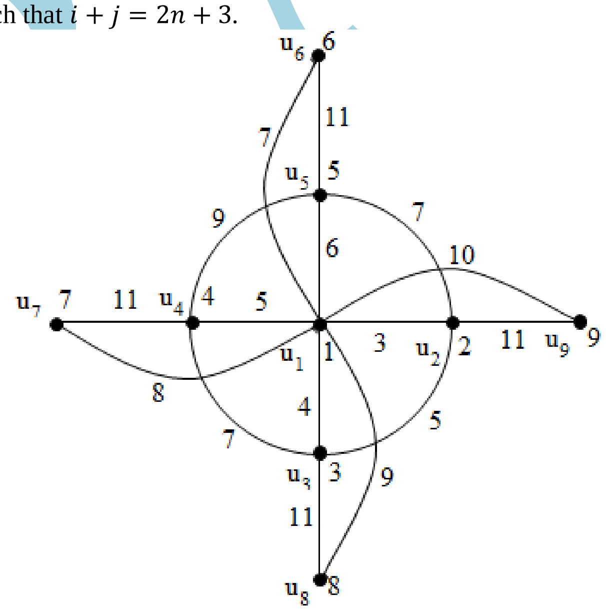


Figure 8

For example, Lucky Edge Labeling of fl_4 is given in figure 8 and η(fl_4)= 11.
Thus fl_n has Lucky Edge labeling and the labeling is {3, 4,…. $2n + 3$)}.
The Lucky Number η is $2n + 3$.
Hence Flower Graph fl_n is a Lucky Edge Graph.

Theorem: 3.6
P_n^2 is a Lucky Edge Graph and the Lucky number is 9.

Proof:
Let $G = P_n^2$ be the graph.
Let V(G) = { $u_i : 1 \leq i \leq n$}
E(G) = {$(u_iu_{i+1}) : 1 \leq i \leq n$-1}∪{$(u_iu_{i+2}) : 1 \leq i \leq n$-2}
Let f : V[G] →{1, 2, 3, 4, 5} defined by

$$f(u_i) = \begin{cases} 1 & i \equiv 1 \pmod 5 \\ 2 & i \equiv 2 \pmod 5 \\ 3 & i \equiv 3 \pmod 5 \\ 4 & i \equiv 4 \pmod 5 \\ 0 & i \equiv 5 \pmod 5 \end{cases}, \quad 1 \le i \le n.$$

Thus the induced edge labeling are

$$f^*(u_i u_{i+1}) = \begin{cases} 3 & i \equiv 1 \pmod 5 \\ 5 & i \equiv 2 \pmod 5 \\ 7 & i \equiv 3 \pmod 5 \\ 9 & i \equiv 4 \pmod 5 \\ 6 & i \equiv 5 \pmod 5 \end{cases}, \quad 1 \le i \le n-1.$$

$$f^*(u_i u_{i+2}) = \begin{cases} 4 & i \equiv 1 \pmod 5 \\ 6 & i \equiv 2 \pmod 5 \\ 8 & i \equiv 3 \pmod 5 \\ 5 & i \equiv 4 \pmod 5 \\ 7 & i \equiv 5 \pmod 5 \end{cases}, \quad 1 \le i \le n-2.$$

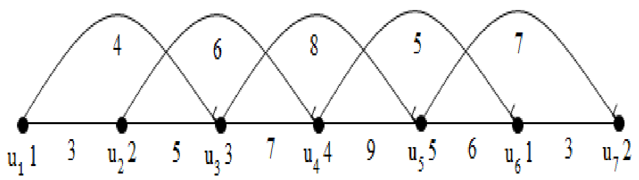


Figure 9

For example, Lucky Edge Labeling of P_7^2 is given in figure 9 and $\eta(P_7^2)$= 9.
Thus P_n^2 has Lucky Edge labeling and the labeling is {3, 4, 5, 6, 7, 8, 9} and Lucky Number $\eta(P_n^2)$= 6.
Hence P_n^2 is a Lucky Edge Graph.

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