

Application of Triangular Fuzzy Numbers in Decision Making

S. Jackson¹, T.Marimuthu², T.Aathivignesh³

¹ Assistant Professor, ^{2,3} PG student

^{1,2,3} P.G&.Research Department of Mathematics, V.O.Chidambaram College, Thoothukudi, India.

Abstract- The fuzzy set theory has been established in 1965 by Lofti A. Zadeh[6] from the University of Berkeley. Under certain conditions the fuzzy set formed in the real numbers is called Fuzzy Number. Due to the uncertainty of information and the complexity of the decision-making problem, it is difficult for decision makers to express their preferences by using exact numbers. so to reduce the complexity we can use the fuzzy numbers in such cases. Fuzzy Numbers plays a vital role in many applications decision making, Fuzzy Logic and approximate reasoning etc. In this paper We used the Triangular Fuzzy Number to find the best team in a Group Dance Competition between Five Teams.

Key words: Fuzzy Number, Triangular Fuzzy Number, Decision making.

1. INTRODUCTION

Mathematics is mainly used for computation purposes. Mathematical computation using two-valued logic has become obsolete. This is the age of logical development and its computational efficiency. The logic is being considered as one of the basic and easily applicable tools that can be easily imparted in software programming also. In 1965, L.A. Zadeh[6], an Electronics and Communication Engineer from California University of USA, generalized this logic by adding all values in the unit interval [0,1] and this is considered as the truth degree set / membership values. This generalized logic is named as 'fuzzy logic'[4]. This has stimulated the minds of the mathematicians and makes them to realize this generalization. The significance of fuzzy variables is that they facilitate gradual transitions between states and consequently, possess a natural capability to express and deal with observations and measurement uncertainties. Decision making is the process of selecting a possible course of action from all the available alternatives. In almost all such problems the multiplicity of criteria for judging the alternatives is pervasive. With regard to such problems, the decision maker wants to attain more than one objective or goal in Selecting the course of action while satisfying the constraints dictated by environment; process and resources.

Fuzzy numbers[3] are very special fuzzy subsets of the real numbers. Among various types of fuzzy sets of special significance there are fuzzy sets which are defined on the set R of real numbers. Membership functions of these sets which have the form $A : R \rightarrow [0, 1]$ clearly have a quantitative meaning and may under certain conditions, be viewed as fuzzy numbers or fuzzy intervals. To view them in this way, they should capture our intuitive conceptions of approximate numbers or intervals, such as numbers that are close to a given real number or numbers that are around a given interval of real numbers. The main aim of this research is to use the

Triangular Fuzzy Number to find the best team in a Group Dance Competition .

2.PRELIMINARIES:

Definition2.1[2]: Fuzzy number

A fuzzy number 'A' is a fuzzy set on real line R must satisfy the following conditions

(i) $\mu_A(x)$ is piecewise continuous, There exist a at least on $x_0 \in R$ with

$$\mu_A(x_0)=1$$

(ii) μ_A must be normal and convex.

Definition2.2 [3]: Triangular fuzzy number

A Triangular fuzzy number of a set A is defined as $A=(a_1,a_2,a_3)$ and its membership is given by,

$$\mu_A(x)= \begin{cases} \frac{x-a_1}{a_2-a_1} & \text{for } a_1 \leq x \leq a_2 \\ \frac{a_2-x}{a_3-a_2} & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

Definition2.3 [3]: Triangular fuzzy number matrix

The elements of Triangular fuzzy number matrix is $A=(a_{ij})_{m \times n}$ $a_{ij}=(a_{ijL},a_{ijM},a_{ijN})$ be the ij^{th} element of A.

Definition2.4[3]: Triangular fuzzy membership matrix

The membership function of $a_{ij}=(a_{ijL},a_{ijM},a_{ijN})$ is defined as $a_{ij}=(a_{ijL}/10,a_{ijM}/10,a_{ijN}/10)$.

If $0 \leq a_1 \leq a_2 \leq a_3 \leq 10$, where $0 \leq a_1/10 \leq a_2/10 \leq a_3/10 \leq 1$. Let $A=(a_{ij})_{m \times n}$ and $B=(b_{ij})_{m \times n}$ are two triangle fuzzy number matrices of same order $n \times n$.

Definition2.5[3]: Addition

$(A+B)=(a_{ij} + b_{ij})_{m \times n}$ Where $(a_{ij} + b_{ij})=(a_{ijL} + b_{ijL}, a_{ijM} + b_{ijM}, a_{ijN} + b_{ijN})$ is the ij^{th} element $(A+B)$.

Definition2.6[3]: Subtraction

$(A-B)=(a_{ij} - b_{ij})_{m \times n}$ Where $(a_{ij} - b_{ij})=(a_{ijL} - b_{ijL}, a_{ijM} - b_{ijM}, a_{ijN} - b_{ijN})$ is the ij^{th} element $(A-B)$.

Definition2.7[3]: Maximum operation on Triangular fuzzy number

The maximum operation is given by $\max(A,B) = (\sup\{a_{ij}, b_{ij}\})$ Where $\sup\{a_{ij}, b_{ij}\}=(\sup\{a_{ijL}, b_{ijL}\}, \sup\{a_{ijM}, b_{ijM}\}, \sup\{a_{ijN}, b_{ijN}\})$ is the ij^{th} element of $\max(A,B)$

Definition2.8 [1]: Arithmetic mean (AM) for Triangular fuzzy number

Let $A=(a_1, a_2, a_3)$ be Triangular fuzzy number $AM(A) = (a_1 + a_2 + a_3)/3$

Definition2.9[2]: Relativity function

Let x and y be a variables defined on a universal set X . The relativity function is denoted as $f(x/y) = \{\mu_y(x) - \mu_x(y)\} / \{\max\{\mu_y(x), \mu_x(y)\}$ where $\mu_y(x)$ is the membership function of x with respect to y for triangle fuzzy number and $\mu_x(y)$ is the membership function of y with respect to x got triangle fuzzy number. Here $\mu_y(x) - \mu_x(y)$ is calculated using subtraction operation and $\max\{\mu_y(x), \mu_x(y)\}$ is calculated by definition (7).

Definition2.10[2]: Comparison matrix

Let $A=\{x_1, x_2, \dots, x_{i-1}, x_i, \dots, x_n\}$ be a set of n -variables defined on X . Form a matrix of relativity values $f(x_i/x_j)$ where x_i 's for $i=1$ to n -variable defined on a matrix X . The matrix $C=(c_{ij})$ is a square matrix of order n is called comparison matrix or C -matrix with $AM f(x_i/x_j) = AM\{\mu_{x_j}(x_i) - \mu_{x_i}(x_j)\} / AM(\max\{\mu_{x_j}(x_i), \mu_{x_i}(x_j)\})$ Where AM represents arithmetic mean.

3.APPLICATION OF TRIANGULAR FUZZY NUMBERS:

The given are the scores ensured by five teams named A,B,C,D,E in a group dance competition judged based on the criteria listed below

Criteria	Possible points
Variety of styles	10
Co-ordination	10
Creativeness	10

Let us consider the set $\{e_1, e_2, e_3\}$ as universal set, where e_1, e_2, e_3 denotes the Variety of styles, Co-ordination, Creativeness, respectively. The matrix A represents the scores in the form of triangular fuzzy number matrix.

STEP 1:

Consider the triangle fuzzy number matrix from the imprecise estimation needed for the problem using the definition of "Triangular fuzzy number matrix"

	a	b	c	d	e
A	(5,5,7)	(4,6,7)	(7,7,8)	(4,4,5)	(6,6,5,7)
B	(5,5,6)	(5,5,5)	(6,7,7)	(6,7,7.5)	(5,6,6)
C	(4,6,7)	(5,6,6)	(4,4.5,7)	(5,7,8)	(4,6,7.5)
D	(7,8,8)	(8,8,9)	(7,8,8)	(7,7.5,7.5)	(4,4,5)
E	(7,7,8)	(6,6.5,7)	(6,7,7)	(6,6,6.5)	(4,4,5.5)

STEP 2: :

Convert the given matrix into membership function using the definition of "Triangular fuzzy membership matrix"

	a	b	c	d	e
A	(0.5,0.5,0.7)	(0.4,0.6,7)	(0.7,0.7,0.8)	(0.4,0.4,0.5)	(0.6,0.65,0.7)
B	(0.5,0.5,0.6)	(0.5,0.5,0.5)	(0.6,0.7,0.7)	(0.6,0.7,0.75)	(0.5,0.6,0.6)
C	(0.4,0.6,0.7)	(0.5,0.6,0.6)	(0.4,0.45,0.7)	(0.5,0.7,0.8)	(0.4,0.6,0.75)
D	(0.7,0.8,0.8)	(0.8,0.8,0.9)	(0.7,0.8,0.8)	(0.7,0.75,0.75)	(0.4,0.4,0.5)
E	(0.7,0.7,0.8)	(0.6,0.65,0.7)	(0.6,0.7,0.7)	(0.6,0.6,0.65)	(0.4,0.4,0.55)

$\mu_a(A)=(0.5,0.5,0.7), \mu_b(A)=(0.4,0.6,0.7), \mu_c(A)=(0.7,0.7,0.8), \mu_d(A)=(0.4,0.4,0.5), \mu_e(A)=(0.6,0.65,0.7)$

$\mu_a(B)=(0.5,0.5,0.6) \mu_b(B)=(0.5,0.5,0.5) \mu_c(B)=(0.6,0.7,0.7) \mu_d(B)=(0.6,0.7,0.75) \mu_e(B)=(0.5,0.6,0.6)$

$\mu_a(C)=(0.4,0.6,0.7) \mu_b(C)=(0.5,0.6,0.6) \mu_c(C)=(0.4,0.45,0.7) \mu_d(C)=(0.5,0.7,0.8) \mu_e(C)=(0.4,0.6,0.75)$

$$\begin{aligned} \mu_a(D) &= (0.7, 0.8, 0.8) & \mu_b(D) &= (0.8, 0.8, 0.9) & \mu_c(D) &= (0.7, 0.8, 0.8) & \mu_d(D) &= (0.7, 0.75, 0.75) & \mu_e(D) &= (0.4, 0.4, 0.5) \\ \mu_a(E) &= (0.7, 0.7, 0.8) & \mu_b(E) &= (0.6, 0.65, 0.7) & \mu_c(E) &= (0.6, 0.7, 0.7) & \mu_d(E) &= (0.6, 0.6, 0.65) & \mu_e(E) &= (0.4, 0.4, 0.55) \end{aligned}$$

STEP 3:

Calculate the relativity values $f(x_i/x_j)$ by the Definition (9)

$$\begin{aligned} f(A/a) &= \{\mu_a(A) - \mu_a(A)\} / (\max\{\mu_a(A), \mu_a(A)\}) = 0 \\ f(A/b) &= \{\mu_b(A) - \mu_a(B)\} / (\max\{\mu_b(A), \mu_a(B)\}) = (0.4, 0.6, 0.7) - (0.5, 0.5, 0.6) / \max\{(0.4, 0.6, 0.7), (0.5, 0.5, 0.6)\} = 0.056 \\ f(A/c) &= \{\mu_c(A) - \mu_a(C)\} / (\max\{\mu_c(A), \mu_a(C)\}) = (0.7, 0.7, 0.8) - (0.4, 0.6, 0.7) / \max\{(0.7, 0.7, 0.8), (0.4, 0.6, 0.7)\} = 0.227 \\ f(A/d) &= \{\mu_d(A) - \mu_a(D)\} / (\max\{\mu_d(A), \mu_a(D)\}) = (0.7, 0.8, 0.8) - (0.4, 0.4, 0.5) / \max\{(0.4, 0.4, 0.5), (0.7, 0.8, 0.8)\} = -0.435 \\ f(A/e) &= \{\mu_e(A) - \mu_a(E)\} / (\max\{\mu_e(A), \mu_a(E)\}) = (0.7, 0.7, 0.8) - (0.6, 0.65, 0.7) / \max\{(0.6, 0.65, 0.7), (0.7, 0.7, 0.8)\} = -0.114 \end{aligned}$$

Similarly,

$f(B/a) = -0.056$	$f(B/b) = 0$	$f(B/c) = 0.15$	$f(B/d) = -0.18$	$f(B/e) = -0.128$
$f(C/a) = -0.227$	$f(C/b) = -0.15$	$f(C/c) = 0$	$f(C/d) = -0.13$	$f(C/e) = -0.122$
$f(D/a) = 0.435$	$f(D/b) = 0.18$	$f(D/c) = 0.13$	$f(D/d) = 0$	$f(D/e) = -0.297$
$f(E/a) = 0.114$	$f(E/b) = 0.128$	$f(E/c) = 0.122$	$f(E/d) = 0.297$	$f(E/e) = 0$

STEP 4:

Calculate the comparison matrix from the values of $f(x_i/x_j)$ using the definition "Comparison Matrix". The comparison matrix $C = (C_{ij}) = AM(f(x_i/x_j))$ is given by

$$C = \begin{bmatrix} 0 & 0.056 & 0.227 & -0.435 & -0.114 \\ -0.056 & 0 & 0.15 & -0.18 & -0.128 \\ -0.227 & -0.15 & 0 & -0.13 & -0.122 \\ 0.435 & 0.18 & 0.13 & 0 & -0.297 \\ 0.114 & 0.128 & 0.122 & 0.297 & 0 \end{bmatrix}$$

STEP 5:

Find the minimum value from each row C_i .

$$C_i = \text{minimum of } i^{\text{th}} \text{ row}$$

A	B	C	D	E
-0.435	-0.18	0.227	-0.297	0

STEP 6: The maximum value of the column C_i is the required solution.

Team "E" is the winner of the contest.

Using triangular fuzzy number matrix we conclude that Team "E" is the winner of the given dance competition.

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