A Study on Behaviors of Waiting Line Model to Poisson distribution

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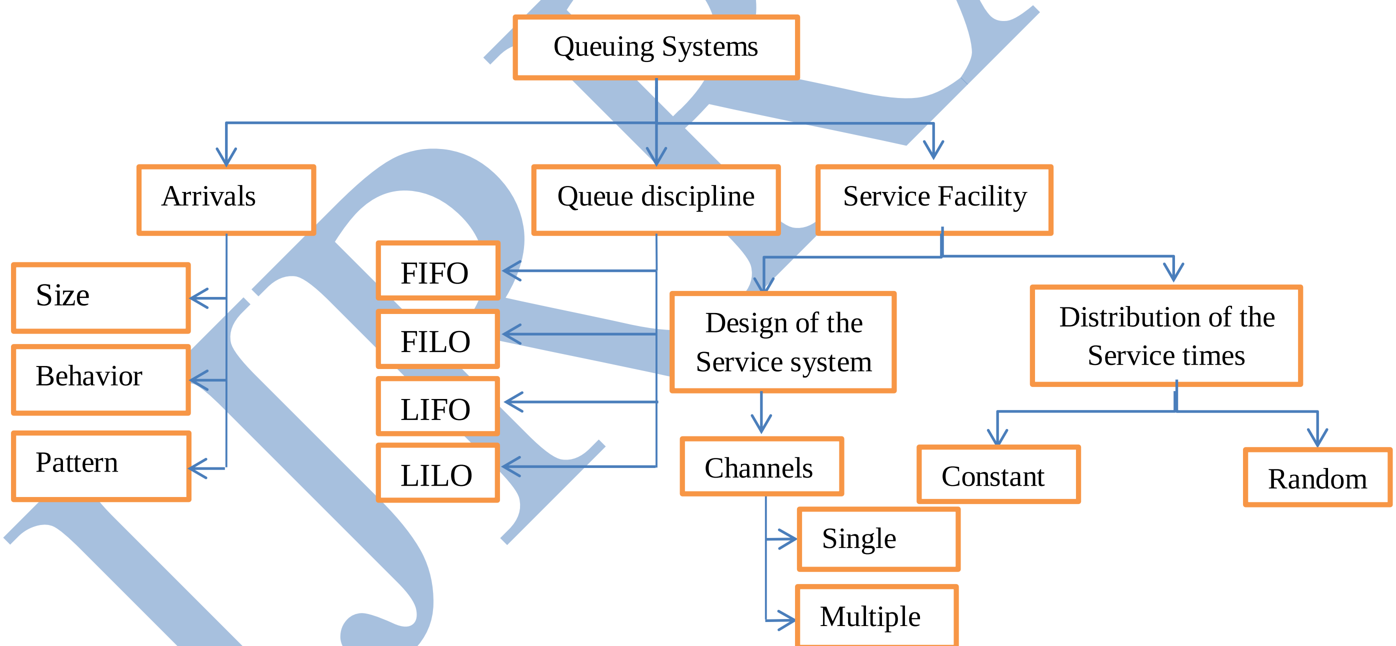
***Abstract*—Queuing theory often represented as a body of knowledge about waiting lines. It plays an important role in operations and operations manager. The characteristic of Arrivals are population size, behavior and statistical distribution. For queue discipline, we have the characteristic like limited (or) unlimited length. Similarly for the service facility we have its design and distribution as characters.**

***Keywords*— Queuing Theory, Waiting Line, Multi-channel Queuing System, Poisson distribution, Single Channel Queuing System.**

I. Introduction

Waiting lines are most common situations in Car repair shop, Fuel filling stations, Ticket counters, etc, The performance of the service system depends on the waiting – line length and average waiting time.



The above diagram is a simple form of a queuing system which has the following explanations.

II. Characteristics of Arrivals

Initially the arrival characteristic has either finite (or) infinite number of arrival population. Number of people arriving into a Bus stand will be an example for unlimited source. Rather number of Buses arriving into a Bus stand may be an example for limited source [1].Random arrival will exists when they are independent of each other. These arrivals cannot be predicted exactly. Most of the queuing problems make use of the Poisson distribution to estimate the number of arrivals per unit time.

The Poisson distribution can be established by using the formula,

$$P(x) = \frac{e^{-\lambda} \lambda^{x}}{x!} \quad for \quad x = 0,1,2,3,\ldots$$

Where,

P(x) = Probability of x arrivals

X = number of arrival per unit time

λ = average arrival rate

e = 2.7183 (which is the base of the natural logarithms)

In queuing models the behavior of arrivals will be of two types namely

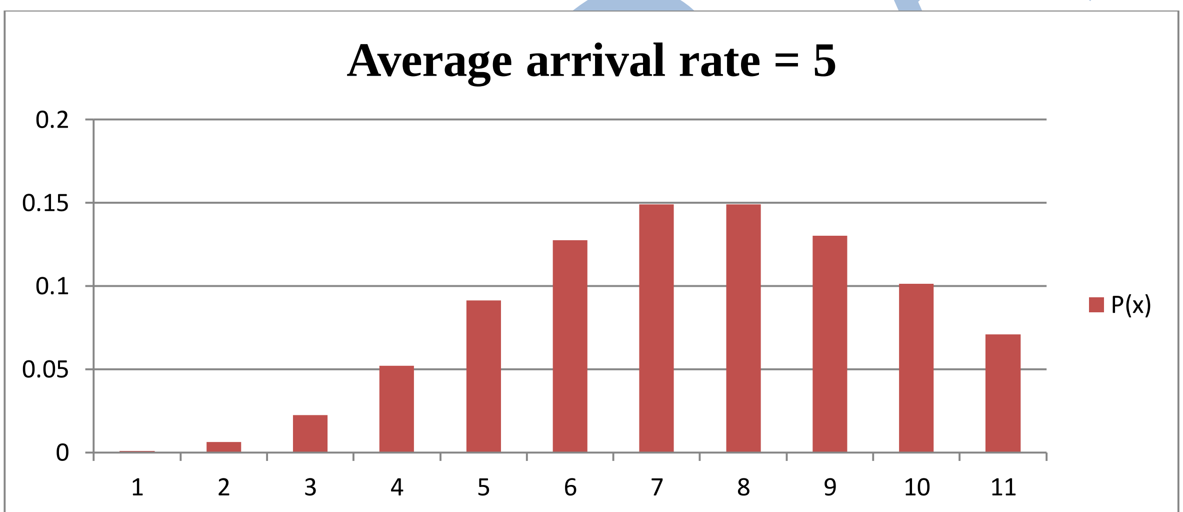
(i) Patient customers

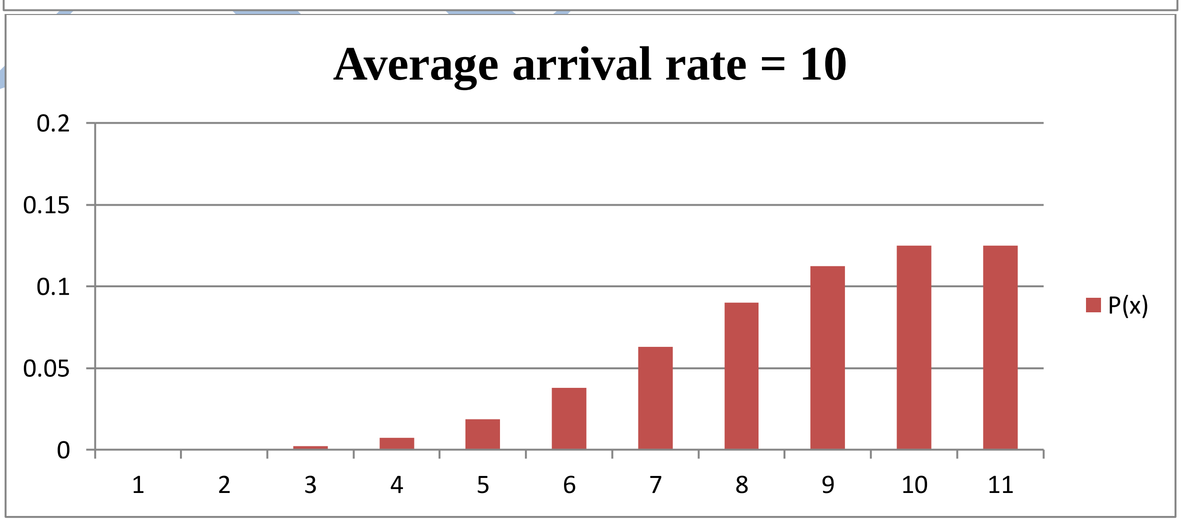
(ii) Impatient customers

Patient customers are people who can wait in the queue until they are served and don't switch between lines. Impatient customers don't want to wait and they may switch between lines. These two situations are the important factor for the need of taking analysis in waiting lines.

We now examine the Poisson distribution for λ = 5 and 7.

Poisson Distribution		
	λ = 5	λ = 10
X	P(x)	P(x)
0	0.006738	0.000454
1	0.033689	0.000454
2	0.084222	0.00227
3	0.140369	0.007566
4	0.175462	0.018915
5	0.175462	0.037831
6	0.146218	0.063051
7	0.104441	0.090073
8	0.065276	0.112592
9	0.036264	0.125102
10	0.018132	0.125102





III. Analysis

The characteristics of waiting line:

The second component in queuing system is waiting line, which may be limited of unlimited.First In First Out rule is the major discipline in queuing systems. This discipline is not valid in some peculiar situations. For example emergency care unit in hospitals, Super market, etc.,

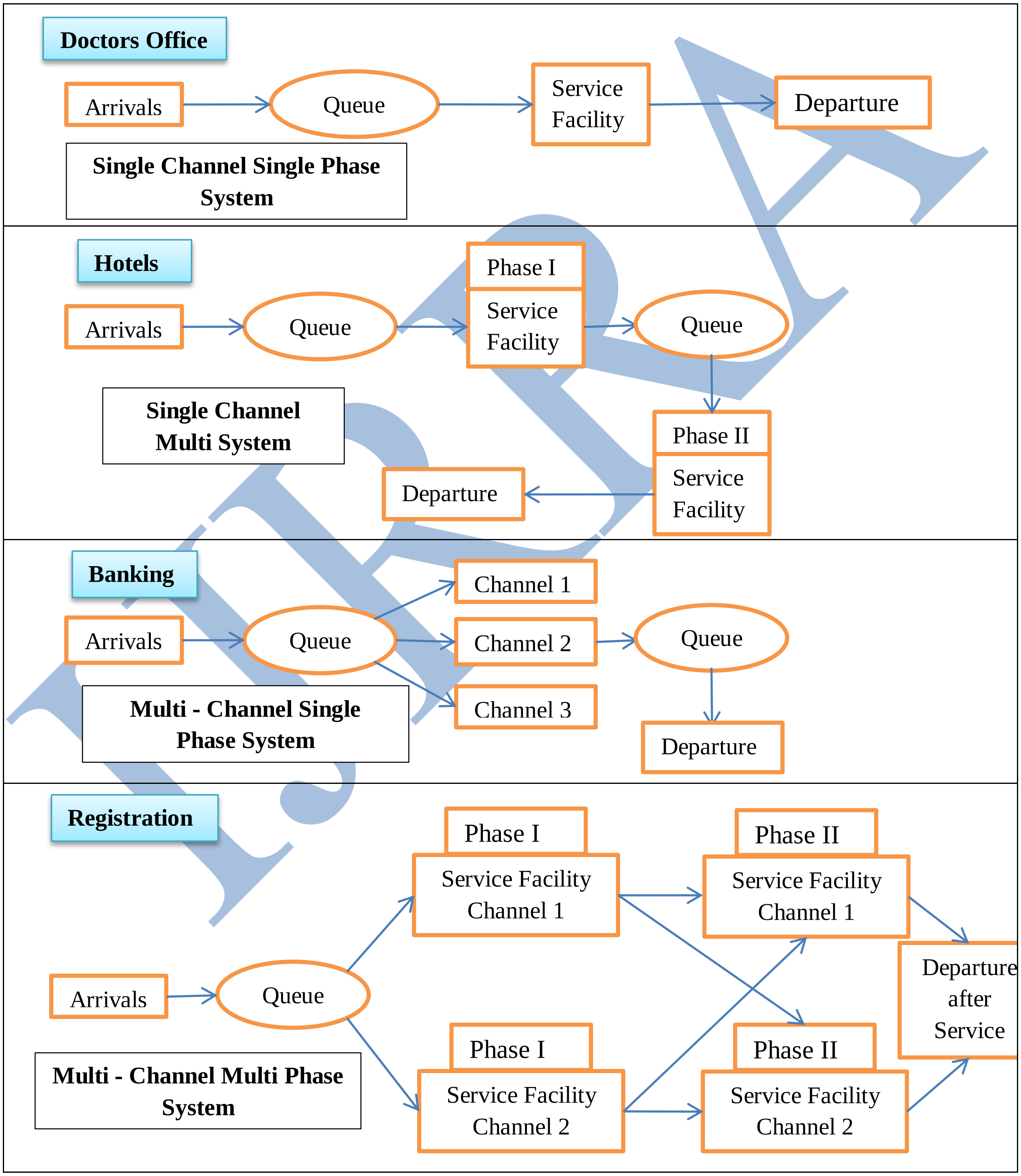
In Emergency care the patients injured critically will be treated first even though they arrive at last. And in super markets, customers buying a minimum number of items will be served first rather customers buying many items have to spend more time to get their stationaries.

The characteristics of Service:

In banking sectors, because of the single channel system there may be a long queue of customers. Nowadays multi-channel system is utilized to reduce the waiting time of the customers in the queue. Multi – channel system makes use of counters to reduce the queue length.

Service time distribution:

The time taken by the machinery to do a service is always have constant time. Whereas a continuous probability distribution often describes the random service time.



Performance Measures of Queues:

Analysis of a queuing system is a method of finding systems performances like, [3]

(i) Average time in the queue
(ii) Average queue length
(iii) Average tie in the system
(iv) Average number of customers in the system

Various Queue models:

There are several varieties in queue models. The most widely utilized models are 3, they are,M/M/1, M/M/S, M/D/1.

But there are some common factors for the above three which are,

(i) Poisson distribution arrivals
(ii) FIFO
(iii) Single – service phase.

Formulae for the above given models:

	L_s	W_s	L_q	W_q	ρ	P_0
M/M/1	$\frac{\lambda}{\mu-\lambda}$	$\frac{1}{\mu-\lambda}$	$\frac{\lambda^2}{\mu(\mu-\lambda)}$	$\frac{\lambda}{\mu(\mu-\lambda)}$	$\frac{\lambda}{\mu}$	$1-\frac{\lambda}{\mu}$
M/M/S	$\frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^M}{(M-1)!\,(M\mu-\lambda)^2}P_0+\frac{\lambda}{\mu}$	$\frac{L_s}{\lambda}$	$L_s-\frac{\lambda}{\mu}$	$\frac{L_q}{\lambda}$	–	$\frac{1}{\sum_{n=0}^{M-1}\left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right]+\frac{1}{M!}\left(\frac{\lambda}{\mu}\right)^M\frac{M\mu}{M\mu-\lambda}}$
M/D/1	$L_q+\frac{\lambda}{\mu}$	$W_q+\frac{\lambda}{\mu}$	$\frac{\lambda^2}{2\mu(\mu-\lambda)}$	$\frac{\lambda}{2\mu(\mu-\lambda)}$	–	–

IV. Conclusion:

In this module several queuing systems have discussed on behaviors of waiting line period using Poisson distribution. To analyze these models we have given models and formulae with some illustrations.

V. References

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Illustrations:

M/M/1							
λ	μ	L_s	W_s	L_q	W_q	P_0	ρ
1	2	1	1	0.5	0.5	0.5	0.5
2	3	2	1	1.333333333	0.66666667	0.333333	0.666667
3	4	3	1	2.25	0.75	0.25	0.75
4	5	4	1	3.2	0.8	0.2	0.8
5	6	5	1	4.166666667	0.83333333	0.166667	0.833333
6	7	6	1	5.142857143	0.85714286	0.142857	0.857143
7	8	7	1	6.125	0.875	0.125	0.875
8	9	8	1	7.111111111	0.88888889	0.111111	0.888889
9	10	9	1	8.1	0.9	0.1	0.9

10	11	10	1	9.090909091	0.90909091	0.090909	0.909091

M/M/S							
λ	μ	M	L_s	W_s	L_q	W_q	P_0
1	1	2	0.5	0.5	-0.5	-0.5	0.333333
2	3	3	0.01801802	0.009009009	-0.6486486	-0.32432	0.577982
3	4	4	0.00374169	0.001247228	-0.7462583	-0.24875	0.566179
4	5	5	0.00061915	0.000154788	-0.7993808	-0.19985	0.554554
5	6	6	8.7122E-05	1.74243E-05	-0.8332462	-0.16665	0.545294
6	7	7	1.0724E-05	1.78731E-06	-0.8571321	-0.14286	0.538439
7	8	8	1.1751E-06	1.6787E-07	-0.8749988	-0.125	0.533331
8	9	9	1.1609E-07	1.45112E-08	-0.8888888	-0.11111	0.529411
9	10	10	1.0443E-08	1.16032E-09	-0.9	-0.1	0.526316
10	11	11	8.6231E-10	8.62311E-11	-0.9090909	-0.09091	0.52381

M/M/S					
λ	μ	L_s	W_s	L_q	W_q
1	2	1	1	0.5	0.5
2	3	2	1	1.333333333	0.66666667
3	4	3	1	2.25	0.75
4	5	4	1	3.2	0.8
5	6	5	1	4.166666667	0.83333333
6	7	6	1	5.142857143	0.85714286
7	8	7	1	6.125	0.875
8	9	8	1	7.111111111	0.88888889
9	10	9	1	8.1	0.9
10	11	10	1	9.090909091	0.90909091