

Mathematical Modelling and Availability Optimization of feeding system in a sugar plant

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Abstract—The paper discusses the availability analysis of feeding system of sugar industry. It comprises of various subsystems so it is a complex and repairable system. It includes feeding system, crushing system, and steam generation, refining system, evaporation system and crystallization. One of the main parts of a sugar industry is feeding system. The supply system of sugar cane comprises of chain conveyor, choppers, chain conveyors and crushing system. If one unit fails, the failure of feeding system takes place.

The mathematical formulation based on Markov birth-death process using probabilistic approach will be used to develop a performance evaluation model to analyze the availability. The first order differential equations are developed to serve purpose. The equations are solved by using normalizing conditions to determine steady state availability. The quantitative analysis of courses of actions and states of nature is dealt. This paper's result is useful in analysis of availability and determining maintenance strategies in sugar industry.

Keywords— Steady state availability, Feeding system, Maintenance strategy, Markov birth-death process, Mathematical model.

I. INTRODUCTION

The sugar industry has continuous operations which convert raw materials into final products. It is difficult to achieve high productivity with minimum standby units, production losses, and minimum cost of repair. The various industrial engineering and operational research concepts are used to achieve high system availability and reliability. The factual knowledge is given by quantitative analysis in the form of failure and repair rates of various subsystems. Heavy production losses are due to random failures, wrong productions techniques and less operative skills. The effective maintenance planning and control brings back the failed system into working state in minimum down time.

The maintenance the system failure free for maximum duration, the factory operating conditions and repair strategies play important part. Each working subsystem's quantitative analysis is done to achieve this. The real system is modeled mathematically and analyzed in actual operating conditions to quantify the system performance in terms of availability. To analyze the system performance, various mathematical models are developed. Markov Birth-Death process is proposed for the present study in a sugar industry. The different differential equations are developed and solved under normalizing conditions to analyze the overall availability of a sugar industry.

II. DESCRIPTION OF FEEDING SYSTEM

The feeding system is having four sub-systems, (i) cutting system (e) is having n units in series which are combination of chains to carry sugar cane and cutters to cut it in pieces and failure of any one causes failure of system (ii) Crushing system (F) in which failure of any one of conveyors and crushers leads

to failure of juice production and of the system. (iii) Bagasse carrier system (G), failure of which effect the supply of fuel to boiler and results in reduction in efficiency of system (iv) the heat generating system (H) consists of boilers, failure of which reduces efficiency of system.

III. TERMINOLOGIES USED

Some terminologies used in this study are described below:

(i) Availability: it is the probability that a system will perform its required function at a given instant of time or over a stated period of time when operated and maintained in prescribed manner. It can be classified as mission availability, point availability and steady state availability.

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$$\text{Availability; } A(T) = (1/T) \int_0^T A(t) dt$$

$$\text{Steady state availability; } A = \lim_{t \rightarrow \infty} A(t)$$

(ii) Reliability. It is measured that the system will perform its required function under given conditions for stated time interval.

(iii) Markov process: when reliability block diagram cannot be easily found, then tools like Markov process, semi Markov and Petri nets are developed. Markov process, stochastic process exhibits the memory less property. When the system follows an exponential distribution, it is a powerful technique to analyze reliability and availability of complex systems.

(iv) Genetic Algorithm: It is a combination of search algorithms which are based on fact of natural selection and natural genetics. These algorithms are powerful for getting improvement in search and computationally simple. The advantage of GA over other technique is that it is not restricted by condition on search spaces.

IV. ASSUMPTIONS AND NOTATIONS

- (i) Failure is not considered during repair at system down state.
- (ii) Repaired rates are as good as new.
- (iii) Failure free time and repair time are stochastically independent.

Notations: $\lambda_1, \lambda_2, \lambda_3$ represents full, reduced and failed state respectively.

E, F, G, H are operative states of all four subsystems and e, f, g, h, failed states of systems E, F and reduced state GH.

- (i) α_E, α_F are repair rates of system E, F.
- (ii) β_E, β_F are complete failure rates of system E, G.
- $\delta_G, \delta_H, \lambda_G, \lambda_H$ transition rate of G, H into \bar{G}, \bar{H} into GH, respectively.

1 represents operative state 4,7,10 reduced state 2,3,5,6,8,9,11 and 12 represents failed state.

$P_n(t)$ represents probability that the system is in state n at time t.

V. MODELLING USING MARKOV PROCESS

The system is modeled as we consider that time involved in repair as well as failure free time must be distributed exponentially.

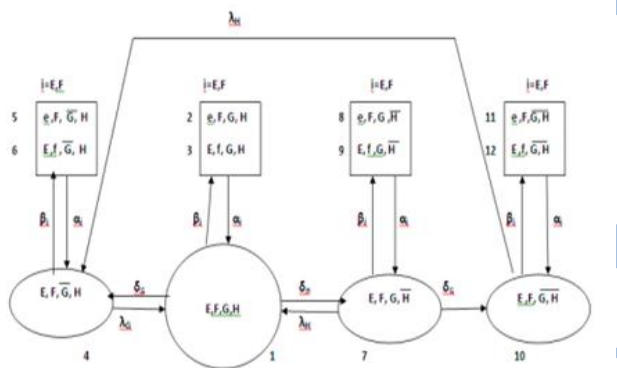


Figure 1: Transition Diagram of Feeding System

The differential equations are easily derived from transition diagram.

$$d/dt [P_1(t)] + (\beta_E + \beta_F + \delta_G + \delta_H) P_1(t) = \alpha_E P_2(t) + \alpha_F P_3(t) + \lambda_G P_4(t) + \lambda_H P_7(t),$$

$$d/dt [P_4(t)] + (\beta_E + \beta_F + \lambda_G) P_4(t) = \alpha_E P_5(t) + \alpha_F P_6(t) + \delta_G P_1(t) + \lambda_H P_10(t),$$

$$d/dt [P_7(t)] + (\beta_E + \beta_F + \delta_G + \lambda_H) P_7(t) = \alpha_E P_8(t) + \alpha_F P_9(t) + \delta_H P_1(t),$$

$$d/dt [P_10(t)] + (\beta_E + \beta_F + \lambda_H) P_10(t) = \alpha_E P_11(t) + \alpha_F P_12(t) + \delta_G P_7(t),$$

$$d/dt [P_n(t)] + \alpha_m P_n(t) = \beta_m P_1(t) \text{ where } (m, n) \text{ belongs to } \{(E, 2) (F, 3)\}$$

$$d/dt [P_n(t)] + \alpha_m P_n(t) = \beta_m P_4(t) \text{ where } (m, n) \text{ belongs to } \{(E, 5) (F, 6)\}$$

$$d/dt [P_n(t)] + \alpha_m P_n(t) = \beta_m P_7(t) \text{ where } (m, n) \text{ belongs to } \{(E, 8) (F, 9)\},$$

$$d/dt [P_n(t)] + \alpha_m P_n(t) = \beta_m P_{10}(t) \text{ where } (m, n) \text{ belongs to } \{(E, 11) (F, 12)\}.$$

By using the condition, the steady state availability can be obtained as:

$$t \rightarrow \infty, P_n(t) \rightarrow P_n \text{ and } d/dt[P_n(t)] \rightarrow 0 \forall n.$$

The equations are then transformed into

$$(\beta_E + \beta_F + \delta_G + \delta_H) P_1 = \alpha_E P_2 + \alpha_F P_3 + \lambda_G P_4 + \lambda_H P_7,$$

$$(\beta_E + \beta_F + \lambda_G) P_4 = \alpha_E P_5 + \alpha_F P_6 + \delta_G P_1 + \lambda_H P_{10},$$

$$(\beta_E + \beta_F + \delta_G + \lambda_H) P_7 = \alpha_E P_8 + \alpha_F P_9 + \delta_H P_1,$$

$$(\beta_E + \beta_F + \lambda_H) P_{10} = \alpha_E P_{11} + \alpha_F P_{12} + \delta_G P_7,$$

$\alpha_m P_n = \beta_n P_1$ where (m, n) belongs to $\{(E, 2) (F, 3)\}$; $\alpha_m P_n = \beta_n P_4$ where (m, n) belongs to $\{(E, 5) (F, 6)\}$; $\alpha_m P_n = \beta_n P_7$ where (m, n) belongs to $\{(E, 8) (F, 9)\}$; $\alpha_m P_n = \beta_n P_{10}$ where (m, n) belongs to $\{(E, 11) (F, 12)\}$

Solving the system of equations recursively in terms of P_1 , we get the equations as follows:

$$P_2 = \beta_E / \alpha_E (P_1), \quad P_3 = \beta_F / \alpha_F (P_1),$$

$$P_4 = [\delta_G(\delta_G + \delta_H + \lambda_H) / \lambda_G (\delta_G + \lambda_H)] P_1,$$

$$P_5 = [\beta_F \delta_G(\delta_G + \delta_H + \lambda_H) / \alpha_E \lambda_G (\delta_G + \lambda_H)] P_1,$$

$$P_6 = [\beta_F \delta_G(\delta_G + \delta_H + \lambda_H) / \alpha_F \lambda_G (\delta_G + \lambda_H)] P_1,$$

$$P_7 = [\delta_H / (\delta_G + \lambda_H)] P_1,$$

$$P_8 = [\beta_E \delta_H / \alpha_E (\delta_G + \lambda_H)] P_1,$$

$$P_9 = [\beta_F \delta_H / \alpha_F (\delta_G + \lambda_H)] P_1$$

$$P_{10} = [\delta_G \delta_H / \lambda_H (\delta_G + \lambda_H)] P_1,$$

$$P_{11} = [\beta_F \delta_G \delta_H / \alpha_E \lambda_H (\delta_G + \lambda_H)] P_1,$$

$$P_{12} = [\beta_F \delta_G \delta_H / \alpha_F \lambda_H (\delta_G + \lambda_H)] P_1$$

Now, using the normalizing Condition,

$$\sum_{n=1}^{12} P_n = 1$$

$$P_1 = [1 + (\beta_E / \alpha_E) + (\beta_F / \alpha_F) + \delta_G(\delta_G + \delta_H + \lambda_H) / \lambda_G (\delta_G + \lambda_H) + \beta_E \delta_G(\delta_G + \delta_H + \lambda_H) / \alpha_E \lambda_G (\delta_G + \lambda_H) + \beta_F \delta_G(\delta_G + \delta_H + \lambda_H) / \alpha_F \lambda_G (\delta_G + \lambda_H) + \delta_H / (\delta_G + \lambda_H) + \beta_E \delta_H / \alpha_E (\delta_G + \lambda_H) + \beta_F \delta_H / \alpha_F (\delta_G + \lambda_H) + \delta_G \delta_H / \lambda_H (\delta_G + \lambda_H) + \beta_E \delta_G \delta_H / \alpha_E \lambda_H (\delta_G + \lambda_H) + \beta_F \delta_G \delta_H / \alpha_F \lambda_H (\delta_G + \lambda_H)]^{-1}$$

As the sum of state probabilities of all working states of the system leads to the system availability, the steady state availability AV_{St} is given by the equation as follows:

$$AV_{St} = P_1 + P_4 + P_7 + P_{10} = [1 + \delta_G(\delta_G + \delta_H + \lambda_H) / \lambda_G (\delta_G + \lambda_H) + \delta_H / (\delta_G + \lambda_H) + \delta_G \delta_H / \lambda_H (\delta_G + \lambda_H)] P_1$$

In view of the value of P_1 , the expression for steady state availability reduces to

$$AV_{St} = 1 / [(1 + (\beta_E / \alpha_E) + (\beta_F / \alpha_F))] \approx 1 - [(\beta_E / \alpha_E) + (\beta_F / \alpha_F)]$$

VI. AVAILABILITY ANALYSIS OF FEEDING SYSTEM

The performance of availability of feeding system is mainly affected by failure and repair rates of each subsystem in a sugar plant. The high availability is ensured by system parameters of feeding system, the availability evaluation model includes all possible states of nature and course of action i.e. repair priorities. Table 1 and 2 represent effect of different combinations of failure and repair rates on availability of various subsystems of feeding system. On basis of this analysis, best possible combination of failure and repair rates may be selected i.e. optimum maintenance strategies.

α_E	0.15	0.25	0.35	0.45	0.55	0.65	
β_E							
0.001	0.989333	0.992000	0.993143	0.993778	0.994182	0.994462	$\beta_F=0.001$ $\alpha_F=0.25$
0.002	0.982667	0.988000	0.990286	0.991556	0.992364	0.992923	
0.003	0.976000	0.984000	0.987429	0.989333	0.990545	0.991385	
0.004	0.969333	0.980000	0.984571	0.987111	0.988727	0.989846	
0.005	0.962667	0.976000	0.981714	0.984889	0.986909	0.988308	
0.006	0.956000	0.972000	0.978857	0.982667	0.985091	0.986769	

Table 1: Availability matrix for cutting subsystem of feeding system

Table 1 and graph in figure 2(a) and 2(b) show the effect on availability of feeding system by using various combinations of failure rate and repair rates of cutting subsystem. In the analysis, it is observed that for some known values of failure and repair rates of crushing ($\beta_F=0.001$, $\alpha_F=0.25$), as failure rate of cutting subsystem increases from 0.001 to 0.006, the system availability decreases by 76%. Similarly as repair rate of cutting subsystem increases from 0.15 to 0.65, the system availability increases by 51%.

α_F	0.15	0.25	0.35	0.45	0.55	0.65	
β_F							
0.001	0.990476	0.993143	0.994286	0.994921	0.995325	0.995604	$\beta_E=0.001$ $\alpha_E=0.35$
0.002	0.983810	0.989143	0.991429	0.992698	0.993506	0.994066	
0.003	0.977143	0.985143	0.988571	0.990476	0.991688	0.992527	
0.004	0.970476	0.981143	0.985714	0.988254	0.989870	0.990989	
0.005	0.963810	0.977143	0.982857	0.986032	0.988052	0.989451	
0.006	0.957143	0.973143	0.980000	0.983810	0.986234	0.987912	

Table 2: Availability matrix for crushing subsystem of feeding system

Table 2 and graph in figure 3(a) and 3(b) show the effect on availability of feeding system by using various combinations of failure rate and repair rates of crushing subsystem. In the

analysis, it is observed that for some known values of failure and repair rates of cutting ($\beta_E=0.001$, $\alpha_E=0.35$), as failure rate of crushing subsystem increases from 0.001 to 0.65, the system availability decreases by 76.9%. Similarly as repair rate of crushing subsystem increases from 0.15 to 0.65, the system availability increases by 51.2%.

From the analysis, it is very much clear that for subsystems G and H, the failure and repair rates do not affect the working capacity when run for long time.

VII. CONCLUSION

It is concluded that for the analysis of availability of various subsystems of feeding system in a sugar plant, the performance evaluating model is used effectively. It also explains the relationship between various failure and repair rates for each subsystem of feeding system in a sugar plant. It also gives different availability levels for various combinations of failure and repair rates for each subsystem. Best possible combination of failure and repair priorities may be selected for each subsystem. The optimal maintenance strategies may be determined to ensure maximum availability of feeding system in a sugar plant. For each subsystem, the optimum values of failure and repair rates of each subsystem are given in Table 3. Beyond these optimum values of failure and repair rates, there is very less increase in availability levels. Therefore, we have selected the optimum values for highest possible availability level. So, findings are discussed with concerned sugar plant management. These results are beneficial to sugar plant management for analysis of availability and decide repair priorities of various subsystems of feeding system in sugar plant to enhance the performance of the system.

Sr. No.	Failure rates	Repair rates	Maximum availability Level
1	$\beta_E=0.001$	$\alpha_E=0.65$	0.994462
2	$\beta_F=0.001$	$\alpha_F=0.65$	0.995604

Table 3: Optimal values of failure and repair rates

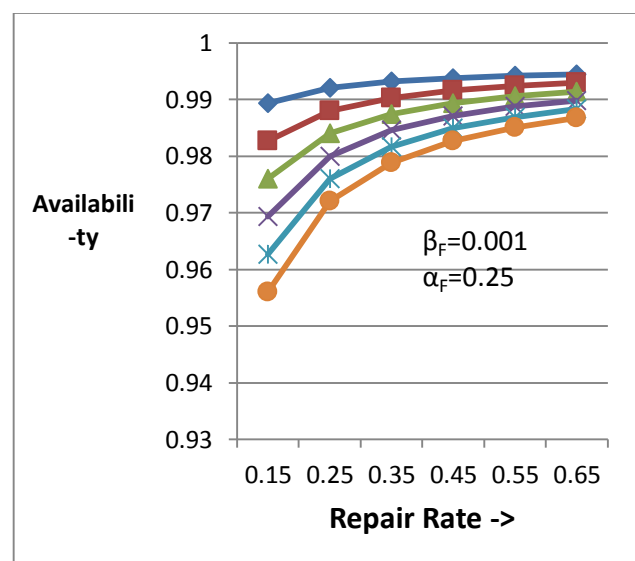


Figure 2(a): Effect of Repair rate of cutting subsystem on feeding system availability

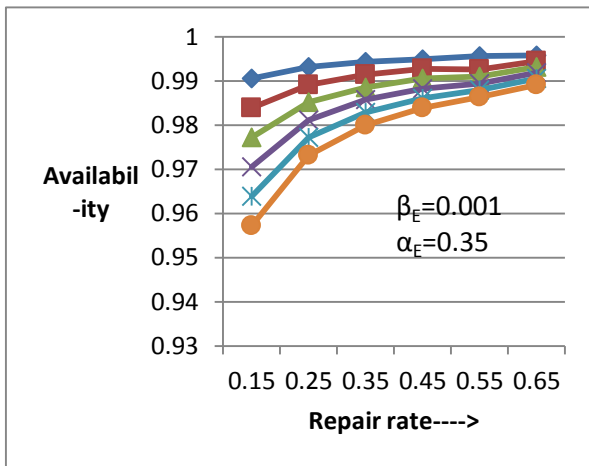


Figure 2(b): Effect of Failure rate of cutting subsystem on feeding system Availability

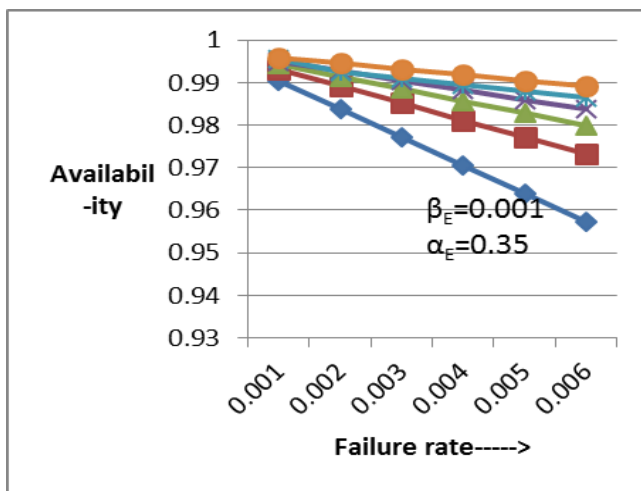


Figure 3(a): Effect of Repair rate of crushing subsystem on feeding system Availability

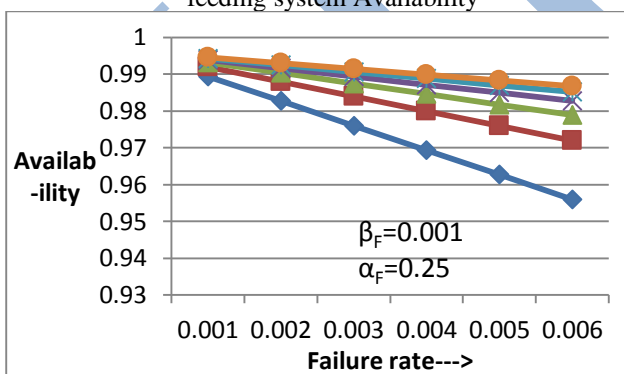


Figure 3(b): Effect of Failure rate of crushing subsystem on feeding system Availability

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