# A Stochastic model with Routine Inspection, Maintenance and Replacement

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***Abstract:*** **The incessantly uncertain structure on the science and technology face has observed appearance of many new or complex system and refined systems. For technology to stay the answer to the requirements made by both society and an industry, it is very significant that the technology germinate and make suitable in the fast changing world. This paper analyzes a stochastic model of a system having two units-one is operative and second is cold standby. Here a routine inspection is conducted on operating unit after a certain fixed period. After inspection either the unit is maintained or the unit is assumed to be failed after inspection. The repair and replacement of unit is based on the guarantee period of the failed unit In this paper the model is studied to determine the various reliability measure by using Markov Process, renewal process, MTSF/MTBF Markov chain. Here the routine maintenance time , repair and replacement time are taken as bivariate exponential.**

**Keywords: Regenerative Point, MTSF, Availability, Busy period, Cold standby, maintenance, replacement policy.**

## I. INTRODUCTION

The present-day systems that are being designed and manufactured are extremely complex in nature and quantitative reliability assessment by actual test data is practically impossible. Then because of these problems, it becomes difficult to illustrate whether a system thus produced will provide a high degree of continuous service or not. There are several technological systems those having automatic control systems. These control systems amid other systems include software system, banking systems, satellite control systems, nuclear weapon control systems. There are big losses of investment in economic terms systems due to the any non-performance of these system. Now-a-day struggle is not only in furnish the cost-effective, economic, productive systems but with these concerns the importance of reliability has becomes essential or crucial. In general,the effectiveness of highest degree is expected from a system. Different researcher takes various types of sets of assumptions related with maintenance, repair and replacement. But only a very few researcher examined the model by taking the assumption of inspection under different conditions. Various author like Jaing R and Jardine(2005),Tuteja R.K and Gulshan, Jui-Hsiang(2001), discuss the benefit analysis of a two-dissimilar cold standby system with repair and maintenance. This paper deals with a stochastic model having two non-identical units. Here a routine inspection is carried out on the operating unit after a fixed time period. It is also assumed that the operative unit is not inspected if another unit is failed. After inspection either the unit is maintained or assumed to be failed after inspection. The decision of repair and replacement of failed unit is done by taking the concept of guarantee period . It is also considered that the unit under maintenance would not fail. In this paper system is analyzed to determine various reliability measures by using mathematical tools MTSF/MTBF Markov chain, Markov Process. It is assumed that a repaired and replaced unit is good as new.

## II. DESCRIPTION OF SYSTEM AND ASSUMPTION

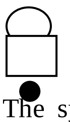
In this paper, an operative unit is inspected after a certain period of its operation and it is decided whether unit can operate further or needs certain maintenance.

- The system consists of two identical units - Initially one unit is operative and second unit is kept as cold standby.
- System is considered in Up-state if one unit is working and in down state if no unit is working.
- Each unit of the system has two modes-normal operative or failed.
- Here a routine inspection is conducted on the operating unit after a certain fixed period.
- It is assumed that operative unit is not inspected if another unit is failed.
- After inspection ,either the unit is maintained or the unit is assumed to be failed after inspection.
- A unit under maintenance would not fail.
- Check the guarantee of the failed unit ,either it is in under the guarantee period or not.
- If the unit is in the guarantee period ,the failed unit is repaired and if the unit is not under the guarantee period then it is replaced by new one.
- A repaired and replaced unit is as good as new.
- All the random variable are independent.

## III. NOTATIONS

E: Set of regenerative states
Ē: Set of non-regenerative states
$U_i$: Routine inspection
O: Unit is in operative state
S: Unit is in cold standby state
α : Constant inspection rate of unit
i(t),I(t): pdf and cdf of inspection time of a failed unit
λ: Constant failure rate of a unit
g(t),G(t): pdf and cdf of repair time of a failed unit
β : probability that unit is in under maintenance
$U_m$ Maintenance of unit
$U_M$ : Maintenance of unit is continuous
$F_r$: Failed unit under repair
$F_{gc}$: Failed unit under guarantee check
$F_{wgc}$: Failed unit waiting for guarantee check
m(t) Maintenance rate
$F_{rp}$: Failed unit under replacement
$r_p(t)$: replacement time
r(t): Repair time
© Symbol for Laplace convolution
® symbol for Laplace Stieltjes Convolution



Up-state
Down-state
Regenerative Point

The system can be in any of the following states with respect of the above symbols:-

$S_0=(O,S)$ $S_5=(F_{rp}, O)$
$S_1=(U_i,O)$ $S_6=(F_R, F_{wgc})$
$S_2=(U_m, O)$ $S_7=(F_{RP}, F_{wgc})$
$S_3=(F_{gc},O)$ $S_8=(F_{GC}, F_{wgc})$
$S_4=(F_r,O)$ $S_9=(U_M, F_{wgc})$

## IV. TRANSITION PROBABILITIES

The epoch of entry into states $\{S_0, S_1, S_2, S_3, S_4, S_5\}$ are regenerative states. The transition probabilities from the states $S_i$ to $S_j$ are given by $Q_{ij}$ and in the states states $p_{ij}$ denotes the transition probability from states $S_i$ to $S_j$ are given under

$p_{01}=1$ $p_{40}=r^*(\lambda)$
$p_{10}=i^*(\beta+\lambda)$ $p_{46}=\{1-r^*(\lambda)\}$
$p_{12}=\beta\{1-i^*(\beta+\lambda)\}/((\beta+\lambda))$ $p_{57}=\{1-r_p^*(\lambda)\}$
$p_{13}=\lambda\{1-i^*(\beta+\lambda)\}/((\beta+\lambda))$ $p_{50}=r_p^*(\lambda)$
$p_{20}=m^*(\lambda)$ $p_{87}=bg^*(\beta)$
$p_{29}=\{1-m^*(\lambda)\}$ $p_{86}=ag^*(\beta)$
$p_{34}=ag^*(\lambda)$ $p_{89}=\{1-g^*(\beta)\}$
$p_{35}=bg^*(\lambda)$ $p_{73}=1$
$p_{38}=\{1-g^*(\lambda)\}$ $p_{93}=1$
$p_{63}=1$ $p_{4}{}^{(6)}{}_{3}=\{1-r^*(\lambda)\}$
$p_{5}{}^{(7)}{}_{3}=\{1-r_p^*(\lambda)\}$ $p_{2}{}^{(9)}{}_{3}=\{1-m^*(\lambda)\}$

It can be easily verified that

$p_{01}=1$ $p_{63}=1$
$p_{10}+p_{12}+p_{13}=1$ $p_{73}=1$
$p_{20}+p_{29}=1$ $p_{93}=1$
$p_{34}+p_{35}+p_{38}=1$ $p_{50}+p_{57}=1$
$p_{40}+p_{46}=1$ $p_{86}+p_{87}+p_{89}=1$
$p_{4}{}^{(6)}{}_{3}=p_4$ $p_{5}{}^{(7)}{}_{3}=p_{57}$
$p_{2}{}^{(9)}{}_{3}=p_{29}$ $p_{34}+p_{35}+p_{3}{}^{(8,6)}{}_{3}+p_{3}{}^{(8,7)}{}_{3}+p_{3}{}^{(8,9)}{}_{3}=1$
$p_{3}{}^{(8,6)}{}_{3}+p_{3}{}^{(8,7)}{}_{3}+p_{3}{}^{(8,9)}{}_{3}=p_{38}$

## V. MEAN SOJOURN TIMES

Mean Sojourn Times may be defined by

$$\mu_i=\lim_{x\to\infty}\int_0^t P[t:0<t>T]dt$$

So that in steady state we have following relations

$\mu_0=1/\alpha$ $\mu_1=\{1-i^*(\beta+\lambda)\}/(\beta+\lambda)$
$\mu_2=[1-m^*(\lambda)]/\lambda$ $\mu_3=[1-g^*(\lambda)]/\lambda$
$\mu_4=[1-r^*(\lambda)]/\lambda$ $\mu_5=[1-r_p^*(\lambda)]/\lambda$

The unconditional mean time taken by the system to transit from any states $S_i$ to $S_j$ is mathematically given by

$$m_{ij}=\int_0^\infty t\,dQ_{ij}(t)=-q_{ij}^{*'}(s)/_{at\ s=0}$$

So that

$m_{01}=1/\alpha$ $m_{20}=-m^{*'}(\lambda)$
$m_{10}=-g^{*'}(\beta+\lambda)$ $m_{29}=[\{1-m^*(\lambda)\}/\lambda]+m^{*'}(\lambda)$
$m_{12}=[\beta\{1-i^*(\beta+\lambda)\}/(\beta+\lambda)^2]+\beta i^{*'}(\beta+\lambda)/(\beta+\lambda)$
$m_{34}=-ag^{*'}(\lambda)$ $m_{35}=-bg^{*'}(\lambda)$
$m_{13}=[\lambda\{1-i^*(\beta+\lambda)\}/(\beta+\lambda)^2]+\lambda i^{*'}(\beta+\lambda)/(\beta+\lambda)$
$m_{40}=-r^{*'}(\lambda)$ $m_{38}=[\{1-g^*(\lambda)\}/\lambda]+g^{*'}(\lambda)$
$m_{46}=[\{1-r^*(\lambda)\}/\lambda]+r^{*'}(\lambda)$ $m_{50}=-r_p^{*'}(\lambda)$
$m_{87}=-bg^{*'}(\beta)$ $m_{57}=[\{1-r_p^*(\lambda)\}/\lambda]+r_p^{*'}(\lambda)$
$m_{86}=-ag^{*'}(\beta)$ $m_{89}=[\{1-g^*(\beta)\}/\lambda]+g^{*'}(\beta)$

It can be easily verified that

$m_{01}=\mu_0$ $m_{10}+m_{12}+m_{13}=\mu_1$
$m_{20}+m_{29}=\mu_2$ $m_{34}+m_{35}+m_{38}=\mu_3$
$m_{40}+m_{46}=\mu_4$ $m_{50}+m_{57}=\mu_5$
$m_{86}+m_{87}+m_{89}=\mu_8$

## VI. MEAN TIME TO SYSTEM FAILURE

The mean time to system failure is given by the equations

$\Omega_0(t)=Q_{01}(t)$ ® $\Omega_1(t)$
$\Omega_1(t)=Q_{10}(t)$ ® $\Omega_0(t)+Q_{12}(t)$®$\Omega_2(t)+Q_{13}(t)$ ® $\Omega_3(t)$
$\Omega_2(t)=Q_{20}(t)$ ® $\Omega_0(t)+Q_{29}(t)$
$\Omega_3(t)=Q_{34}(t)$ ® $\Omega_4(t)+Q_{35}(t)$ ® $\Omega_5(t)+Q_{38}(t)$
$\Omega_4(t)=Q_{40}(t)$ ® $\Omega_0(t)+Q_{46}(t)$
$\Omega_5(t)=Q_{50}(t)$ ® $\Omega_0(t)+Q_{57}(t)$

Solving above equation by taking Laplace Stieltjes transformations and solving for $\Omega_0^{**}(s)$,we get

$$\Omega_0^{**}(s)=\frac{N(s)}{D(s)}$$

Where

$N(s)=q_{01}q_{12}q_{29}+q_{01}q_{13}q_{38}+q_{01}q_{13}q_{34}q_{46}+q_{01}q_{13}q_{35}q_{57}$
$D(s)=1-q_{01}q_{50}\,q_{13}q_{35}-q_{01}q_{40}\,q_{13}q_{34}-q_{01}q_{10}-q_{01}q_{12}q_{20}$

$$MTSF=\Omega_0=\lim_{s\to\infty}[\{1-\Omega_0^{**}(s)\}/s]=\{D'(0)N'(0)\}/D(0)$$

Where

$D'(0)-N'(0)=[\mu_0+\mu_1+\mu_2p_{12}+\mu_3p_{13}+\mu_4p_{13}p_{34}+\mu_5p_{13}p_{35}]$
$D(0)=1-p_{10}-p_{12}p_{20}-p_{13}p_{34}p_{40}-p_{13}p_{35}p_{50}$

## VII. AVAILABILITY OF THE SYSTEM

The point wise availability $\text{₳}_i(t)$ of the system is given by

$\text{₳}_0(t) = Q_{01}(t)$ © $\text{₳}_1(t) + \mathbf{M}_0(t)$

$\text{₳}_1(t) = Q_{10}(t)$ © $\text{₳}_0(t) + Q_{12}(t)$ © $\text{₳}_2(t) + Q_{13}(t)$ © $\text{₳}_3(t) + \mathbf{M}_1(t)$

$\text{₳}_2(t) = Q_{20}(t)$ © $\text{₳}_0(t) + Q_2^{(9)}{}_3$ © $\text{₳}_3(t) + \mathbf{M}_2(t)$

$\text{₳}_3(t) = Q_{34}(t)$ © $\text{₳}_4(t) + Q_{35}(t)$ © $\text{₳}_5(t) + (Q_3^{(8,6)}{}_3 + Q_3^{(8,7)}{}_3 + Q_3^{(8,9)}{}_3)$ © $\text{₳}_3(t) + \mathbf{M}_3(t)$

$\text{₳}_4(t) = Q_{40}(t)$ © $\text{₳}_0(t) + Q_4^{(6)}{}_3(t)$ © $\text{₳}_3(t) + \mathbf{M}_4(t)$

$\text{₳}_5(t) = Q_{50}(t)$ © $\text{₳}_0(t) + Q_5^{(7)}{}_3(t)$ © $\text{₳}_3(t) + \mathbf{M}_5(t)$

Now taking Laplace transform of these equations and solving them for $\text{₳}_0^*(s)$, we get

$$\text{₳}_0^*(t) = \frac{N_{1(s)}}{D_{1(s)}}$$

The steady states availability is given by

$$\text{₳}_0^{**} = \lim_{s\to\infty} (s\text{₳}_0^*(s)) = \frac{N_{1^{(0)}}}{D'_{1^{(0)}}}$$

Where

$N_1(0) = [(\mu_0+\mu_1+p_{12}\mu_2)(1- p_{38} - p_{34}p_{46} - p_{35}p_{57}) + (\mu_3+p_{34}\mu_4+p_{35}\mu_5)(p_{12}p_{29}+p_{13})]$

and

$D_1(0)=0$ $\mathbf{M}_0(t)=\mu_0(t)$

$\mathbf{M}_1(t)=\mu_1(t)$ $\mathbf{M}_2(t)=\mu_2(t)$

$\mathbf{M}_3(t)=\mu_3(t)$ $\mathbf{M}_5(t)=\mu_5(t)$

$\mathbf{M}_6(t)=\mu_6(t)$

$D_1'(0) = (p_{34}p_{40}+p_{35}p_{50})(\mu_0+\mu_1+p_{12}\mu_2) + (1-p_{10} - p_{12}p_{20})(1- m_{34}p_{40} - m_{35}p_{50}+p_{34}p_{46}+p_{35}p_{57})$

## VIII. MAINTENANCE TIME

Let $K_i$ is the Maintenance time starting from a regenerative states $S_i$ at t=0 is given by

$K_0(t) = Q_{01}(t)$ © $K_1(t)$

$K_1(t) = Q_{10}(t)$ © $K_0(t)+Q_{12}(t)$ © $K_2(t) + Q_{13}(t)$ © $K_3(t)$

$K_2(t) = Q_{20}(t)$ © $K_0(t) + Q_2^{(9)}{}_3$ © $K_3(t) + \text{Ꝁ}$

$K_3(t) = Q_{34}(t)$ © $K_4(t) + Q_{35}(t)$ © $K_5(t) + (Q_3^{(8,6)}{}_3 + Q_3^{(8,7)}{}_3 + Q_3^{(8,9)}{}_3)$ © $\text{₳}_3(t)$

$K_4(t) = Q_{40}(t)$ © $K_0(t) + Q_4^{(6)}{}_3(t)$ © $K_3(t)$

$K_5(t) = Q_{50}(t)$ © $K_0(t) + Q_5^{(7)}{}_3(t)$ © $K_3(t)$

The Maintenance time is given by

$$K_0^*(t) = \frac{N_{2(s)}}{D_{1(s)}}$$

$$K_0^{**} = \lim_{s\to 0} (sK_0^*(s)) = \frac{N_{2(0)}}{D'_{1(0)}}$$ Where

$N_4(0) = p_{01}p_{13}(W_3+W_5p_{35})(p_{26}p_{68}+p_2^{(7)}{}_8)$

$D_1'(0)$ is already defined

## IX. ROUTINE INSPECTION TIME

Let $I_i$ is the Maintenance time starting from a regenerative states $S_i$ at t=0 is given by

$I_0(t) = Q_{01}(t)$ © $I_1(t)$

$I_1(t) = Q_{10}(t)$©$I_0(t)+Q_{12}(t)$©$I_2(t) + Q_{13}(t)$ © $I_3(t) + \acute{\mathbf{W}}_1$

$I_2(t) = Q_{20}(t)$ © $I_0(t) + Q_2^{(9)}{}_3$ © $I_3(t)$

$I_3(t) = Q_{34}(t)$ © $I_4(t) + Q_{35}(t)$ © $I_5(t) + (Q_3^{(8,6)}{}_3 + Q_3^{(8,7)}{}_3 + Q_3^{(8,9)}{}_3)$ © $I_3(t)$

$I_4(t) = Q_{40}(t)$ © $I_0(t) + Q_4^{(6)}{}_3(t)$ © $I_3(t)$

$I_5(t) = Q_{50}(t)$ © $I_0(t) + Q_5^{(7)}{}_3(t)$ © $I_3(t)$

The Maintenance time is given by

$$I_0^*(t) = \frac{N_{3(s)}}{D_{1(s)}}$$

$$I_0^{**} = \lim_{s\to 0} (sI_0^*(s)) = \frac{N_{3(0)}}{D'_{1(0)}}$$

Where

$N_3(0) = \acute{\mathbf{W}}\, [p_{34} + p_{35} - p_{34}\, p_4^{(6)}{}_3 - p_{35}\, p_5^{(7)}{}_3)]$

Where

$D_1'(0)$ is already defined

## X. 10. Repair Time

Let $R_i$ is the Repair time starting from a regenerative states $S_i$ at t=0 is given by

$R_0(t) = Q_{01}(t)$ © $R_1(t)$

$R_1(t) = Q_{10}(t)$ ©$R_0(t) + Q_{12}(t)$©$R_2(t) + Q_{13}(t)$ © $R_3(t)$

$R_2(t) = Q_{20}(t)$ © $R_0(t) + Q_2^{(9)}{}_3$ © $R_3(t)$

$R_3(t) = Q_{34}(t)$ © $R_4(t) + Q_{35}(t)$ © $R_5(t) + (Q_3^{(8,6)}{}_3 + Q_3^{(8,7)}{}_3 + Q_3^{(8,9)}{}_3)$ © $R_3(t)$

$R_4(t) = Q_{40}(t)$ © $R_0(t) + Q_4^{(6)}{}_3(t)$ © $R_3(t) + \text{Ŧ}$

$R_5(t) = Q_{50}(t)$ © $R_0(t) + Q_5^{(7)}{}_3(t)$ © $R_3(t)$

The Repair time is given by

$$R_0^*(t) = \frac{N_{4(s)}}{D_{1(s)}}$$

$$R_0^{**} = \lim_{s\to 0} (sR_0^*(s)) = \frac{N_{4(0)}}{D_1'(0)}$$

Where

$N_4(0) = \text{Ŧ}\; p_{34}\,[p_{12}p_2^{(9)}{}_3 + p_{13}]$

## XI. REPLACEMENT TIME

Let $X_i$ is the Repair time starting from a regenerative states $S_i$ at t=0 is given by

$X_0(t) = Q_{01}(t)$ © $X_1(t)$

$X_1(t) = Q_{10}(t)$ ©$X_0(t) + Q_{12}(t)$ ©$X_2(t)+Q_{13}(t)$© $X_3(t)$

$X_2(t) = Q_{20}(t)$ © $X_0(t) + Q_2^{(9)}{}_3$ © $X_3(t)$

$X_3(t) = Q_{34}(t)$ © $X_4(t) + Q_{35}(t)$ © $X_5(t) + (Q_3^{(8,6)}{}_3 + Q_3^{(8,7)}{}_3 + Q_3^{(8,9)}{}_3)$ © $X_3(t)$

$X_4(t) = Q_{40}(t)$ © $X_0(t) + Q_4^{(6)}{}_3(t)$ © $X_3(t)$

$X_5(t) = Q_{50}(t)$ © $X_0(t) + Q_5^{(7)}{}_3(t)$ © $X_3(t) + \text{Ṝ}$

The Repair time is given by

$X_0^*(t) = [N_5(s) / D_1(s)]$

$X_0^{**} = \lim_{s\to 0} (sX_0^*(s)) = [N_5(s) / D'_1(s)]$

### Busy Period Analysis

Inspection Time + Maintenance Time + Repair Time + Replacement Time

## XII. CONCLUSION:

On the basis of above we can see the expected results through the graphs as following. If we take repair rate and inspection time as negative binomial distributions as $r(t)=\theta e^{-\theta t}$ $g(t)=\pi e^{-\pi t}$ $m(t)=\delta e^{-\delta t}$ $i(t)=\gamma e^{-\gamma t}$ $r(t)=\mu e^{-\mu t}$

Then we get,

| | |
|---|---|
| $p_{01}=1$ | $\mu_0=1/\alpha$ |
| $p_{73}=1$ | $\mu_1=1/\beta+\lambda+\gamma$ |
| $p_{10}=\gamma/\beta+\lambda+\gamma$ | $\mu_2=1/\lambda+\delta$ |
| $p_{12}=\beta/\beta+\lambda+\gamma$ | $\mu_3=1/\lambda+\pi$ |
| $p_{13}=\lambda/\beta+\lambda+\gamma$ | $\mu_4=1/\lambda+\theta$ |
| $p_{63}=1$ | $\mu_5=1/\lambda+\mu$ |
| $p_{20}=\delta/\lambda+\delta$ | $m_{57}=\lambda/(\lambda+\mu)^2$ |
| $p_{29}=\lambda/\lambda+\delta$ | $m_{34}=a\pi/(\lambda+\pi)^2$ |
| $p_{34}=a\pi/\lambda+\pi$ | $m_{35}=b\pi/(\lambda+\pi)^2$ |
| $p_{35}=b\pi/\lambda+\pi$ | $p_{38}=\lambda/\lambda+\pi$ |
| $p_{40}=\theta/\lambda+\theta$ | $p_{46}=\lambda/\lambda+\theta$ |
| $p_{89}=\beta/\beta+\pi$ | $p_{93}=1$ |
| $p_{87}=b\pi/\beta+\pi$ | $p_{86}=a\pi/\beta+\pi$ |
| $p_{50}=\mu/\lambda+\mu$ | $p_{57}=\lambda/\lambda+\mu$ |

## XIII. REFERENCES:-


[1]. Nakagawa, T. and Osaki, S. (1975). Stochastic behaviour of two unit parallel redundant system with preventive maintenance, Microlectron Reliab. 14, p. 457-461.

[2]. Jui-Hsiang Chian and John Yuan, (2001) , optimal maintenance policy for a deteriorating production system under inspection,,

[3]. Tuteja, R.K., and Gulshan, T. "cost – benefit analysis of a two- server-two unit warm standby system with different type of failure", Microelectron. Reliab., 1992, VOL.32,p1353-1359.

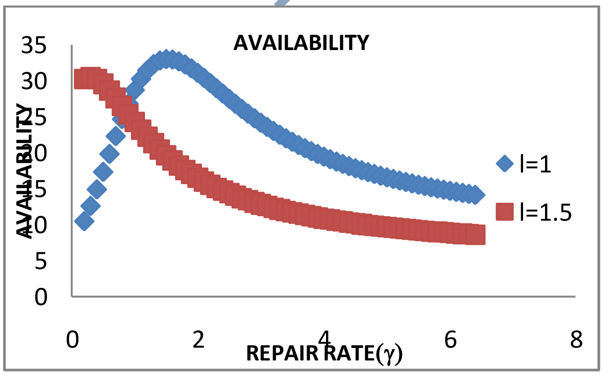
[4]. Brown, M., Proschan, F. (1983) "Imperfect repair", Journal of Applied Probability,20: 851–859.

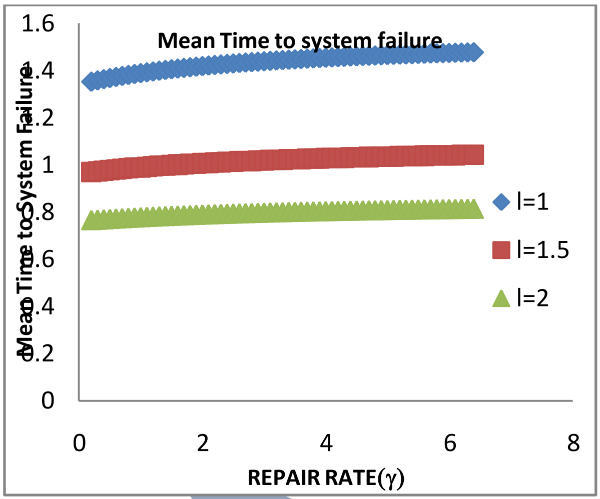
[5]. Jiang, R. and Jardine, A.K.S. (2005) "Two Optimization Models of theOptimum Inspection Problem",Journal of the Operational ResearchSociety,56: 1176–1183.

[6]. LEWIS E. E. Introduction to Reliability Engineering. John Wiley and Sons. Co. 1987.

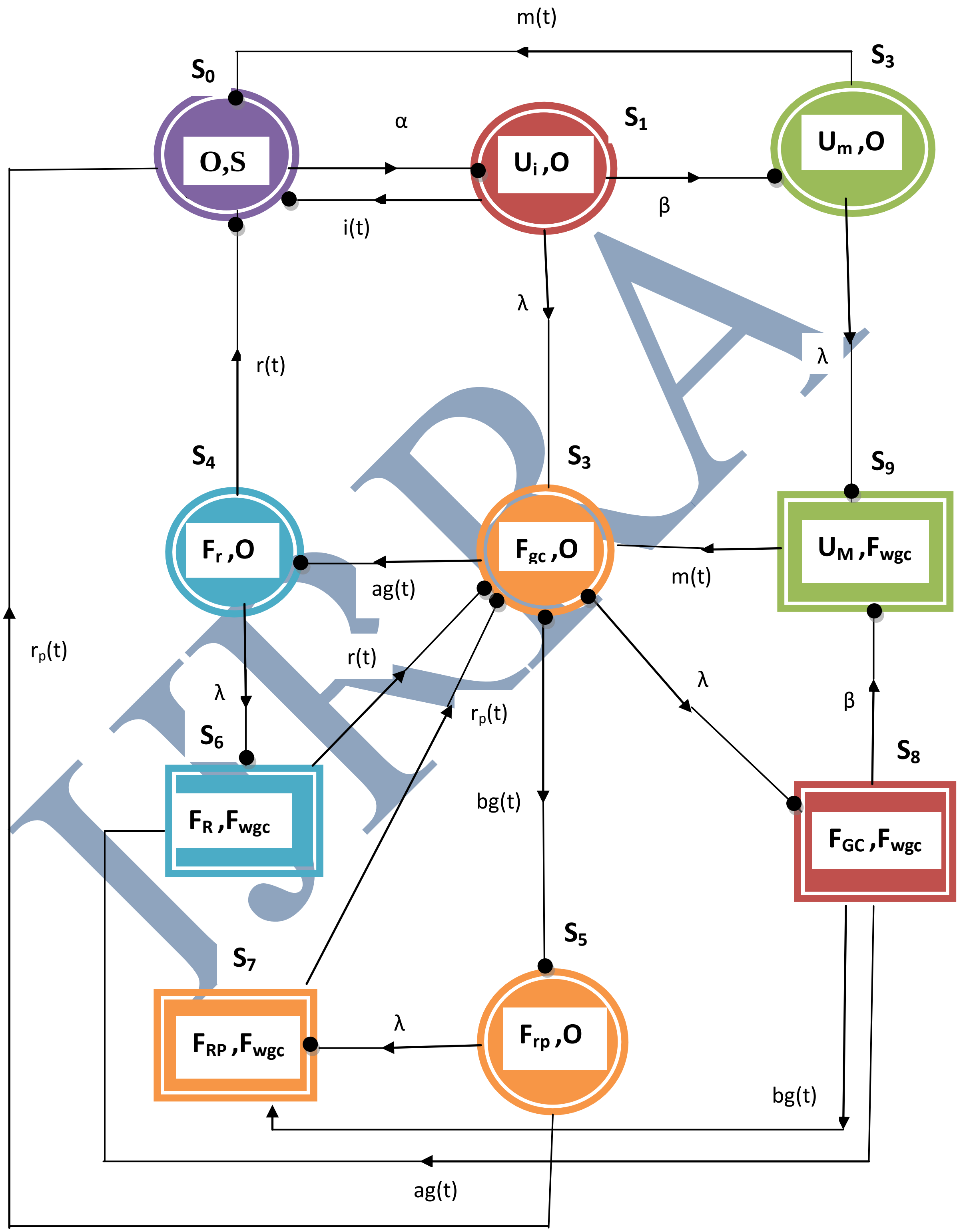
[7]. H. Wang. A survey of maintenance policies of deteriorating systems. European Journal of Operational Research, 139(3):469–489, 2002

[8]. Barlow R.E., Proschan F. Mathematical Theory of Reliability. SIAM, 1996.

## State Transition Diagrams




Regenerative State:-$S_0$,$S_1$,$S_2$ ,$S_3$,$S_5$,$S_6$,$S_8$ Non-regenerative State:-$S_4$ ,$S_7$