

Estimation Techniques for CFO

Anju¹, Amit Ahlawat²

¹Hindu College of Engineering, Sonapat

²Shri Baba Mastnath Engineering College Rohtak

Abstract: Orthogonal frequency division multiplexing (OFDM) is a special case of multicarrier transmission where a single DataStream is transmitted over a number of lower rate subcarriers. In July 1998, the IEEE standardization group decided to select OFDM as the basis for their new 5-GHz standard aiming a range of data stream from 6 up to 54 Mbps. This new standard is the first one to use OFDM in packet-based communications. In wireless communication, concept of parallel transmission of symbols is used to achieve high throughput and better transmission quality. Orthogonal Frequency Division Multiplexing (OFDM) is one of the techniques for parallel transmission. The idea of OFDM is to split the total transmission bandwidth into a number of orthogonal subcarriers in order to transmit the symbols using these subcarriers in parallel. In this paper we will discuss the basics of OFDM technique, role of OFDM in this era, its benefits and losses and also some of its application.

Keywords: Orthogonal Frequency Division Multiplexing (OFDM), BER, ISI, PAPR, DVB, DAB

I. INTRODUCTION

CFO estimation can be performed either in the time or the frequency domain as discussed ahead.

1. TIMES-DOMAIN ESTIMATION TECHNIQUES FOR CFO

For CFO estimation in the time domain, cyclic prefix (CP) or training symbol is used. Each of these techniques is described as below.

1.1 CFO Estimation Techniques Using Cyclic Prefix (CP)

With perfect symbol synchronization, a CFO of ϵ results in a phase rotation of $2\pi n\epsilon/N$ in the received signal. Under the assumption of negligible channel effect, the phase difference between CP and the corresponding rear part of an OFDM symbol (spaced N samples apart) caused by CFO ϵ is $2\pi n\epsilon/N = 2\pi$. Then, the CFO can be found from the phase angle of the product of CP and the corresponding rear part of an OFDM symbol, for example

$$\hat{\epsilon} = (1/2\pi) \arg \{ y_j^* [n] y_j [n + N] \}, n =$$

In order to reduce the noise effect, its average can be taken over the samples in a CP interval as

$$\hat{\epsilon} = \frac{1}{2\pi} \arg \left\{ \sum_{n=-N_G}^{-1} y_j^* [n] y_j [n + N] \right\}$$

(1.1)

Since the argument operation $\arg(\cdot)$ is performed by using $\tan^{-1}(\cdot)$, the range of CFO estimation in Equation (5.1) is $[-\pi, +\pi)/2\pi[-0.5, +0.5)$ so that $|\hat{\epsilon}| < 0.5$ and consequently, integral CFO cannot be estimated by this technique. Note that $y^*[n]y[n+N]$ becomes real only when there is no frequency offset. This implies that it becomes imaginary as long as the

CFO exists. In fact, the imaginary part of $y_j^* [n] y_j [n + N]$ can be used for CFO estimation. In this case, the estimation error is defined as

$$e_\epsilon = \frac{1}{L} \sum_{n=1}^L \text{Im} \{ y_j^* [n] y_j [n + N] \}$$

(1.2)

where L denotes the number of samples used for averaging. Note that the expectation of the error function in Equation (1.2) can be approximated as

$$E\{e_\epsilon\} = \frac{\sigma_d^2}{N} \sin\left(\frac{2\pi\epsilon}{N}\right) \sum_{k \text{ corresponding to useful carriers}}^L |H_k|^2 \approx K\epsilon$$

(1.3)

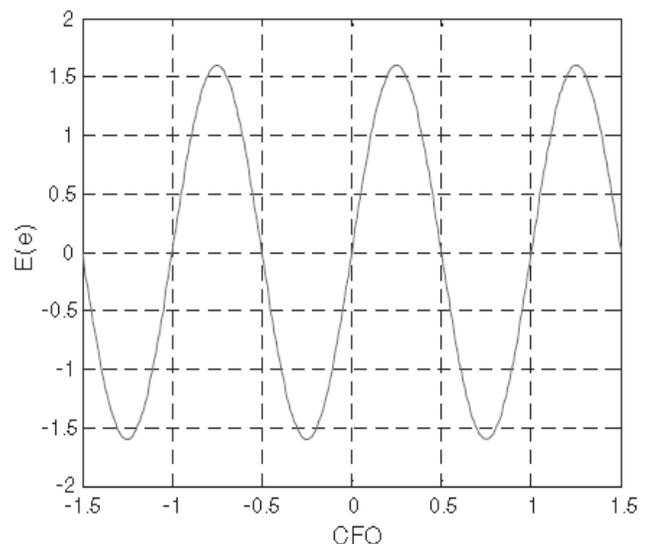


Figure 1.1 Characteristic curve of the error function Equation (1.3).

where σ_d^2 is the transmitted signal power, H_k is the channel frequency response of the k th subcarrier, and K is a term that comprises transmit and channel power. Figure 5.1 show that the error function in Equation 15.3) has an S-curve around the origin, which is required for synchronization. Note that frequency synchronization can be maintained by controlling VCO in accordance with the sine function in Equation (1.3).

This particular approach also provides $|\hat{\epsilon}| < 0.5$ as with Equation (1.1).

1.2 CFO Estimation Techniques Using Training Symbol

We have seen that the CFO estimation technique using CP can estimate the CFO only within the range $\{|\epsilon| \leq 0.5\}$. Since CFO can be large at the initial synchronization stage, we may need estimation techniques that can cover a wider CFO range. The range of CFO estimation can be increased by reducing the distance between two blocks of samples for correlation. This is made possible by using training symbols that are repetitive with some shorter period. Let D be an integer that represents the ratio of the OFDM symbol length to the length of a repetitive pattern. Let a transmitter send the training symbols with D repetitive patterns in the time domain, which can be generated by taking the IFFT of a comb-type signal in the frequency domain given as

$$X_j[k] = \begin{cases} A_m, & \text{if } k = D \cdot i, i = 0, 1, \dots, \{N/D\} \\ 0, & \text{otherwise} \end{cases} \quad (1.4)$$

Where A_m represents an M -ary symbol and N/D is an integer. As $x_1[n]$ and $x_1[n+N/D]$ are identical (i.e., $y_j^*[n]y_j[n+N/D] = |y_j[n]|^2 e^{j2\pi\epsilon n}$), a receiver can make CFO estimation as follows:

$$\hat{\epsilon} = \frac{D}{2\pi} \arg \left\{ \sum_{n=0}^{N/D-1} y_j^*[n]y_j[n+N/D] \right\} \quad (1.5)$$

The CFO estimation range covered by this technique is $\{|\epsilon| \leq D/2\}$, which becomes wider as D increases. Note that the number of samples for the computation of correlation is reduced by $1/D$, which may degrade the MSE performance. In other words, the increase in estimation range is obtained at the sacrifice of MSE (mean square error) performance. As the estimation range of CFO increases, the MSE performance becomes worse. By taking the average of the estimates with the repetitive patterns of the shorter period as

$$\hat{\epsilon} = \frac{D}{2\pi} \arg \left\{ \sum_{m=0}^{D-2} \sum_{n=0}^{N/D-1} y_j^*[n+mN/D]y_j[n+(m+1)N/D] \right\} \quad (1.6)$$

The MSE performance can be improved without reducing the estimation range of CFO.

II. FREQUENCY-DOMAIN ESTIMATION TECHNIQUES FOR CFO

If two identical training symbols are transmitted consecutively, the corresponding signals with CFO of ϵ are related with each other as follows:

$$y_2[n] = y_1[n]e^{j2\pi N\epsilon n/N} \leftrightarrow Y_2[k] = Y_1[k]e^{j2\pi\epsilon k} \quad (1.7)$$

Using the relationship in Equation (1.7), the CFO can be estimated as

$$\hat{\epsilon} = \frac{1}{2\pi} \tan^{-1} \left\{ \frac{\sum_{k=0}^{N-1} \text{Im}[Y_1^*[k]Y_2[k]]}{\sum_{k=0}^{N-1} \text{Re}[Y_1^*[k]Y_2[k]]} \right\} \quad (1.8)$$

This is a well-known approach by Moose. Although the range of CFO estimated by Equation (1.8) is $|\epsilon| \leq \pi/2\pi = 1/2$, it can be increased D times by using a training symbol with D repetitive patterns. The repetitive patterns in the time-domain signal can be generated by Equation (1.4). In this case, Equation (1.8) is applied to the subcarriers with non-zero value and then, averaged over the subcarriers. As discussed in the previous subsection, the MSE performance may deteriorate due to the reduced number of non-zero samples taken for averaging in the frequency domain. Note that this particular CFO estimation technique requires a special period, usually known as a preamble period, in which the consecutive training symbols are provided for facilitating the computation in Equation (1.8). In other words, it is only applicable during the preamble period, for which data symbols cannot be transmitted.

We can think about another technique that allows for transmitting the data symbols while estimating the CFO. As proposed by Classen, pilot tones can be inserted in the frequency domain and transmitted in every OFDM symbol for CFO tracking. Figure 2.2 shows a structure of CFO estimation using pilot tones. First, two OFDM symbols, $y_1[n]$ and $y_1+p[n]$, are saved in the memory after synchronization.

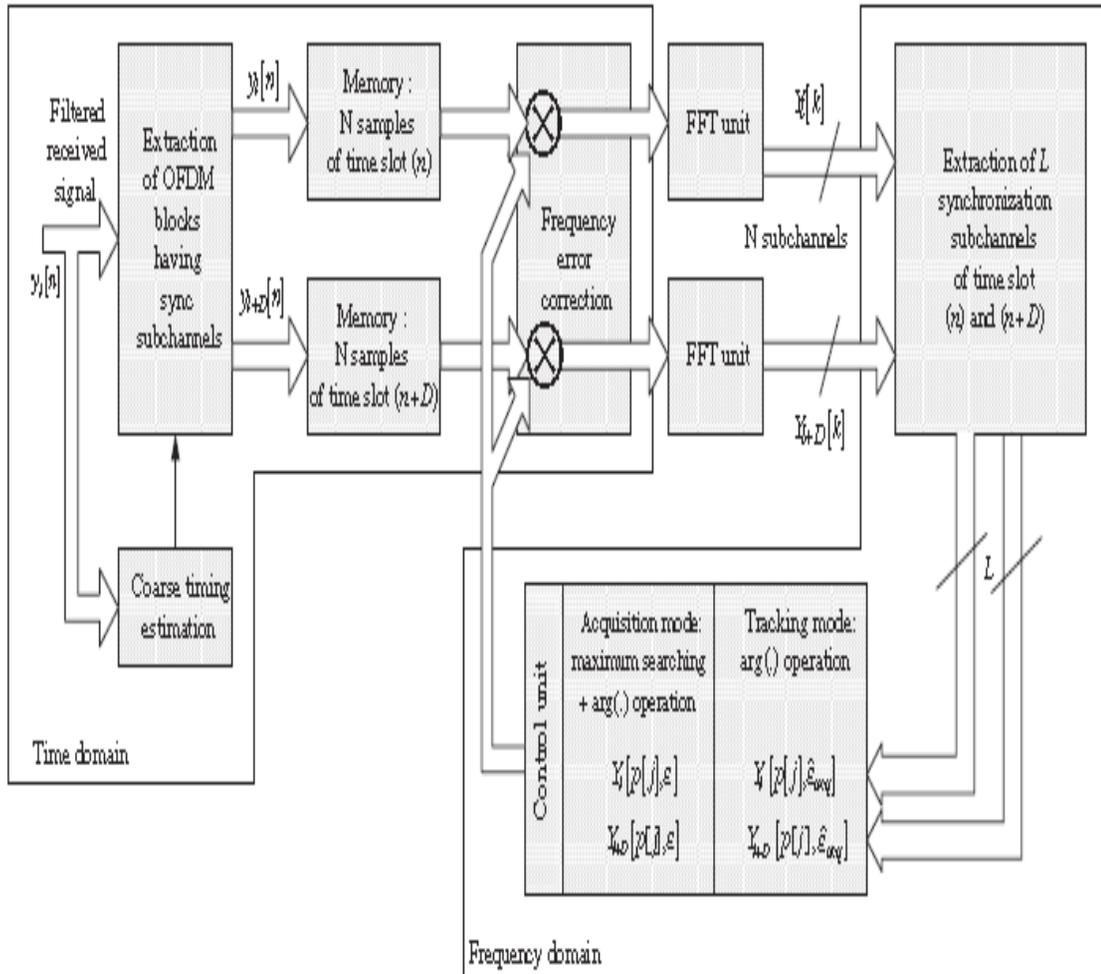


Figure 2.2 CFO synchronization scheme using pilot tones.

Then, the signals are transformed into $\{Y_l[k]\}_{k=0}^{N-1}$ and $\{Y_{l+D}[k]\}_{k=0}^{N-1}$ via FFT, from which pilot tones are extracted. After estimating CFO from pilot tones in the frequency domain, the signal is compensated with the estimated CFO in the time domain. In this process, two different estimation modes for CFO estimation are implemented: acquisition and tracking modes. In the acquisition mode, a large range of CFO including an integer CFO is estimated. In the tracking mode, only fine CFO is estimated. The integer CFO is estimated by

$$\hat{\epsilon}_{acq} = \frac{1}{2\pi \cdot T_{sub}} \max_{\epsilon} \left\{ \left| \sum_{j=0}^{L-1} Y_{l+D}[p[j], \epsilon] Y_l^*[p[j], \epsilon] X_{l+D}^*[p[j]] X_l[p[j]] \right| \right\} \quad (1.9)$$

where L , $p[j]$, and $X_l[p(j)]$ denote the number of pilot tones, the location of the j^{th} pilot tone, and the pilot tone located at $p[j]$ in the frequency domain at the l^{th} symbol period, respectively. Meanwhile, the fine CFO is estimated by

$$\hat{\epsilon}_f = \frac{1}{2\pi \cdot T_{sub} \cdot D} \arg \left\{ \sum_{j=1}^{L-1} Y_{l+D}[p[j], \hat{\epsilon}_{acq}] Y_l^*[p[j], \hat{\epsilon}_{acq}] X_{l+D}^*[p[j]] X_l[p[j]] \right\} \quad (1.10)$$

In the acquisition mode, $\hat{\epsilon}_{acq}$ and $\hat{\epsilon}_f$ are estimated and then, the CFO is compensated by their sum. In the tracking mode, only $\hat{\epsilon}_f$ is estimated and then compensated.

III. SIMULATION RESULTS

Figure 3.1 shows the calculated degradation of the SNR due to the frequency offset.

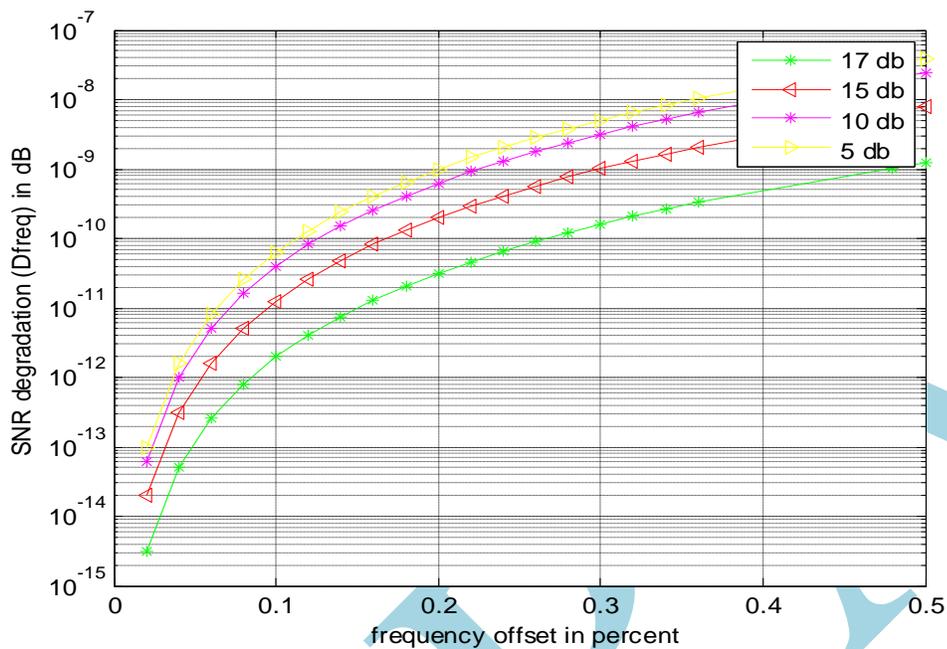


Figure 3.1 SNR degradation caused by frequency offset

For smaller SNR values, the degradation is less than for bigger SNR values as shown in Figure 3.1. In Figure 3.2 CFO estimation using Classen Method is simulated with following parameters

$CFO = [0.15 \ 0.25 \ 0.35 \ 0.45]$;
 $FFT \ size = 128$; %
 $Modulation = 'QAM'$

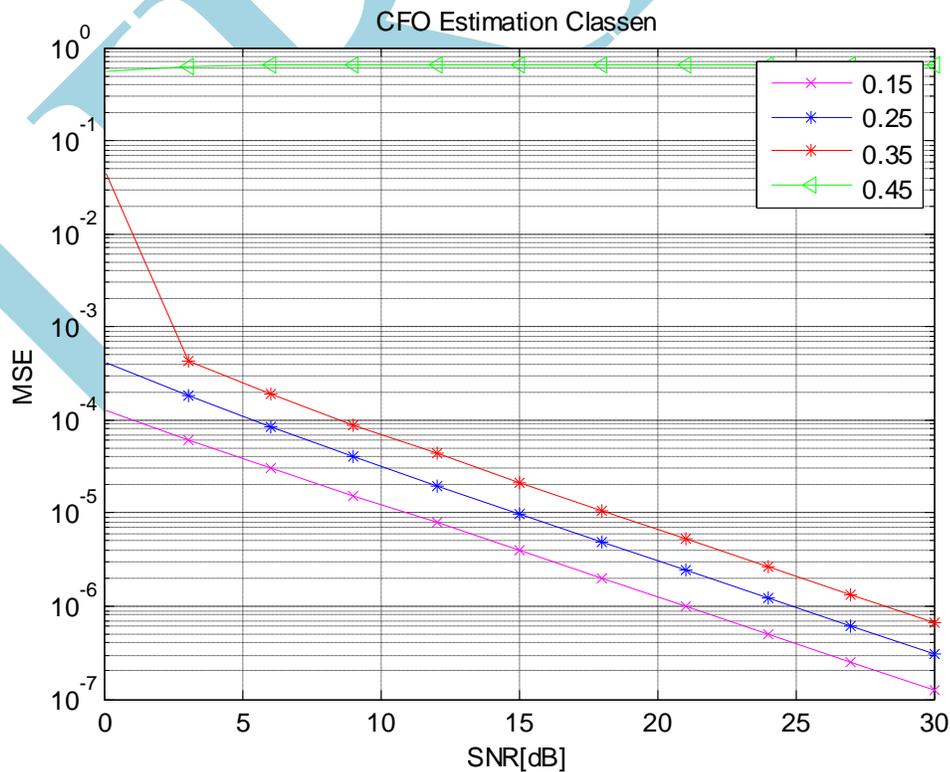


Figure 3.2 CFO Estimation using Classen Method

At low frequency offset error is less and at high frequency offset 0.45 this technique does not perform well and gives more error. In Figure 3.2 CFO estimation using Time-domain CP based method is simulated with following parameters CFO = [0.15 0.25 0.35 0.45];

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