Performance Analysis of Adaptive Channel Equalizer Using Different Algorithms

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Abstract: **This paper analyses the performance of Adaptive Channel Equalizer when designed using different algorithms. Adaptive equalization in digital communication is a process of compensating the disruptive effects caused mainly by intersymbol interference in a band limited channel and plays a vital role for enabling higher data rate in modern digital communication systems.**

Keywords - LMS, RLS, ISI, adaptive, auto-correlation, orthogonality, filter co-efficients, transfer function.

I. INTRODUCTION

An adaptive channel equalizer is a transversal adaptive filter which is used to combat intersymbol interference. Generally the channels used in digital data transmission systems are linear time invariant systems. The most severe source of signal distortion in these channels is intersymbol interference. This occurs when the bandwidth of the transmitted signal is more than the coherent bandwidth of the channel, which makes the original signal dispersed in time due to selective frequency fading. To overcome this problem, a channel equalizer is placed at the receiver end, which would cancel out ISI.

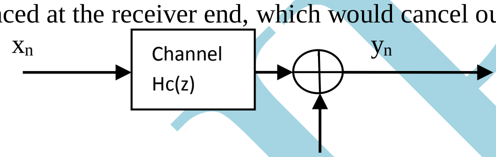

Fig 1: signal passing through a channel

In figure 1, Hc(z) is the transfer function for the channel and v_n is the channel noise assumed to be additive white Gaussian noise. The transfer function Hc(z) incorporates the effects of the modulator and the demodulator. It filters the signal as well as removes the channel distortions.

The purpose of a channel equalizer is to undo the distorting effects of the channel and recover, from the received waveform y_n, the signal x_n that was transmitted. Typically, a channel equalizer will be an FIR filter with enough taps to approximate the inverse transfer function of the channel. A basic equalizer system is shown below:

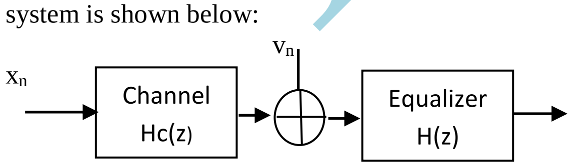

Fig 2: Channel with an equalizer

As channel characteristics vary with time, equalizers are also required to adapt to changing channels. For this, it is designed with an adaptive algorithm.

II. LMS ADAPTIVE ALGORITHM

The LMS (Least Mean Square) algorithms are based on the criterion to minimize the mean square error. That is

$$E[e_n^2]=E[(x_n-\hat{x}_n)^2]=\text{minimum}$$

Where, $\hat{x}_n=h_0y_n+h_1y_{n-1}+h_2y_{n-2}+\dots\dots\dots+h_my_{n-m}$

The weight-adjustment algorithm becomes,

$$h(n+1)=h(n)+2\mu e_n y_n$$

Here μ is the step size of LMS algorithm. μ is selected so that, $0\leq\mu\leq1/R$.

Where, R is autocorrelation of y(n).that is,

$$R=E[y_n y^T_n]$$

The output error e_n is fed back and used to control the adaptation of the filter weights h(n). The filter tries to de-correlate the secondary signal y(n) from the error e_n. If the weight has more or less reached its optimum value, then

$$h(n+1)=h(n)$$

And e_n y(n) becomes zero .

The adaptive implementation of equalizer requires the following sequential steps:

1. At time n, the filter coefficients h(n) is available
2. Compute filter output $\hat{x}_n[k]=h(n)y(n)$
3. Compute estimation error, $e[k]=x(k)-\hat{x}[k]$
4. Compute next filter coefficient, $h[n+1]=h(n)+2\mu y(n)e_n$
5. Get next input sample at time n+1
6. Repeat steps 1 to 5 until error is small enough.
7. The computation of the estimate at next time instant should be made with the new weight, i.e.

$$\hat{x}(n+1)=h(n+1)y(n+1)$$

III. RLS ADAPTIVE ALGORITHM

The recursive adaptive algorithms are based on the exact minimization of the performance index. In LMS algorithm the adaptive weights are not optimal at each time instant while in RLS algorithm, the weights are optimal at each time instant n. These algorithms are based on the exact minimization of least squares criteria. The adaptive RLS algorithms are recursive analogs of the block processing methods of linear prediction .They may be used in place of LMS in any adaptive application. Due to their fast convergence they are used in fast start up channel equalizers. Their main disadvantage is that they require a fair amount of computation, $O(M^2)$ operations per time update. In rapidly varying channels, they may be too costly for implementation. The fast reformulation of RLS algorithms combines the computational efficiency of LMS and the fast convergence of RLS.

Here the estimation criteria, E[e(n)]=min , is replaced with a least squares weighted time average that includes all estimation errors from initial time instant to the current time n, that is e(k), k=0,1,………..n:

$E_n = e^2(0)+e^2(1)+......................+e^2(n)$

Where, $e(k)=x(k)-\hat{x}(k)$

and $\hat{x}(k)$ is the estimate of x(k) produced by adaptive filter.

The estimate of x(k) is computed as:

$\hat{x}(k)= h^T y(k)$

Now, $E_n= e^2(n) +f e^2(n-1)+f^2e^2(n-2)+\ldots\ldots+f^n e^2(0)$
Where the forgetting factor f is positive and less than one. The orthogonality equation is,
dEn /dh=0

The conventional RLS algorithm is:

1. Compute the error signal $e_n = x_n-\hat{x}_n$
2. Adjust the weights using the euation:
   h(n)=h(n-1)+e(n/n-1)k(n)
3. k(n) is calculated as
4. k(n)=u(n)k(n/n-1)
5. u(n)=1/[1+v(n)]
6. v(n)= k(n/n-1)T y(n) ,
7. k(n/n-1)= f -1 P(n-1) y(n)
8. $P(n) = R(n)^{-1}$
9. $R(n)= \sum_{k=0}^{n} f^{(n-k)} y(k)y^T(k)$
10. $h(n) = R(n)^{-1}r(n)$ where,
   $r(n)=\sum_{k=0}^{n} f^{(n-k)} x(k)y(k)$

IV. THE KALMAN ADAPTIVE ALGORITHM

For the asymptotic Kalman filter, the observations are available from the infinite past to the present, namely, $\{y_i , -\infty < i \le n\}$. In a time-recursive form, the time-varying Kalman filter for estimating xn is based on the finite observation subspace Yn.

The basic equation is:

$$x_{n+1}=ax_n+w_n$$

This is made truly recursive by means of recursively computing the required gain Gn from one time instant to the next.

.$\hat{x}_n$ and $\hat{x}_{n/(n-1)}$ are the optimal estimates of x_n based upon the observation subspaces Y_n and Y_{n-1} with the initial condition $\hat{x}_{0/-1}$= 0. Iterating the state and measurement models i.e.

$$x_{n+1}=ax_n+w_n \quad \text{and}$$

$$y_n=cx_n+w_n$$

Starting at n-0, the following two results are obtained

$\hat{x}_{(n+1)/n} = a\hat{x}_{n/n}$ and

$\hat{y}_{n/(n-1)} = c\hat{x}_{n/(n-1)}$

The proof of both is based on the linearity property of estimation of the state equation. For example,

$$\hat{x}_{(n+1)/n}=a\widehat{x_{n}+w_n} = a\hat{x}_{n/n} + \hat{w}_{n/n} = a\hat{x}_{n/(n-1)}$$

where $\hat{w}_{n/n}$ was set to zero because w_n does not depend on any of the observations Y_n. The iteration of state equation leads to the expression

$$x_n=a^n x_0+a^{n-1}w_1+a^{n-2}w_2+\ldots\ldots\ldots+aw_{n-2}+w_{n-1}$$

Thus the observation subspace Y_n will depend only on

$$\{x_0,w_0,w_1,\ldots.w_{n-1},v_0,v_1,v_2,\ldots\ldots\ldots..v_n\}$$

Assuming that x_0 is uncorrelated with w_n it follows that w_n will be uncorrelated with all random variables in the above set and thus with Y_n.

The Kalman adaptive algorithm can be summarized as:

0. Initialize by $\hat{x}_{0/-1} = 0$ and $P_{0/-1}=E[x_0^2]$.

1. At time n,$\hat{x}_{n/(n-1)}$ and $P_{n/(n-1)}$ and the new measurement Y_n are available.

2. Compute $\hat{y}_{n/(n-1)}=c\hat{x}_{n/(n-1)}$ and the gain G_n using relation

V. $G_n=\frac{E[\epsilon_n x_n]}{E[\epsilon_n^2]} = \frac{cP_{n/(n-1)}}{R+c^2P_{n/(n-1)}}$

3. Correct the predicted estimate

$\hat{x}_{n/n}=\hat{x}_{n/(n-1)} + G_n \epsilon_n$

And compute its mean-square prediction error $P_{n/n}$ using relation

$$P_{n/n}=P_{n/(n-1)}-G_n cP_{n/(n-1)} =$$

$$P_{n/(n-1)}-\frac{c^2P_{n/(n-1)}}{R+c^2P_{n/(n-1)}}=\frac{RP_{n/(n-1)}}{R+c^2P_{n/(n-1)}}$$

4. Predict the next estimate $\hat{x}_{(n+1)/n} = a\hat{x}_{n/n}$ and compute the mean-square prediction error $P_{n+1/n}$ using relation $P_{n+1/n} = a^2P_{n/n} + Q$

5. Go to the next time instant, n → n + 1.

The optimal predictor $\hat{x}_{n/(n-1)}$ satisfies the Kalman filtering equation, i.e.

$$\hat{x}_{(n+1)/n}=a\hat{x}_{n/n}=a(\hat{x}_{n/(n-1)}+G_n\epsilon_n)=a\hat{x}_{n/(n-1)}+aG_n(y_n-c\hat{x}_{n/(n-1)}).$$

I. SIMULATION RESULTS

The models for Adaptive Channel Equalizer using different algorithms have been developed using Simulink tool and thereby simulated. The results have been obtained in terms of the noise still left in the output which is measured by the difference between output signal and desired signal. The chosen values of parameters of the three filters are:

LMS Filter :

Filter length=45,step size=0.5,leakage factor=1.

RLS Filter :

Filter length=45,forgetting factor=1.

kalman Filter :

Filter length=5, process noise variance=0.3, initial correlation matrix=0.5.

In an observation for 10 time units , the obtained values are as given below:

LMS Filter :

Noise remains **zero** for most of the time It attains a value of one for one time unit.

RLS Filter :

Noise remains **zero** for most of the time .It attains a value of 0.27 for 1 time unit.

Kalman Filter:

Noise remains **z**ero for most of the time. It attains a value of one for 1 time unit and 0.5 for 1 time unit.

As the noise remains almost **zero** in the output signal, the output signal is approximately same as the desired signal, which is the delayed input signal. Thus, the channel distortions have been removed to a large extent. The output signal is an **equalized** signal. The different filters, used, are acting as equalizers with corresponding algorithm. The noise signals for the three algorithms are as shown below:

LMS Algorithm :

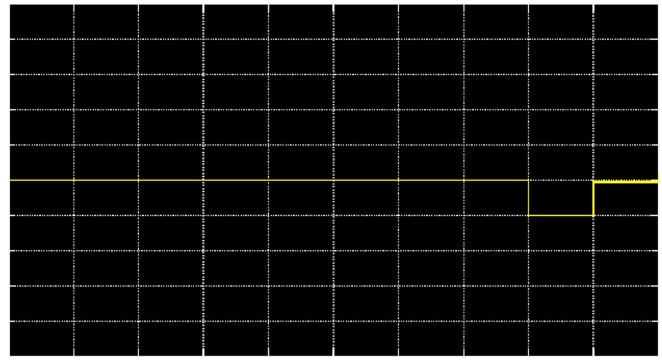

FIG 3: NOISE SIGNAL FOR A LMS BASED CHANNEL EQUALIZER

RLS Algorithm:

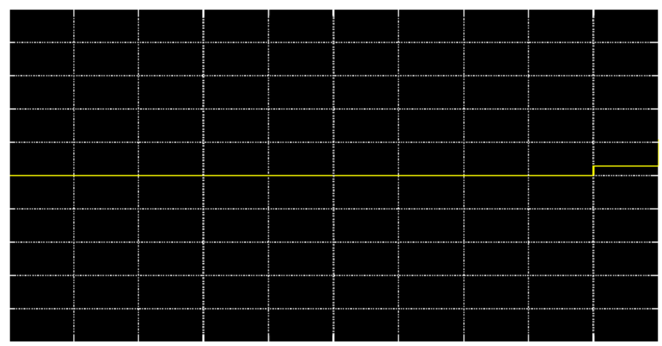

FIG 4: NOISE SIGNAL FOR A RLS BASED CHANNEL EQUALIZER

Kalman Algorithm:

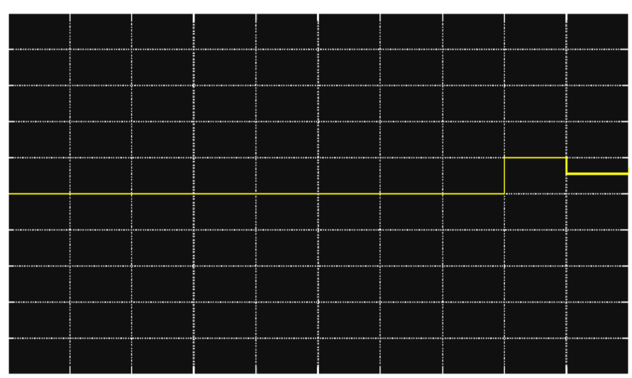

FIG 5: NOISE SIGNAL FOR A KALMAN BASED CHANNEL EQUALIZER

VI. CONCLUSION

The results, so obtained, imply that the RLS algorithm gives the best equalized output among the three algorithms. Kalman algorithm is the worst among them as it gives larger noises and for longer times.

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