

Analysis of Image Compression Using Fractal Wavelet Techniques

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Abstract —In this paper we show the two implementations of fractal (Pure-fractal and Wavelet fractal image compression algorithms) which have been applied on the images in order to investigate the compression ratio and corresponding quality of the images using peak signal to noise ratio (PSNR). And in this paper we also set the threshold value for reducing the redundancy of domain blocks and range blocks, and then to search and match. By this, we can largely reduce the computing time. In this paper we also try to achieve the best threshold value at which we can achieve optimum encoding time.

Keywords: Fractal image coding; Wavelet; Iterated Function System; Wavelet; Mean Square Error; Compression Ratio.

I. INTRODUCTION

In 1988 M. Barnsley and Jacquin introduced the FRACTAL image compression techniques are the product of the study of iterated function systems (IFS). For recent years, the application of fractal image coding has become more and more popular. These techniques involve an approach to compression quite different from standard transform coder-based methods. Transform coders model images in a very simple fashion, namely, as vectors drawn from a wide-sense stationary random process. They store images as quantized transform coefficients. Fractal block coders, as described by Jacquin, assume that “image redundancy can be efficiently exploited through *self-transformability* on a blockwise basis” [1]. They store images as contraction maps of which the images are approximate fixed points. Images are decoded by iterating these maps to their fixed points.

Fractal coding is based on fractal geometry, it has a character of big compression ratio and a fast decoding speed, but it cannot be used for real time processing. It is its blocks searching and matching that makes its long time. As wavelet can get good space frequency multi resolution, the energy mainly concentrated in low frequency sub images, and the images with same directions but different resolutions have self similarity, which is consistent with fractal’s nature properties. The combination of wavelet and fractal is firstly proposed by Pentland and Horowitz. They wanted to find the redundancy of sub images decomposed after wavelet. Later, Rinaldo and Calvagno proposed a new method. First, decompose a image by wavelet, and then code the sub image with minimum resolution, and predict the other sub images.

Finally, we’ll finish the compression. Jin Li introduced a new method. They firstly computed the bytes of fractal predicting, and only predicted when economization. But the methods above are all time consumption, and the reconstructed images are not always good. This paper proposed a new blocks searching method based on fractal. Firstly, we transform the image by wavelet, then divide it into blocks. Before matching, we first reduce the amount of domain blocks and the range blocks to lessen the block pools, then following the contractive mapping transformation.

II. RELATED WORK

Relation between fractal image coding and wavelets is not a new one. The first mention of the connection was by Pentland and Horowitz in [11]. The algorithm described in [11], however, consists of a within sub-band fixed vector quantizer that uses cross-scale conditioning for entropy coding vector indices, and is only loosely related to Jacquin-style schemes we examine here. An important paper linking wavelets and fractal image coding is that of Rinaldo and Calvagno [12]. The coder in [12] uses blocks from low frequency image sub bands as a vector codebook for quantizing blocks in higher frequency sub bands. The main focus of [12] is to develop a new coder rather than to analyze the performance of fractal block coders in general. While the procedure in [12] is inspired by the Jacquin-style coders examined in this paper, it differs in important ways. We discuss these differences in Section V. The link between fractal and wavelet-based coding described in Section III-B below was reported independently and nearly simultaneously by this author [13], by Krupnik, Malah, and Karnin [14], and by van de Walle [2]. This paper contains a substantial extension and generalization of the algorithms, analyses, and ideas presented in the previous three papers.

III. FRACTAL IMAGE CODING

3.1 Collage Theorem

Collage theorem is the technique core of fractal coding. For a certain image X , we can choose a certain number of contractive mapping, such as N , and we can get number N sets by transformed for N times, in which every set is a small image. If the reconstructed image collaged by these N small images is very similar to X , we get the right IFS. Supposed $\{RT: w_i, i = 1, 2, \dots, P\}$ is a contractive transform set, IFS, and R is a real set. To any $V \in RT$, $\epsilon > 0$ if the largest contractive gene $s \in (0, 1)$, and $h(V, W(V)) < \epsilon$ is satisfied, we will get $h(V, A) < \epsilon / (1 - s)$.

A is the attractor of IFS, and $h(A, B)$ is the Hausdorff distance. Collage theorem supplies a up bound Value between V and IFS attractor, which represents the degree of approximation, the up bound value of collage error. Collage theorem provides the theoretical basis for image compression with IFS. A binary image can be considered as a R^2 mentionable tight subset. And a gray image can be considered to be carried out by sampling and quantization from an original gray curve. Even we cannot make the original image be the attractor of a IFS, $\{W(V) : R : w_i, i = 1, 2, \dots, P\}$, we can regard V as a good approach, if $W(V)$ is much close to V , and $W_i (i=1, 2, \dots, P)$ is a contractive mapping.

3.2 Partition

X should be divided into some range blocks (R_i) and some domain blocks (D_i), and a D_i should contain more pixels than a R_i to ensure the mapping, $W_i : D_i \rightarrow R_i$, is contractive. Generally, if a R_i is $b \times b$, a D_i should be $2b \times 2b$.

3.3 Computation of IFS

Three dimensional affine transformations can be expressed as:

$$W_i = \begin{pmatrix} a_i & b_i & 0 \\ c_i & d_i & 0 \\ 0 & 0 & s_i \end{pmatrix}$$

The transformation above is a synthetic of two. It is the matching process of R_i and D_i including geometric transformation and gray transformation.

$W_i \Rightarrow$ is the geometric transformation.

$W_i(z) = S_i(z) + O_i$ is the gray transformation.

IV. WAVELET COMPRESSION

Wavelet Theory deals with both discrete and continuous cases. Continuous wavelet transform (CWT) is used in the analysis of sinusoidal time varying signals [6]. CWT is difficult to implement and the information that has been picked up may overlap and results in redundancy. If the scales and translations are based on the power of two, DWT is used in the analysis. It is more efficient and has the advantage of extracting non overlapping information

about the signal. 2-D transform can be obtained by performing two 1-D transform. Signal is passed through low pass and high pass filters L & H , then decimated by a factor of 2, consisting 1 level transform, thus splitting the image into four sub-bands referred as LL , HL , LH & HH (Approximation, Horizontal Detail, Vertical Detail, and Diagonal Detail respectively). Further decomposition is achieved by acting upon four sub-bands. The inverse transform is obtained by up sampling all the four sub bands by a factor of 2 and then using reconstruction filter. Higher scales correspond to more stretched wavelet. [7, 8].

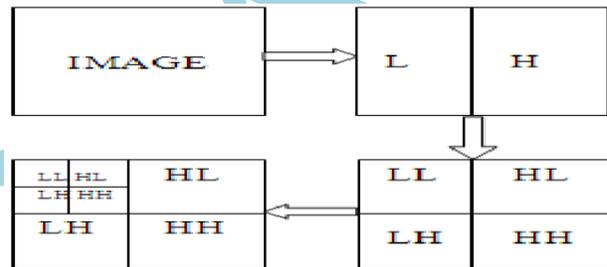


Figure1. Two levels Wavelet Decomposition applied on an image

V. WAVELET-FRACTAL IMAGE COMPRESSION ALGORITHM

The motivation for Wavelet-fractal image compression stems from the existence of self-similarities in the multi-resolution wavelet domain. Fractal image compression in the wavelet domain can be considered as the prediction of a set of wavelet coefficients in the higher frequency subbands from those in the lower frequency subbands. Unlike Pure-fractal estimation, an additive constant is not required in wavelet domain fractal estimation, as the wavelet tree does not have a constant offset. Down sampling of domain tree, matches the size of a domain tree with that of a range tree. The scale factor is then multiplied with each wavelet coefficient of domain tree to reach its correspondence in range tree. The authors of [8] answered the question “why fractal block coders work” comprehensively referring the fundamental limitations of the Pure-fractal compression algorithms [8]. Let D_l denote the domain tree, which has its coarsest coefficients in decomposition level l , and let R_{l-1} denote the range tree, which has its coarsest coefficients in decomposition level $l-1$. The contractive transformation (T) from domain tree D_l to range tree R_{l-1} , is given by $T(D_i) = \alpha \times S.D_i$ where S denotes sub sampling and α is the scaling factor. Let $x = (x_1, x_2, x_3, x_4, \dots, x_n)$ be the ordered set of coefficients of a range tree and $y = (y_1, y_2, y_3, y_4, \dots, y_n)$ the ordered set of coefficients of a down sampled domain tree. Then, the mean squared error is given by Equation (5).

$$MSE = \sum_{i=1}^n (R_i - T(D_i))^2 = \sum_{i=1}^n (x_i - \alpha xy) \dots\dots(2)$$

And the optimal α is obtained by Equation (3).

$$= \frac{(\sum_{i=1}^n x_i * y_i)}{\sum_{i=1}^n 1}$$

We should search in the domain tree to find the best matching domain block tree for a given range block tree. The encoded parameters are the position of the domain tree and the scaling factor. It should not be left unmentioned that in this algorithm; the rotation and flipping have not been implemented. To increase the accuracy of scale factors, new scheme of Wavelet fractal compression is introduced [9]. In this approach, α in contrast to the previous method which had to be calculated for each block tree individually, is computed for each level separately, hence the more α s and the better quality achieved.

VI. EXPERIMENTAL RESULTS

The table I has shown below which gives the results of previous scheme and proposed scheme of Fractal Wavelet Compression technique. Both of this schemes of fractal wavelet Compression Technique is tested for 512 x 512 original image of lenna. The Results of performance is shown in table I given below. In this paper we shown that by choosing the threshold value we can reduce the redundancy among domain and range blocks before matching, because there are lots of similar blocks in the block pools. By setting the threshold value maximum number of domain blocks will be eliminated, and few domain blocks will be left. Due to this very small time will be consumed. And finally we obtain the result with very less encoding and decoding time in few seconds and with high compression ratios moreover with good quality of image.



Figure2. Original And Reconstructed image without threshold value

Table I shows that there is very large difference in encoding time in fractal wavelet compression technique. When we implement the coding of fractal wavelet

compression technique without any threshold value in MATLAB then we got the Peak signal to noise ratio is 36.7167 which is good but we got the encoding time very high that is 118.2810 (approx. 2 min) and the decoding time is 14.9690sec which is quite low then encoding time. If we set the particular threshold value which is very low i.e. -3.5527e-15 then we got the peak signal to noise ratio 28.6468 and encoding time is 24.4370 sec which is very low as compare to previous scheme and the decoding time is 14.2810sec which is little small then the previous scheme. Now further if we change the threshold value from negative to positive i.e. +3.5527e-15 then we analyze the major change again in encoding time. At this threshold value the encoding time obtained is very low i.e. 5.1880sec which is quite low. But there is no change in the PSNR value and decoding time as the negative value of threshold. So we can say that on the second value of threshold we obtain the best result if we concern only with encoding and decoding time. Original image and reconstructed image shown in figure 2, which is obtained when we implement the coding without threshold value. In which it is shown that the reconstructed image is of very good quality.

7.1 For threshold value= -3.5527e-15



Figure3. Original And Reconstructed image

7.2 For threshold value= +3.5527e-15



Figure4. Original And Reconstructed image

VII. CONCLUSION

In this paper we show that for the various values of Threshold in the fractal Wavelet image compression the Encoding Time reduces a lot. Several fractal image compression algorithms in spatial and wavelet domains were implemented. In the previous work fractal-wavelet compression [3] directly divided the original image into range blocks and domain blocks, then affine transform the range blocks and match with domain blocks. Finally compressing and coding. This work already reduced the encoding time in large amount from hours to few minutes. By which we can reduce the redundancy of domain blocks and range blocks, the reconstructed image is not as good as the original, but the computing time is largely reduced i.e. from few minutes to few seconds. In this paper, we choose MSE to judge the similarity of all the blocks. As the distribution of gray is different from image blocks, there may be some residual by using MSE. In addition, we choose PSNR to judge the quality of reconstructed image. PSNR is the most common and widely used measuring method. Recent researches show that the PSNR does not always has the same visual quality as what human see.

VIII. REFERENCES

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