

Analysis and Prediction of Nonstationary Financial Time Series using OSELM

Archana Thakran¹, Ram Pal Singh², and Akshay Bhasin³

¹Department of Computer Science, ARSD, University of Delhi, Delhi, INDIA.

²Department of Computer Science, DDUC, University of Delhi, Delhi, INDIA.
rprana@gmail.com

³Department of Computer Science and Engg,BIT, Mesra, INDIA.
akshay8595@yahoo.com

Abstract. In this paper, we study the application of Online Sequential Extreme Learning Machine(OSELM) for forecasting time series which are chaotic, nonlinear complex and nonstationary inherently. OSELM is a learning algorithm for single layer feed forward neural networks(SLFNs) which can learn the data one-by-one or chunk-by-chunk with fixed or varying chunk size with much faster rate than the conventional learning algorithms. It has been shown in literature that OSELM computation-ally faster with better generalization ability than other existing popular machine learning methods. The experiments are performed on different time series like IBM and SantaFe-A with different activation functions namely sigmoid, sin, hardlim and rbf. It is observed that experimental results are very competitive and close to the actual values of real life nonstationary such as IBM and SantaFe-A. Simulation results demonstrate the effectiveness and efficiency of method in dealing with complex nonlinear time series forecasting.

Keywords: ELM, Financial Time Series,OSELM,Regression

1 Introduction

For practical situation in dynamic complex system modelling, nonlinear time series prediction and signal processing play very important role. Artificial neural networks (ANNs) were widely used for pattern recognition, classification and regression problems. Despite of various advantages of (ANNs) such as better approximation capabilities, simple network structure still suffers from disadvantages like presence of local minima's, imprecise learning rate, selection of hidden neurons and overfitting etc. Therefore, in order address existing problems of ANNs, OSELM has been implemented and applied to predict nonstationary (where statistical parameters like mean and standard deviation alter over a period of time) time series like financial time series IBM and SantaFe-A (laser generated time series data were recorded from a Far-Infrared-Laser in a chaotic state). In order to handle this type of time series, the optimal number of hidden nodes and activation functions to be determined beforehand by users. Financial time series forecasting is a challenging real time regression problem. For past decade,

gradient descent method [1] has been used in various learning algorithms. However, the gradient descent method suffers from major problems like very slow learning rate, may converge to local minima's, therefore requires large number of iteration steps to obtain better learning performance. Another technique proposed by Vapnik et al. was Support Vector Machine(SVM) [2] which maps data set from input space to a high dimensional feature space through some pre-terminated non linear mapping.

Latest technique Extreme Learning Machine (ELM) suggested by Huang et al. [3] which is a learning algorithm for Single Hidden layer Feed forward networks (SLFNs). In ELM formulation, input weights and hidden layer biases are arbitrarily generated and fixed. Later we solve the system of linear equations in terms of unknown weights connecting hidden layer to output layer. The solution of these equations is obtained using Moore-Penrose generalised Pseudo Inverse [5]. Since output weights are analytically determined using the least squares method thereby allowing a great reduction in training time. Another variants of ELM introduced by Liang et al.[6][7] is Online Sequential Learning Machine (OS-ELM) which is simple and efficient learning algorithm that can handle training data one by one or chunk by chunk of varying length and discard data samples for which training has already been done and it uses recursive least squares solution for algorithm.

In this paper, Section 2 discusses briefly OSELM algorithm and study its applicability for given time series. Section 3 covers the preprocessing of data sets by accumulated generating operation (AGO). It also briefly discusses various activation functions that are used. Section 4 gives experimental results and Section 5 discusses conclusion.

2 Online Sequential ELM (OSELM)

In this research paper, OSELM has been used to analyse the given time series as training process and predict the future values by training OSELM using training samples. Online sequential ELM introduced by Liang et al.[6] was an improvement and it can handle the real time data more efficiently. In certain situations the entire data set may not be available initially but keeps arriving at different intervals of time which OSELM can handle such data effectively. OSELM can learn from the training samples by taking one element at a time or by grouping the samples into varying size chunks[6].

Salient Features of OSELM are:

- { It can learn from the recently arrived training samples instead of entire historic training samples. OSELM discards the samples for which the training is complete.
- { Training samples are clubbed into chunks of varying or fixed size and the training samples are fed sequentially to the learning algorithm.
- { OSELM has no former knowledge about the number of training samples presented to it.

In OSELM the input weights and biases are randomly generated and fixed and based on this the output weight matrix is determined using the recursive least squares algorithm [8].

$$\tilde{y} = Z^T Y \quad (1)$$

where $\text{rank}(Z) = P$, P is the number of hidden neurons. Sequential solution of Eqn.(1) results in implementing OSELM.

Learning Algorithm of OSELM: Given training sample $N = \{(x_i, y_i) | x_i \in \mathbb{R}^M; y_i \in \mathbb{R}^N; i = 1, \dots, N\}$ where N training data samples are available initially [3][4][6].

1. Initialization Phase:

Take a small chunk of training data $N_0 = \{(x_i, y_i) | i = 1, \dots, N_0\}$ from the given training set N where $N_0 \ll P$

- (a) Arbitrarily generate the hidden neuron parameters $(w_i, b_i) | i = 1, \dots, P$.
- (b) Compute initial hidden layer output matrix Z_0

$$Z_0 = \begin{bmatrix} G(w_1, b_1, x_1) & \dots & G(w_P, b_P, x_1) \\ \vdots & \ddots & \vdots \\ G(w_1, b_1, x_{N_0}) & \dots & G(w_P, b_P, x_{N_0}) \end{bmatrix}_{N_0 \times P} \quad (2)$$

- (c) Approximate the initial output weight matrix Y_0 where it is given

$$Y_0 = Z_0^T Y_0 \quad (3)$$

where Z_0 and Y_0 is defined as $Z_0^T = (Z_0^T Z_0)^{-1} Z_0^T$, $G_0 = Z_0^T Z_0$ and $Y_0 = f(y_1, \dots, y_{N_0})^T$

- (d) Set $k = 0$.

2. Sequential Learning Phase:

- (a) Take $(k + 1)^{\text{th}}$ chunk of training data

$$N_{k+1} = \{(x_j, y_j) | j = (N_0 + \dots + N_k) + 1, \dots, (N_0 + \dots + N_k) + N_{k+1}\} \quad (4)$$

- where each chunk is of varying sizes N_0, N_1, \dots, N_{k+1} .
- (b) Set $Y_{(k+1)}$ as given by array

$$Y_{(k+1)} = \begin{bmatrix} y_{(P, N_0+1)}^T & \dots & y_{(P, N_0+N_{k+1})}^T \\ \vdots & \ddots & \vdots \\ y_{(1, N_0+1)}^T & \dots & y_{(1, N_0+N_{k+1})}^T \end{bmatrix}_{(N_0+N_{k+1}) \times N} \quad (5)$$

- (c) Compute $Z^{(k+1)}$ as given at bottom of the page¹
- (d) Compute $(k + 1)^{th}$ output weight matrix $^{(k+1)}$ sequentially using the following equation

$$^{(k+1)} = ^{(k)} + G^{(k+1)}Z^{(k+1)T}(Y^{(k+1)} - Z^{(k+1)} ^{(k)}) \quad (7)$$

where $G_{k+1} = G_k + Z_{k+1}^T Z_{k+1}$

- (e) Set $k=k+1$. Goto step 2(a)

3 Experimental Results

3.1 Data Sets

The data sets available for experiment are inherently noisy, chaotic, nonstation-ary and highly random. We have used IBM as nancial time series² and laser generated time series data were recorded from a Far-Infrared-Laser in a chaotic state SantaFe-A³ in the experiments. A data plot was made of a SantaFe-A se-ries and shown in Fig.1. The data size and their ranges have been described in the Table 1.

Data Sets	Time Period	Training Samples	Testing Samples
IBM	1978-2014	9871	3290
SantaFe-A	-	750	250

Table 1: Details of the Data Sets used in Prediction Analysis.

3.2 Preprocessing of Data Sets with AGO

The rst order accumulated generating operation(AGO) on original time se-ries for reducing stochastic volatility and provides smoothness[9]. The 1st order (AGO) may be summarised as follows. Let the original data be $X^{(0)}$ having N samples are given by

$$x^{(0)} = \{x_1^{(0)}, x_2^{(0)}, \dots, x_i^{(0)}, \dots, x_N^{(0)}\} \quad (8)$$

$$Z^{(k+1)} = \begin{matrix} 1. \\ 2. \\ 3. \\ 4. \\ 5. \\ 6. \\ 7. \end{matrix} \begin{matrix} G(w_1; b_1; x_{j=0}^{k, N_{j+1}}) \\ G(w_p; b_p; x_{j=0}^{k, N_{j+1}}) \\ G(w_1; b_1; x_{j=0}^{k, N_{j+1}}) \\ G(w_p; b_p; x_{j=0}^{k, N_{j+1}}) \\ G(w_1; b_1; x_{j=0}^{k, N_{j+1}}) \\ G(w_p; b_p; x_{j=0}^{k, N_{j+1}}) \\ G(w_1; b_1; x_{j=0}^{k, N_{j+1}}) \end{matrix} \quad (6)$$

¹ [http:// nance.yahoo.com/home](http://nance.yahoo.com/home)
² [http:// nance.yahoo.com/home](http://nance.yahoo.com/home)
³ www-psych.stanford.edu/~andreas/Time-Series/SantaFe.html

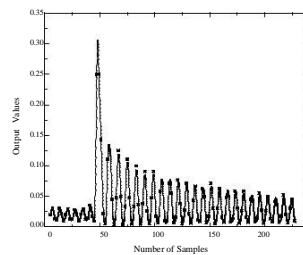


Fig. 1: Time Series of a SantaFe-A

where $x^{(0)}_i$ is the time series sample in original series $x^{(0)}$ at time step of i . For our analysis all data values to be positive. Eqn.(8) is transformed into a new sequence

$$x^{(1)} = [x_1^{(1)}; x_2^{(1)}; \dots; x_i^{(1)}; \dots; x_N^{(1)}] \quad (9)$$

by the 1st order AGO where

$$x_k^{(1)} = \sum_{i=1}^k x_i^{(0)}$$

$$x_k^{(1)} = f(x_{k-1}^{(1)}) + x_k^{(0)}; \quad k = 2; \dots; N$$

$$\text{with } x_1^{(1)} = x_1^{(0)}$$

Further, inverse accumulated generation operation (IAGO) where series in learning model is converted back to its original to achieve the predicted values. AGO prediction model based on OSELM can be discussed in detailed. Liang et al. based on ELM proposed online sequential ELM (OSELM) which has improvement of batch learning based ELM to sequential learning. Simulation results are in favour of OSELM with better generalization ability and fast processing compared to other existing methods. Particularly with OSELM, the processes of learning and prediction of time series are sequentially performed. In OSELM based model, the learning process of k^{th} step, the input and output

of OSELM are set as $x^{(1)}(k, i) (i = 1; 2; \dots; n)$ and $x^{(1)}(k)$, which are output of AGO operator applied on the original series $x^{(0)}(k, i) (i = 1; 2; \dots; n)$ and $x^{(0)}(k)$, respectively and thereby input to learning model. By learning mapping process from $x^{(1)}(k, i) (i = 1; 2; \dots; n)$ and $x^{(1)}(k)$, the connecting weight parameters are iteratively adjusted by OSELM method and prediction can be made immediately.

In this paper, online adjusted learning is done by another model OSELM for different activation functions in hidden layer nodes and results are summarized in the given Tables III-XI. The blocks that represent both AGO and IAGO operators give accumulated generation and inverse accumulated generation in this scheme. At k^{th} step, learning model output $x^{(1)}(k+1)$ is obtained by implementing $x^{(1)}(k, i) (i = 1; 2; \dots; n)$ as inputs to the learning machine, where

$x^{(1)}(k i)$ are the AGO results of original $x^{(0)}(k i)(i = 1; 2; ; n)$. The ultimate prediction result $x^{(0)}(k + 1)$ is then achieved by implementing IAGO: $x^{(0)}(k + 1) = x^{(1)}(k + 1) - x^{(1)}(k)$.

3.3 Normalization

In all our experiments the original data sets are normalised with zero error and standard deviation equals to 1 and data is adjusted into the range [0,1]. To perform normalization the following expression is used as

$$x^1 = \frac{x - x_{\min}}{x_{\max} - x_{\min}} \quad (10)$$

3.4 Performance Analysis

To analyse the prediction performance results obtained by using the learning algorithm OSELM with statistical metrics, namely, root mean square error (RMSE), normalized mean square error (NMSE), mean absolute difference (MAD) and directional symmetry (DS). The definitions of these criteria are listed in the Table 2. The values of NMSE and RMSE give the measures of the deviation between predicted values and actual values and smaller the values of NMSE and RMSE, the closer are the actual values to the predicted values of time series. The directional symmetry (DS) metrics provides the correctness of predicted direction having higher value percentage indicates better predictor. In this experimental investigation, OSELM has been used alongwith different activation functions provide better performance on different time series used for prediction under AGO and normalization smoothness parameter. The machine learning algorithm for on said time series are implemented in the experimental results using MATLAB code ⁴. For the sake of comparison of different activation functions like sigmoid, sin, hardlim and rbf are used in experiments.

3.5 Simulation Results

The effectiveness of proposed prediction system is determined by employing non-linear complex IBM and SantaFe-A time series data set. All results are simulated and calculated for OSELM with AGO operator with MATLAB 10 environment with 2.80GHz (CPU) and with 4GB memory (RAM). OSELM based method shows better prediction performance than that of other existing methods with smaller prediction errors and prediction accuracies of OSELM is more or less at the same level with faster processing speed but better than support vector re-gression based methods. Results obtained from OSELM compared with existing methods as given in Tables II-V.

In order to further demonstrate the effectiveness of OSELM with existing learning methods, financial time series IBM and SantaFe-A time series and their respective ranges for training and testing data set size are shown in Table I. In

⁴ <http://www.ntu.edu.sg/home/egbhuang>

the typical examples, OSELM is tested one-by-one and chunk-by-chunk learning with 20 sample chunk size. As given in Table I, IBM data samples size starting from year 1978 to 2014 has been used. Out of which 9871 data sample is used for training process and the remaining 3290 data samples used for testing process. Second time series in Table I is SantaFe-A laser generated time series data were recorded from a Far-Infrared-Laser in a chaotic state. For this time series training process data samples size is 750 and testing process samples size is 250 and performance based on determined criteria is explicitly given in Eqns.(9-12). Performance results on learning algorithms with activation functions sigmoid (sig) and sine (sin) with respective time series is given in Tables II-V.

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (A_i - P_i)^2} \quad (11)$$

$$NMSE = \frac{1}{2n} \sum_{i=1}^n \frac{(A_i - P_i)^2}{X_i} \quad (12)$$

where $P = \frac{1}{n} \sum_{i=1}^n P_i$

$$MAD = \frac{1}{n} \sum_{i=1}^n |A_i - P_i| \quad (13)$$

$$DS = \frac{100}{n} \sum_{i=1}^n (d_i) \quad (14)$$

It should be noted that A and P represent actual and predicted values, n is the total number of data points

AF	PD	LA	LM	Time fsecg	RMSE		Nodes
					Training	Testing	
sigmoid	Normalisation	ELM	Batch	0.0625	0.0189	0.0727	3
		OSELM	20-by-20	0.1875	0.0194	0.0496	5
	AGO	ELM	Batch	0.9531	0.0029	0.0050	139
		OSELM	20-by-20	0.1094	0.0061	0.0004	11
sin	Normalisation	ELM	Batch	0.0313	0.0199	0.0567	3
		OSELM	20-by-20	0.1250	0.0183	0.1487	3
	AGO	ELM	Batch	0.1875	0.0054	0.0004	31
		OSELM	20-by-20	0.1712	0.0058	0.0004	13
hardlim	Normalisation	ELM	Batch	2.8437	0.0157	0.2125	183
		OSELM	20-by-20	2.7187	0.0354	0.1495	153
	AGO	ELM	Batch	1.4688	0.0178	0.0026	111
		OSELM	20-by-20	5.1406	0.0239	0.0031	181
rbf	Normalisation	ELM	Batch	0.0313	0.0179	0.1723	7
		OSELM	20-by-20	0.2187	0.0181	0.2462	3
	AGO	ELM	Batch	0.3750	0.0053	0.0004	41
		OSELM	20-by-20	0.2187	0.1613	0.0106	3

AF: Activation

Table 2: Regression Analysis of IBM nancial time series Function, PD:Preprocessing of Data, LA: Learning Algorithm, LM:Learning Mode.

AF	PD	LA	LM	Time fsecg	RMSE		Nodes
					Training	Testing	
sigmoid	Normalisation	ELM	Batch	0.0006	0.1412	0.1892	35
		OSELM	20-by-20	0.0313	0.1592	0.1914	13
	AGO	ELM	Batch	0.0313	0.0171	0.0299	31
		OSELM	20-by-20	0.0313	0.0200	0.0060	25
sin	Normalisation	ELM	Batch	0.0312	0.1411	0.1914	35
		OSELM	20-by-20	0.0312	0.01693	0.1414	37
	AGO	ELM	Batch	0.0312	0.0203	0.0294	19
		OSELM	20-by-20	0.0781	0.0310	0.0308	3
hardlim	Normalisation	ELM	Batch	0.0625	0.1462	0.1864	81
		OSELM	20-by-20	0.3437	0.1384	0.1888	187
	AGO	ELM	Batch	0.0468	0.0234	0.0287	81
		OSELM	20-by-20	0.3125	0.0265	0.0087	189
rbf	Normalisation	ELM	Batch	0.0312	0.1384	0.1829	37
		OSELM	20-by-20	0.0312	0.2012	0.1324	37
	AGO	ELM	Batch	0.0625	0.0197	0.0047	27
		OSELM	20-by-20	0.0625	0.0011	0.0047	27

Table 3: Regression Analysis of SantaFe-A laser generated time series AF:Activation Function,PD:Preprocessing of Data, LA: Learning Algorithm, LM:Learning Mode.

AF	PD	Models	IBM			SantaFe-A		
			Metrics			Metrics		
			NMSE	MAD	DS	NMSE	MAD	DS
sig	AGO	OSELM	0.0162	0.00032	85.58%	0.03212	0.0259	46.53 %
		NORM	0.0693	0.0408	49.45%	0.0471	0.0159	62.37 %
sin	AGO	OSELM	0.1387	0.0011	89.64%	0.01387	0.0031	61.64%
		NORM	0.2344	0.0594	50.94%	0.01344	0.0594	68.94%

Table 4: Performance Analysis of IBM and SantaFe-A time series using OSELM learning algorithm where PD:Preprocessing of Data.

Data Set	Converged NMSE				Computation Time			
	RLFSVR[10]	SVR[10]	OSELM	ELM	RLFSVR[10]	SVR[10]	OSELM	ELM
IBM	2.658	3.714	0.0162	0.4709	0.235	6.6000	0.1094	0.9531

Table 5: Comparative analysis of the OSELM and ELM learning models with other models RLFSVR and SVR

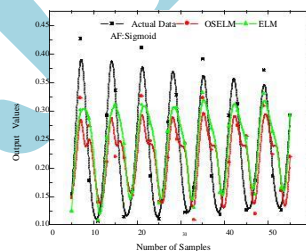


Fig. 2: Predicted Result of SantaFe-A time series using normalised data and learning algorithms OSELM and ELM.

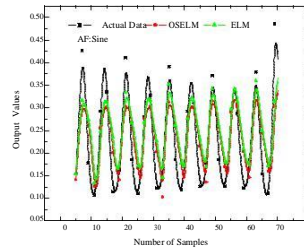


Fig. 3: Predicted Result of SantaFe-A time series using normalised data and learning algorithm OSELM and ELM.

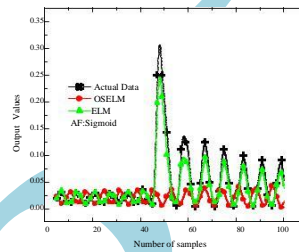


Fig. 4: Predicted Result of SantaFe-A time series using AGO data and learning algorithm OSELM and ELM.

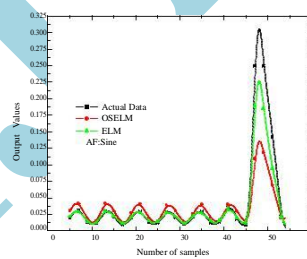


Fig. 5: Predicted Result of SantaFe-A time series using AGO data and learning algorithm OSELM and ELM.

4 Conclusion

The use of OSELM for complex nonlinear time series forecasting is studied in the paper. It is concluded by observing that in most of the cases OSELM performs better than SVR and RLFSVR in terms of NMSE and computational time in predicting the future values of time series and other category of time series like SantaFe-A. The learning algorithm provide promising alternative to BP neural network and support vector machines for time series forecasting. As demonstrated in the experiment, OSELM forecast significantly better with AGO and normalization preprocessing ways for data set than support vector regression based forecasting model. Moreover, several activation functions such as sigmoid, sin, hardlim and radial bias function are used to determine better performance of forecasting model. The performance of learning model has been evaluated by carrying out a series of experiments on various time series. Finally, extreme learning based method has very few parameters need to tune and optimize for obtaining optimum results and there is very minimal chances of either over-fitting or under-fitting of the training data and leads to no significant impact on models performance. The proposed implementation of online sequential ELM has unique advantage that it can adaptively tune weights when some new chunk of data samples are added or deleted.

References

1. Rumelhart. D.E., Hinton. G.E., Williams. R.J., Learning Representing by Back Propagation Errors, Nature, 323, pp.533-536,1986.
2. Vapnik. V.N., The Nature of Statistical Learning, 2nd ed, New York: Springer, 2000.
3. Huang. G-B., Wang. D-H. and Lan. Y., Extreme Learning Machine : A Survey, International Journal of Machine Learning and Cybernetics, vol.2, pp.107-122, 2011.
4. Huang. G-B., Zhu. Q-Y. and Siew. C-K., Extreme Learning Machine : A new Learning Scheme of Feedforward Neural Network, Int. Joint Conf. on Neural Netw. (IJCNN 2004) Budapest, Hungary, pp.25-29, 2004.
5. Rao. C.R. and Mitra. S.K., Generalised Inverse of Matrices and its Applications, Wiley, N.Y, 1971.
6. Liang. N-Y., Huang. G-B, Saratchandran. P. And Sundararajan. N, A fast and accurate Online Sequential learning algorithm for feed forward network, IEEE Trans. for Neural Network, 17(6), pp.1411-1423, 2006.
7. Yibin Ye, Squartini. S. and Piazza. F., Online Sequential Extreme Learning Machine in nonstationary environments, Neurocomputing, vol.116, pp.94-101,2013.
8. Chong. E.K.P. and Zak. S.H., An introduction to optimization, Wiley, New York,2001.
9. Luo Y, Zeng. B and Liao. D, Non-equidistant GRM(1,1) Generated by Accumulated Generating Operation of Reciprocal Number and its Application, International Journal of Computer Science Issues, vol. 10, issue 2, no 2, pp.119-123, 2013.
10. Khemchandani. R, Jayadeva and Chandra. S, Regularised least squares fuzzy support vector regression for nancial time series forecasting, Expert Systems and applications, vol.36, pp.132-138, 2009.