

Optimum Connection Pattern of MUX/DEMUX to enhance fault tolerance of SEN MIN

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Abstract- In big data communication an efficient form of information processing emphasizes on exploring parallel computing system. In this regard, a network is required which is capable of handling huge traffic produced in big data communication and processing. MIN plays an important role in designing of these parallel processors and is a quiet interesting topic among researchers. In this paper, an optimum technique has been introduced to use variable size of MUX/DEMUX at input/output stages of MIN to enhance reliability-cost-ratio. Also an optimum connection pattern has been suggested here to achieve maximum number of disjoint paths with the available hardware. It has also been shown that the connection pattern severely affects the routing scheme and reliability of MIN.

Keywords—Multistage Interconnection Networks (MIN), Reliability, Connection Pattern, Multiplexer (MUX), Demultiplexer (DEMUX), Switching Element (SE), and Cost.

I. INTRODUCTION

With the advancements in the technology, increased computing power is required, which is achieved by using multiple processors connected in parallel [1-3]. These processors need to communicate with each other or to their memory modules connected within a system. MIN behaves as a cost effective medium for communication between above said processors and memory modules which are connected parallel [4]. Hence it eliminates the need of cross-bar networks, which are very expensive and bus networks, which are inefficient in terms of speed and performance. So for effective designing of MIN, a balance has to be established between relatively high speed performance in today's fast computing scenario and reasonable cost [5]. Many researchers have proposed new techniques and designs of MIN to achieve better performance in literature [6-17]. These techniques are based on (i) By increasing number of stages in the network [7], (ii) By increasing size of SE [8, 10], (iii) By using MUX/DEMUX at input/output stage of MIN, thus reducing number of stages by one [11-15] and (iv) By increasing the size of MUX/DEMUX used in the network. Although these techniques have improved the reliability of MIN to a n extent but there are still some gaps which have been observed in the previous findings which are tried to be filled in this research. These gaps are summarized as follows:

- (i) Although it has been acclaimed that by increasing the number or size of SE does not increase the reliability to the desired level and increases the cost of overall network, whereas using MUX/DEMUX at Input/ Output nodes increases reliability with reasonably low cost. Although the effect of connection pattern on reliability of MIN has not been analyzed for SEN MIN yet.
- (ii) Effect of different connection pattern of 2:1 MUX/ 1:2 DEMUX on routing schemes of Gamma-Minus MIN has been analyzed recently, but effect of connection pattern of 4:1 MUX/ 1:4 DEMUX has not been analyzed,

although it has been stated that MUX/DEMUX can be used upto the size of 4:1/1:4 to increase reliability.

- (iii) Various authors have used different connection patterns for MUX and DEMUX at input as well as output stage. Nowhere it has been suggested that why that particular connection pattern has been used and what would be the effect if that connection pattern is altered. These questions are unanswered in literature.
- (iv) Routing scheme of these MIN has also been less emphasized in literature. The effect of connection pattern of MUX and DEMUX on routing scheme as also kept unconsidered.

All these points have motivated to pursue this research. In this paper optimum connection pattern of MUX and DEMUX has been presented to achieve maximum reliability out of the given hardware. Routing scheme of MIN with MUX and DEMUX has also been discussed here. A new method of increasing reliability using higher sizes of MUX and DEMUX has been suggested with not much increase in the hardware cost of MIN.

II. PRELIMINARIES AND BACKGROUNDS

In typical MIN of size $N \times N$ there are total $\log_2 N$ number of stages consisting of Switching Elements (SE) of $c \times c$ size, with N/c number of SE per stage. As an example Shuffle Exchange Network (SEN) of size $N \times N$ with SE of size 2×2 consist of $\log_2 N$ number of stages consisting of $N/2$ SE per stage [7]. SEN is a unique path MIN in its basic topology as shown in Fig. 1, hence does not provide any fault tolerance [8-10]. Lot of enhancements has been suggested in past to improve the fault tolerance capability of SEN by increasing number of paths between its Source Destination Node Pair (S-D) [7-16]. These advancements consists of (i) by increasing number of stages for example SEN+ and SEN+2, (ii) By increasing size of SE for example ASEN, (iii) By introducing multi layers at

middle stage for example Multilayer MIN, and (iv) By introducing various size d MUX and DEMUX at input and output stages respectively, for example SEN-Minus. In first three methods of improving fault tolerance of MIN, extra hardware has been used which increases the overall cost of interconnection network and is undesirable. In last method fault tolerance has been achieved with nominal increment in hardware which does not affect much the overall cost of IN [8, 10]. It has been claimed that this method improves system performability to a great extent in term of reliability, transmission time BW etc. furthermore various sizes of MUX and DEMUX have been employed at input and output stage of SEN MIN acclaimed that by increasing the size of MUX and DEMUX, reliability and fault tolerance capability of MIN increases [11-15].

It has been shown that although reliability of Min increases with higher size of MUX and DEMUX but reliability cost ratio decrease to a great extent which is not desirable. So a limit on the size of MUX and DEMUX upto 4:1 and 1:4 respectively has been establish so as to achieve high reliability at moderate cost [11-15]. But connection pattern of these MUX and DEMUX have not been discussed anywhere. It has been analyzed that the connection pattern of MUX and DEMUX greatly affect the reliability and availability of disjoint paths in particular MIN. So it is important to establish the connection pattern to achieve maximum disjoint paths and high reliability. In this $2 \times 1/1 \times 2$ and $4 \times 1/1 \times 4$ MUX/DEMUX have been analyzed and optimum connection pattern have been suggested which provide maximum number of disjoint paths between each S-D node pair and hence provide high reliability[11-15]. Further routing schemes of SEN MIN with $2 \times 1/1 \times 2$ and $4 \times 1/1 \times 4$ MUX/DEMUX have also been presented whi8ch improves transmission time by using Non-Backtracking algorithm.

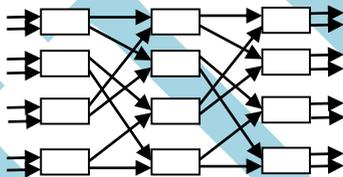


Fig. 1: SEN MIN topology

III. EFFECT OF CONNECTION PATTERN OF MUX/DEMUX ON SEN MIN

In SEN MIN, there is $N/2$ SE at each stage of configuration of 2×2 , so in first configuration (SEN-Minus) 2:1 MUX and 1:2 DEMUX are connected to input and output stages of SEN with 8×8 network size. In second configuration (FTSM) 4:1 MUX and 1:4 DEMUX has been used for 16×16 network size of SEN MIN. There are possibly four connection patterns for MUX and DEMUX to be attached at input and output stage respectively for both configurations. These four possible configurations of SEN MIN network are shown in figure 2(a)-(d) for first configuration i.e. SEN-Minus and from 3(a)-(d) for second configuration i.e. FTSM.

Fig 2 (a): As shown in Fig. 2(a) input '0' and '2' are connected to MUX '0' and '2' and input '1' and '3' are

connected to MUX '1' and '3', whereas input '4' and '6' are connected and input '5' and '7' are also connected. Same connection pattern is used for DEMUX and is known as 0-2 connection pattern. At output side DEMUX '0' and '4' are connected, whereas DEMUX '1' and '5' are connected and so on. This connection pattern is known as 0-4 connection pattern

Fig 2 (b): As shown in Fig. 2 (b) the reverse connection pattern of Fig 2(a) has been used i.e. 0-4 connection pattern has been used at input side and 0-2 connection pattern has been used at output side. It has been shown here that this topology does not achieves full connectivity for all S-D node pairs (For example if a packet is to be transmitted from source '2' to the destination '2' shown by the green line, it does not show this connectivity).

Fig 2 (c): In Fig. 2 (c) 0-2 connection is used at both input as well as output sides of SEN-Minus.

Fig 2 (a): In Fig. 2 (d) 0-4 connection pattern has been used.

Fig 3 (a): In Fig. 3 (a) upper half of the even MUX i.e. '0', '2', '4', '6' are connected and lower half of the even MUX are connected. In similar fashion upper half odd MUX and lower half odd MUX are connected at the input side as shown. This connection pattern is called as 0-2-4-6 pattern. At the output side even out of even DEMUX and odd out of odd DEMUX are connected and rest all even and rest all odd DEMUX are connected together and is called as 0-4-8-12 pattern as shown.

Fig 3 (b): 0-2-4-6 connection is used at both input as well as output sides of FTSM.

Fig 3 (c): 0-4-8-12 connection is used at both input as well as output sides of FTSM.

Fig 3 (d): Reverse connection pattern of Fig. 3(a) has been used i.e. 0-2-4-6 connection pattern at input side and 0-4-8-12 connection pattern at output side is used.

For routing of packet from source node to destination node, it has been assumed that packet is to be transferred from node '0' to node '2'. The red lines in Fig. 2 are showing the routing of the packet from source node '0' to destination node '2' and in Fig. 3 the routing of the packet from source node '0' to destination node '6'. In both the configurations SEN-Minus and FTSM, only first connection pattern used is showing maximum number of disjoint paths between the chosen S-D node pair. The number of paths is same for each configuration but these configurations are either using same node (SE) or Link for more than one path. Hence their reliability and fault tolerance is

reduced which is proved in the further section. From here it can be summarized that if MUX and DEMUX have been used in SEN MIN topology at its input and output stage then the whole network should be divided in to subgroups, where number of SE in each subgroups should be equal to 'N/4'. At input side all even inputs should be connected within that sub group and all odd inputs should be

connected. At the destination side all first output node of each subgroups are connected, all second output nodes are connected and so on. This is the general pattern for connecting any configuration of MUX and DEMUX at input and output stages of SEN MIN, which can provide maximum number of paths with in the network

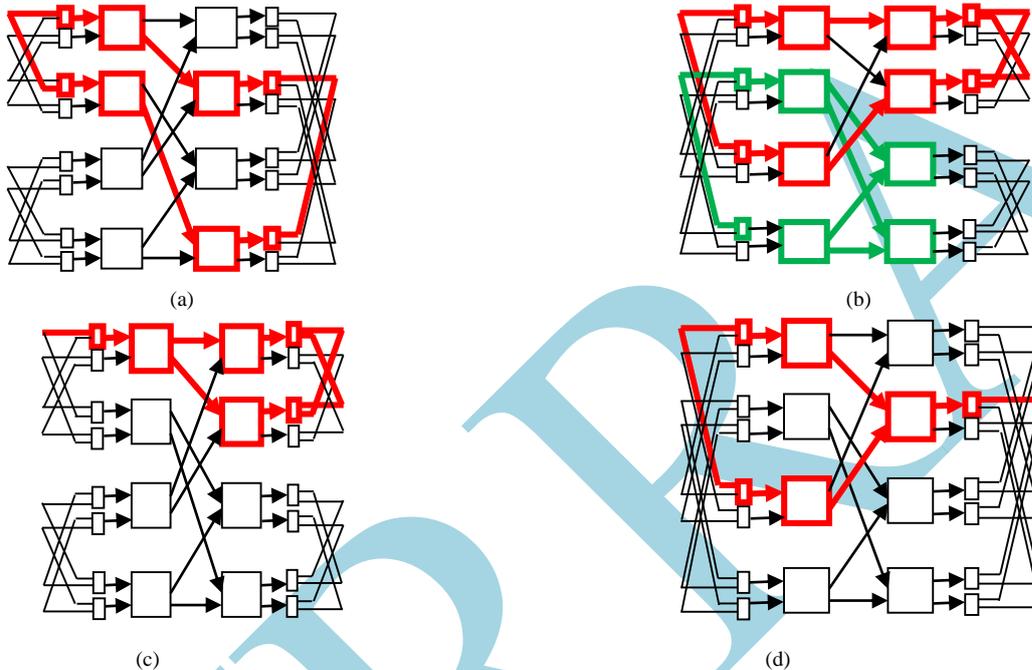
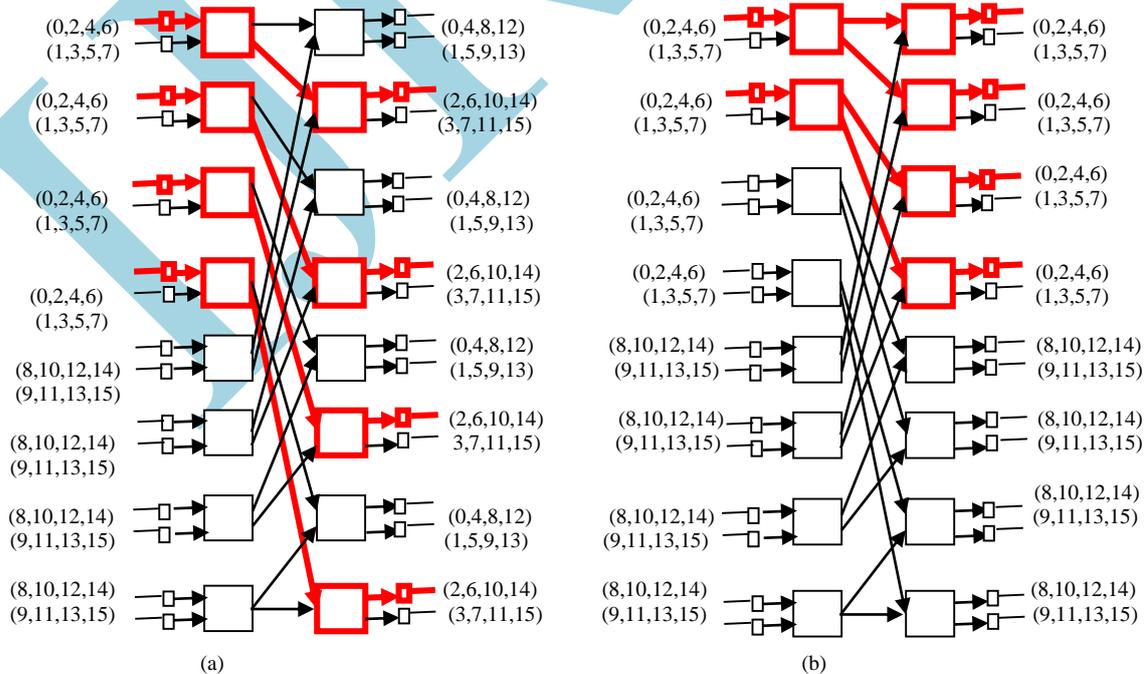


Fig. 2 (a)-(d) SEN-Minus with different connection pattern of MUX and DEMUX at input and output stage respectively. Red lines are showing routing from source terminal '0' to destination terminal '2' and green lines are showing routing of source terminal '2' to destination terminal '2' in all the Figures.



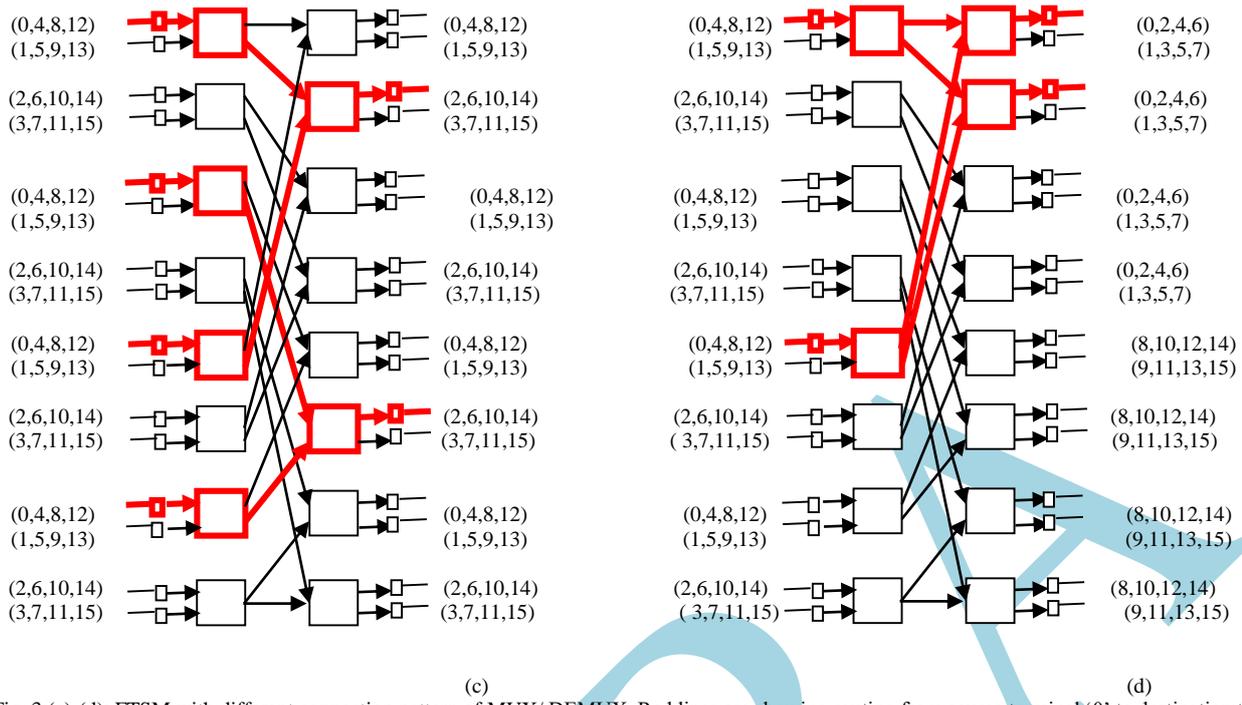


Fig. 3 (a)-(d): FTSM with different connection pattern of MUX/ DEMUX. Red lines are showing routing from source terminal '0' to destination terminal '6'.

IV. TERMINAL RELIABILITY

Terminal Reliability of a system can be defined as the probability of atleast one path of the system configuration works properly, which exists between each S-D node pair [7, 9, 11, 15, 16, 17]. It is an important parameter to evaluate. To evaluate terminal reliability, Reliability Block Diagrams (RBD) method has been used. It has been seen that as the size of MUX and DEMUX increases, the number of paths for each S-D node pair also increases. But reliability is not a function of number of Path availability so reliability of a network has to be analyzed separately. Using RBD of Fig 4 and 5, equations to analyze terminal reliability of these network have been deduced and shown in Table 1. Reliability of these networks has to be calculated by assuming SE reliability as 'r' where 'r' is a function of time and can be expressed as:

where λ is failure rate $\infty .000001$ /sec

$$r = \int_0^t e^{-\lambda t}$$

The assumptions for reliability calculation taken are as follows:

1. Reliability of 2×1 SE is taken as 'r' (w.r.t. the number of cross-points i.e. 2×2 SE has 4 cross-points $4/4 = 1$).
2. Reliability of $1 \times 2/2 \times 1$ MUX/DEMUX is taken as 'r/2'.
3. Similarly for 1×4 and 4×1 MUX/DEMUX is taken as 'r' (for 4 cross-points) and for $1 \times 8/8 \times 1$ MUX/DEMUX is taken as 'r/2' (for 8 cross-points).

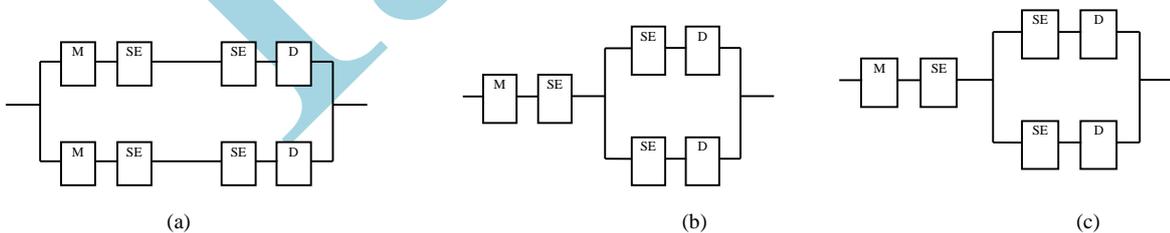
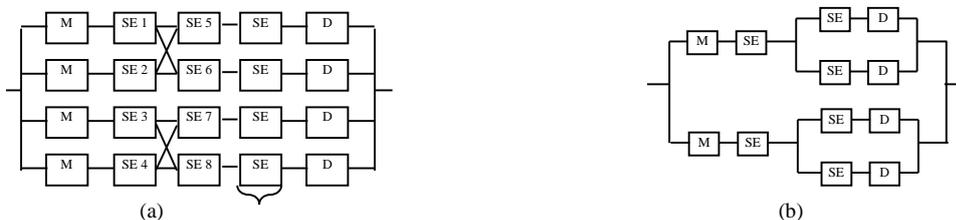


Fig 4 (a)-(c): RBD of 1st, 3rd and 4th configuration of SEN-Minus topology.



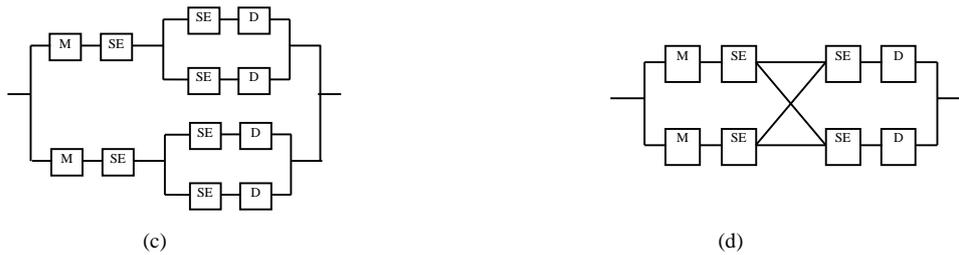


Fig 5(a)-(d): RBD of 1st, 3rd and 4th configuration of SEN-Minus topology.

Table 1: Reliability equations for all SEN MIN configurations for 2x1/1x2, 4x1/1x4 size of MUX/DEMUX

S.No.	Network	Size of MUX/DEMUX Used	Number of Paths	Terminal Reliability Equation
1	SEN-Minus 1	2x1/1x2	2	$[1 - (1 - r^3)^2]$
2	SEN-Minus 2		-	Not applicable
3	SEN-Minus 3		2	$r^{3/2} [1 - (1 - r^{3/2})^2]$
4	SEN-Minus 4		2	$r^{3/2} [1 - (1 - r^{3/2})^2]$
1	FTSM 1	4x1/1x4	8	$1 - \{1 - [1 - [1 - r^2]^2] [1 - [1 - r^2]^2]\}^2$
2	FTSM 2		4	$1 - [1 - r^{3/2} \{1 - (1 - r^{3/2})^2\}]^2$
3	FTSM 3		4	$1 - [1 - r^{3/2} \{1 - (1 - r^{3/2})^2\}]^2$
4	FTSM 4		4	$[1 - (1 - r^{3/2})^2]^2$

Table 2: Reliability Value for all SEN MIN for Reliability of SE (R_{SE}) = 0.99 to 0.90

Network Name	Size of MUX/DEMUX	R _{SE} =0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.90
SEN-Minus 1	2x1/1x2	0.9991	0.9965	0.9924	0.9867	0.9797	0.9713	0.9617	0.9510	0.9393	0.9266
SEN- Minus 2		---	----	----	----	----	----	----	----	----	----
SEN- Minus 3		0.9848	0.9693	0.9534	0.9373	0.9209	0.9042	0.8873	0.8702	0.8530	0.8356
SEN- Minus 4		0.9848	0.9693	0.9534	0.9373	0.9209	0.9042	0.8873	0.8702	0.8530	0.8356
FTSM 1	4x1/1x4	0.9999999	0.999999	0.99999	0.9998	0.9996	0.9993	0.9987	0.9978	0.9966	0.9950
FTSM 2		0.9998	0.9991	0.9978	0.9961	0.9937	0.9908	0.9873	0.9832	0.9784	0.9730
FTSM 3		0.9998	0.9991	0.9978	0.9961	0.9937	0.9908	0.9873	0.9832	0.9784	0.9730
FTSM 4		0.9996	0.9982	0.9960	0.9930	0.9891	0.9843	0.9788	0.9725	0.9655	0.9577

As shown from the Table 2 SEN-Minus 1 and FTSM 1 shows the highest reliability among respective configurations and overall FTSM 1 is the highest reliable network of SEN class of MIN. Hence it can be summarized that the connection pattern which has been presented in this paper is the optimum connection and can be used to achieve high reliability and fault tolerance. The graphical representation of the results achieved in Table 3 is shown in Fig 6 for both networks.

V. CONCLUSION

MIN are responsible for reliable data communication in super computer systems. SEN MIN are most used MIN, which are improving at a very fast rate according to the requirements of super computers. In this paper SEN MIN with different sizes of MUX and DEMUX has been explored. The optimum connection pattern of these MUX and DEMUX has been suggested so as to achieve maximum number of disjoint path in the existing topologies. It has been found that SEN MIN

with maximum size of 4:1/1:4 MUX/DEMUX should be used as it provides the highest reliability with reasonable cost.

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