Landau Criteria of Superfluidity of Two-Component Mixture of Bosons

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Abstract- **Using the quasi-particle energy of a mixture of two-component gas of bosons which is assumed to be in a superfluid state, the Landau criterion for superfluidity has been applied to obtain the magnitude of the Landau's critical velocity (V_c) below which the superfluid state can exist as stable state. Different combinations of bosons have been used to calculate V_c. It is found that, the magnitude of V_c increases with scattering length (*a*) and its magnitude is less than the velocity of sound (V_s) in superfluid liquid Helium (V_s=238m/s), and it varies roughly between 10.95m/s to 15.12m/s. The value of Landau critical velocity for superfluid Helium is of the order of 50m/s to 60m/s depending on the type of inter-particle interaction assumed between the Helium atoms.**

Keywords— Quasi-Particle energy, Landau criterion, Superfluidity, Critical velocity, Mixture of two-component gas.

I. INTRODUCTION

Identical bosons at very low temperatures when sharing the same wave function are said to be in a state called the Bose-Einstein-Condensation (BEC). Such a state was predicted by Einstein in 1925 for an ideal gas of bosons [1]. For quite some time the BEC was treated as a theoretical idea since there was no experimental evidence for the existence of many-body condensed state. More precisely, the phenomena of Bose-Einstein-Condensation is the macroscopic accumulation of bosons in the ground state of the non-interacting (perfect ideal) gas. Such state is also known as zero-momentum-state (ZMS). BEC is a state of matter in which separate atoms or subatomic particles are cooled to near absolute zero (0K, -273.15^{0}C, -459.67^{0}F), condense into a single quantum mechanical entity, that is, one that can be described by a wave-function on a near macroscopic scale.

The origin of these studies lies in the liquefaction of helium. H. Kamerlingh Onnes first liquefied helium in 1908 when he cooled it below the gas or liquid transition temperatures of 4.2K [2]. M. Wolfke and W.H Keesom found out experimentally, in 1927, that at low temperatures around 2.17K, there exists another phase transition [3]. By measuring specific heat (C_V) variation against temperature (T), they found a discontinuity in the specific heat curve, and the graph of specific heat variation looked like the Greek letter λ, and thus the transition point was called λ point. The phase before the λ transition point was called Helium I, and the phase after the λ transition point was called Helium II. The liquid helium below the λ point, Helium II, had remarkable superfluid properties, and these properties were experimentally established by P. Kapitza in 1938 in Moscow [4] and J.F. Allen and A.D. Misener in 1938 [5]. In a simple experiment, Kapitza allowed Helium to flow between two cylinders connected by a thin tube of diameter 0.5µm. He found that only below λ-point, Helium flowed easily through the tube and this means that there was very low viscosity to the flow of the liquid. This frictionless flow through narrow tubes led to the term superfluidity for liquid Helium II, and this is mentioned in Kapitza's 1938 paper.

Now to understand the bosonic superfluid such as Helium II, the BEC theory proposed by Einstein was already available. However, that was not considered as enough since more theoretical insight into the phenomena of superfluidity was required. Fritz London was the first to propose a hypothesis for connection between normal to superfluid phase transition and to Bose-Einstein-Condensation [6]. London could predict quite accurately the existence of the λ temperature based on the mass of the ^{4}He atoms and the density of liquid Helium. Based on the characteristics of BEC, Tisza proposed a two fluid model in which he assumed the co-existence of a thermal and condensate phase in the fluid. Tisza's model could explain to some extent the superfluid fountain effect and the existence of temperature waves (second sound) [7]. Then later, Landau proposed a phenomenological description of a superfluid in terms of weakly interacting mixture of excitations such as phonons and rotons [8]. A breakthrough in the understanding of Landau's excitation spectrum was provided by Bogoliubov, who used field theoretic methods to present a basis in which the Hamiltonian of weakly interacting gas of bosons is diagonalized, to obtain a quasi-particle excitation spectrum that exhibits a phonon-like spectrum with a linear dispersion at very low momenta [9]. In fact, Bogoliubov theory is a description of a weakly interacting Bose gas, and it exhibits both Bose-Einstein-Condensation and phonon (quasi-particle) excitation spectrum. The existence of a gapless mode in quasi-particle energy excitation spectrum of a condensed system (like superfluid helium) is called Goldstone mode.

Experimental observations on Bose-Einstein-Condensation in ultra-cold atomic gases started around 1995 [10, 11, 12]. Extensive historical details about the discovery of the phenomena of superfluidity can be found in the studies of single component boson systems in which the inter-particle interactions were assumed to be weak [13,14]. This was the fundamental basis of the Bogoliubov theory for condensed Bose systems.

II. THEORY

According to Bogoliubov theory, the Hamiltonian of a mixture of bosons can be described by two operators namely, *a* and *b* where *a* stands for one type of boson and *b* represents the second type of bosons. For an interacting system of two species of bosons, the Hamiltonian is written as;

$$\mathrm{H} = \varepsilon(a^{+}a + b^{+}b) + g(a^{+}b + b^{+}a) \qquad (1)$$

where ε is a parameter that determines the kinetic energy of the bosons, $a^{+}a$ and $b^{+}b$ represents the existence of one type and the second type of bosons respectively, g is a parameter that determine the measure of the interaction strength between the particles and $a^{+}b$ and $b^{+}a$ are creation and destruction of bosons as described by the respective operators.

The Hamiltonian of a system that exhibits problems such as quantum phase transition and superfluidity takes the form,

$$\mathrm{H} = \varepsilon(a^{+}a + b^{+}b) + g(a^{+}b^{+} + ba) \qquad (2)$$

The Hamiltonian in Eq. (2) has been diagonalized using Bogoliubov canonical transformations [15] and the quasi-particle spectrum obtained has been studied. Consequently, the interacting system that is likely to undergo phase transition is described.

With improvements of experimental techniques, ultra-cold atomic gases of different species or hyperfine states [16] and binary Bose systems have been studied experimentally [17]. Experiments have been done to create degenerate two-component Bose droplets and mixtures [18, 19, 20, 21]. It needs to be understood as to what parameters and or physical conditions can lead to the existence of superfluidity as a stable state. In one component Bose system, it is the so-called Landau criteria for superfluidity that determines the stability and existence of the superfluid state.

Landau criteria is a cornerstone in our understanding of the dynamical behavior of superfluids. According to Landau's criteria for superfluidity, a Bose-Einstein-Condensate flowing with a group velocity smaller than the velocity of sound (238m/s-first sound) remains in a stable superfluid state and moves without dissipation. In fact, it is what is called Landau's critical velocity (V_c), that determines the existence of superfluid state. If E_P is the quasi-particle energy excitation of one component boson system at the momentum P, then the Landau's critical velocity (V_c), is [22]

$$V_c = \min_P\left(\frac{E_P}{P}\right) \qquad (3)$$

BEC exists in any Bose system as long as temperature is lower than critical temperature. This can be explained from Bose statistics. As for superfluid, there must be interactions between two particles. This will mean that, there exists BEC in an ideal Bose gas, but no superfluidity in an ideal gas. Superfluidity exists in an interacting system at very low temperature, i.e., when $T<T_c$, where T_c is the critical temperature at which the superfluid phase will appear. This means that an ideal gas is not a superfluid, but there is no BEC without superfluid properties. The particles in a superfluid must be coherently correlated, and this will require some interactions between the particles. Such an interaction and or correlation is not needed in BEC. Thus, BEC and superfluidity may have some similarities, but they are not identical phenomena. With regard to this, it can be stated that, all BEC may not be superfluid, but all superfluids may be BEC. Perhaps a better statement may be that, one can have superfluids that are BEC's and there may be BEC's that are not superfluids. Thus, the best method to determine whether the assembly is superfluid or not is to use the criteria for superfluidity proposed by Landau. This is what is done in this paper for a mixture of two-component Bose gases.

When we deal with a two-component mixture of bosons, some important characteristics of the assembly change when it is compared to the one-component system. Here, the mass of each atom in one-component is different from the mass in the second component. Now the transition temperature (T_c) for the Bose-Einstein-Condensation is given by;

$$T_c = \frac{3.31\hbar^2 n^{2/3}}{4\pi^2 kM} \qquad (4)$$

where *n* is the critical particle number density, M is the particle mass of the given boson gas and the value of T_c depends on the mass of each particle. Thus, the transition temperature T_c will be different for each component.

In this paper, the pairs of bosons that are selected to calculate the Landau's critical velocities are $^{41}K+^{41}K$, $^{39}K+^{41}K$, $^{39}K+^{39}K$ and $^{87}Rb+^{41}K$. The quasi-particle energy of a mixture of a two-component gas of bosons of density *n* is given by;

$$E_c = (0.023)\frac{4\pi a_{BB}\hbar^2}{m_{BB}}n \qquad (5)$$

where, a_{BB} is the boson-boson scattering length, m_{BB} is the reduced mass of two types of bosons constituting the mixture and it is given by; $m_{BB} = \frac{m_1 m_2}{m_1 + m_2}$ where m_1 is the mass of one type, and m_2 is the mass of the second type. Also, $\hbar = \frac{h}{2\pi}$, where h is the Planck's constant.

The Landau's criteria for superfluidity is obtained by equating E_c to momentum and velocity such that,

$$E_c = pc \qquad (6)$$

where *P* is the quasi-particle momentum and *c* is the critical velocity (V_c) of flow of an excitation through the superfluid such that the superfluid state remains stable or there is no breakdown of superfluidity.

Combining Eqs. (5) and (6) we get;

$$E_c = m_{BB}c^2 \qquad (7)$$

or

$$c = V_c = \left(\frac{E_c}{m_{BB}}\right)^{\frac{1}{2}} \qquad (8)$$

III. RESULTS AND DISCUSSION

According to Landau's criteria of superfluidity, a Bose-Einstein-condensate flowing with a group velocity (V_c) smaller than the velocity of sound (238 m/s, first sound) remains a stable superfluid state and moves without dissipation. In ^{4}He, which is a strongly interacting system (inter-particle interaction is strong), $V_c \approx 60$m/s [23]. In the calculations shown in Table 1, the value of V_c varies approximately between 10.95m/s and 15.12m/s.

TABLE 1: The values of Energy densities and critical velocities calculated using Eq. (8)

Isotopes	Scattering Lengths in terms of Bohr Radius (a_0)	Energy density in Joules per cubic metres	m_{BB} in Kg	V_c (m/s)
^{41}K+^{41}K	83a_0	4.116E-24	3.4303E-26	10.95397
^{41}K+^{41}K	85a_0	4.216E-24	3.4303E-26	11.08624
^{41}K+^{41}K	87a_0	4.314E-24	3.4303E-26	11.21435
^{39}K+^{41}K	110a_0	5.594E-24	3.3445E-26	12.93289
^{39}K+^{41}K	113a_0	5.747E-24	3.3445E-26	13.10856
^{39}K+^{41}K	116a_0	5.899E-24	3.3445E-26	13.28078
^{39}K+^{39}K	134a_0	6.985E-24	3.2629E-26	14.63125
^{39}K+^{39}K	140a_0	7.298E-24	3.2629E-26	14.95547
^{39}K+^{39}K	143a_0	7.456E-24	3.2629E-26	15.11649
^{87}Rb+^{41}K	100a_0	3.648E-24	4.6631E-26	8.84484

It is evident that as the scattering length increases from 83a_0 to 143a_0 (where a_0 is the Bohr radius) the value of V_c also increases. The scattering length is a measure of strength of the contact-interaction. Hence, for stronger interaction V_C increases in a two-component mixture of bosons. Since the value of V_C for any two-component boson gas is smaller than the V_C for Helium gas (^{4}He), the inter-particle interaction in the two-component gas of bosons is smaller compared to the inter-particle interaction between Helium atoms. It seems that the mass of the particles plays a definite role in determining the value of V_C in the superfluid state. Helium is the lowest mass boson whereas the masses of Rb and K are quite large. Hence strong inter-particle interaction and large mass of the interacting particles leads to reduction of V_c.

IV. CONCLUSION

The calculations of this study shows that, as the energy densities increase with the corresponding increase in magnitude of the scattering lengths, the values of the Landau's critical velocities also increase. This occurs in order to maintain coherence in the superfluid state. However, for interacting particles having large mass such as ^{87}Rb+^{41}K, a reduction of Landau's critical velocity is noted. This indicates that, in a super fluid state, the mass of the bosons plays a definite role in calculation of the Landau's critical velocity, which is a crucial parameter in determining the existence of a superfluid state.

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