Ontalogy of Direction of Arrival Estimation Methods Based On Bias, Resolution, Variance, SNR & Performance

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Abstract- **The DOA estimation in array signal processing is one of the important and emerging research area. The effectiveness of this direction of arrival estimation greatly determines the performance of smart antennas.it works on digitized output from each sensor array. The estimation results for coherent signals, broadband signals and multiple signals are of greater consideration. Various information's of the parameters relating to a particular wave can be obtained by analyzing the incoming wave, that is received by a sensor or N array of sensors the accuracy is the estimation of direction of arrival is very crucial in array signal processing. DOA estimation has vital application in radar, SONAR, seismology, earthquake, astronomy, biomedicine and communication the purpose of this papers is to provide analysis of Bartlett method, MUSIC method, Linear prediction method, Max Likelihood method, Minimum Norm method, CLOSET method, ESPIRIT Method, etc. between several Direction of presented. The informative signal are corrupted by an additive white Gaussian noise (AWGN), to show performance of each method by applying directly algorithms without pre-processing techniques..**

Index Terms- DOA Estimation; MUSIC; MVDR; Min-Norm.

1. INTRODUCTION

In signal processing a set of parameters upon which the received signal depends are continuously monitored. DOA estimation carried out using single fixed antenna has limited resolution. physical size of antenna is inversely proportional to antenna main lobe beam width. It is not feasible to increase size of single antenna to obtain sharper beam width. Hence an array of antenna sensors are used for better performance. It has a vital application in biomedicine, sonar, astronomy, communication radar etc. Various algoritms like ESPIRIT, MUSIC, WFS, MVDR, ML Techniques and others can be used for estimation direction of arrival. High resolution direction finding techniques recently have been introduced like MUSIC, Min Norm and advance MUSIC etc. DOA estimation can be computed using angles of arrival and frequency of arrival. DOA estimation methods considered, include spectral estimation, minimum-variance, directionless response estimator, linear prediction maximum entropy, maximum likelihood.

II. DOA ESTIMATION ALGORITHMS

A. Spectral estimation method

These methods estimate DOA by computing the spatial spectrum P(θ), that is, the mean power received by an array as a function of θ, and then determining the local maxima's of this computed spatial spectrum. Most of these techniques have their roots in time series analysis. A brief overview and comparison of some of these methods are found in [1].

B. Bartlett method

The Bartlett algorithm is a Fourier spectrum analysis method. The goal is the find a set of weights w that maximize the received signal power. The m-element circular array receives signals from several spatially separated users. The received signals usually contain both direct path and multipath signals, which are most likely from different directions of arrival angles [3]. One of the earliest methods of spectral analysis is the Bartlett method, in which a rectangular window of uniform weighting is applied to the time series data to be analysed. For bearing estimation problems using an array, this is equivalent to applying equal weighting on each element [1]. Also known as method of averaged periodograms, Bartlett method computes the power spectrum by a standard formula [5].

C. Eigenvector methods

A class of spectral estimation procedures based on an eigenvector-eigenvalue decomposition of the spatial correlation matrix has been developed recently. These procedures are intimately related to the maximum likelihood and linear-prediction methods just described. The motivation for this approach is to emphasize those choices for E which correspond to signal directions. As the expressions for the maximum-likelihood and linear-prediction estimates have E appearing only in the denominator, the rationale is to reduce the lengths of those E's corresponding to signals and increase those not corresponding to plane-wave signals. The problem is that one does not know, in general, which direction to emphasize; it is these directions that we are trying to determine from the spatial spectra. On the other hand, these directions determine the structure of the spatial correlation matrix, in particular the Eigen structure of matrix [2].

D. Linear Prediction Method

This method estimates the output of one sensor using linear combinations of the remaining sensor outputs and minimizes the mean square prediction error, that is, the error between the estimate and the actual output. Thus, it obtains the array weights by minimizing the mean output power of the array subject to the constraint that the weight on the selected sensor is unity [6]. The linear-predictive method is based on finding the weights a, which minimize the mean-squared prediction error. The linear-predictive spectral estimate commonly used in time series problems can also be used in array processing problems. As before, let Xm0, be the Fourier transform of the output of the moth sensor evaluated at the frequency f0. We assume that this value is estimated by a weighted linear combination of the outputs of the other sensors [2]. The linear prediction (LP) method estimates the output of one sensor using linear combinations of the remaining sensor outputs and minimizes the mean square prediction error, that is, the error between the estimate and the actual output. Thus, it obtains the array weights by minimizing the mean output power of the array subject to the constraint that the weight on the selected sensor is unity [1].

E. Maximum Likelihood Method

Perhaps the most well-known high-resolution array processing algorithm is the so-called maximum-likelihood method first reported by Capon. The derivation of this method does not correspond to the standard approach used in maximum-likelihood estimates. The purpose of the constraint is to fix the processing gain for each directional-look to be unity. Minimizing the resulting beam energy reduces the contributions to this energy from sources and/or noise not propagating in the direction-of-look [2].

These two spectral estimation methods provide spectra having better resolution properties than conventional beamforming. Comparison between these two estimates are often drawn. The maximum-likelihood method is an adaptive beamforming algorithm while linear prediction does not yield weights for beamforming. The linear-predictive method has better resolution properties. However, this increased resolution is accompanied by a ripple in the power estimate PLp(k) when the direction-of-look is not equal to the actual signal bearing [2]. The linear-predictive spectrum will be much greater than the maximum-likelihood estimate when this cosine is small [2]. The MLM estimates the DOAs from a given set of array samples by maximizing the log likelihood function. The likelihood function is the joint probability density function of the sampled data given the DOAs and viewed as a function of the desired variables, which are the DOAs in this case. The method searches for those directions that maximize the log of this function. The ML criterion signifies that plane waves from these directions are most likely to cause the given samples to occur [1]. Maximization of the log-likelihood function is a nonlinear optimization problem, and in the absence of a closed-form solution requires iterative schemes. There are many such schemes available in the literature. The well-known gradient descent algorithm using the estimated gradient of the function at each iteration as well as the standard Newton–Raphson method are well suited for the job [1].

F. Maximum Entropy Method

The maximum entropy (ME) method finds a power spectrum such that its Fourier transform equals the measured correlation subjected to the constraint that its entropy is maximized. The entropy of a Gaussian band-limited time series with power spectrum S(f) [1]. Burg used the principle of maximum entropy to define a class of spectral estimates. In this approach the 2M + 1 correlation values, power spectral estimate is constrained to have a Fourier transform equalling the measured correlation values. Consequently, a set of linear constraints on the spectral estimate P(k) is obtained [2]. The minimization problem defined, may be solved iteratively using the standard gradient LMS algorithm. For more information on various issues of the ME method, see Suitability of the ME method for mobile communications in fast-fading signal conditions has been studied [1].

G. Minimum Variance Distortionless Response Estimator

The minimum variance distortion less response estimator (MVDR) is the maximum likelihood method (MLM) of spectrum estimation [9], which finds the maximum likelihood (ML) estimate of the power arriving from a point source in direction θ assuming that all other sources are interference. In the beamforming literature, it is known as the MVDR beam former as well as the optimal beam former, since in the absence of errors, it maximizes the output SNR and passes the look direction signal undistorted as discussed in for DOA estimation problems, MLM is used to find the ML estimate of the direction rather than the power [9]. Following this convention, the current estimator is referred to as the MVDR estimator [1].

H. Eigen Structure Methods

The eigenvalues of can be divided into two sets when the environment consists of uncorrelated directional sources and uncorrelated white noise. The largest eigenvalues correspond to directional sources, and the eigenvectors associated with these eigenvalues are normally referred to as signal eigenvectors. The – smallest eigenvalues are equal to the background noise power, and the eigenvectors associated with these eigenvalues are known as noise eigenvectors [6]. These methods rely on the following properties of: 1) The space spanned by its eigenvectors may be partitioned into two subspaces, namely, the signal subspace and the noise subspace, and 2) the steering vectors corresponding to the directional sources are orthogonal to the noise subspace. As the noise subspace is orthogonal to the signal subspace, these steering vectors are contained in the signal subspace [1]. Many methods have been proposed that utilize the Eigen structure of the array correlation matrix. These methods differ in the way that available array signals have been utilized, required array geometry, applicable signal model, and so on. Some of these methods do not require explicit computation of the eigenvalues and eigenvectors of the array correlation matrix, whereas in others it is essential. Effective computation of these quantities may be done by methods similar to those described in [1].

I. MUSIC Algorithm

The multiple signal classification (MUSIC) method [10] is a relatively simple and efficient Eigen structure variant of DOA estimation methods. It is perhaps the most studied method in its class and has many variations [1]. Multiple Signal Classification (MUSIC) method [11] is widely used in signal processing applications for estimating and tracking the frequency and emitter location. This method is considered as a generalization of the Pisarenko's one [10]. It is based on spectral estimation which exploits the orthogonality of the noise subspace with the signal subspace. Various estimation algorithms can be used to compute the angle of arrival, but this paper focuses on the most accepted and widely used MUSIC algorithm. The data covariance matrix forms the base of MUSIC algorithm. To find the direction of arrival we need to search through the entire steering vector matrix and then bring out those steering vectors that are exactly orthogonal [7].

J. Spectral MUSIC

In its standard form, also known as spectral MUSIC, the method estimates the noise subspace from available samples. This can be done either by eigenvalue decomposition of the estimated array correlation matrix or singular value decomposition of the data matrix with its N columns being the N array signal vector samples, also known as snapshots. The latter is preferred for numerical reasons [1].

K. Root-MUSIC

For a uniformly spaced linear array (ULA), the MUSIC spectra can be expressed such that the search for DOA can be made by finding the roots of a polynomial. In this case, the method is known as root-MUSIC [23]. Thus, root-MUSIC is applicable when a ULA is used and solves the polynomial rooting problem in contrast to spectral MUSIC's identification and localization of spectral peaks. Root-MUSIC has better performance than spectral MUSIC [1].

L. Constrained MUSIC

This method incorporates the known source to improve estimates of the unknown source direction [23]. The situation arises when some of the source directions are already known. The method removes signal components induced by these known sources from the data matrix and then uses the modified data matrix for DOA estimation. Estimation is achieved by projecting the data matrix onto a space orthogonal complement to a space spanned by the steering vectors associated with known source directions. A matrix operation, the process reduces the signal subspace dimension by a number equal to the known sources and improves estimate quality, particularly when known sources are strong or correlated with unknown sources [1].

M. Beam Space MUSIC

The MUSIC algorithms discussed so far process the snapshots received from sensor elements without any pre-processing, such as forming beams, and thus may be thought of as element space algorithms, which contrasts with the beams space MUSIC algorithm in which the array data are passed through a beamforming processor before applying MUSIC or any other DOA estimation algorithms. The beamforming processor output may be thought of as a set of beams; thus, the processing using these data is normally referred to as beam space processing. A number of DOA estimation schemes are discussed in [16,17], where data are obtained by forming multiple beams using an array. The DOA estimation in beam space has a number of advantages such as reduced computation, improved resolution, reduced sensitivity to system errors, reduced resolution threshold, reduced bias in the estimate, and so on [11, 12, 13, 14, 15]. These advantages arise from the fact that a beam former is used to form a number of beams that are less than the number of elements in the array; consequently, less data to process a DOA estimation are necessary. This process may be understood in terms of array degrees of freedom. Element space methods have degrees of freedom equal to the number of elements in the array, whereas the degrees of freedom of beam space methods are equal to the number of beams formed by the beamforming filter. Thus, the process reduces the array's degrees of freedom. Normally, only M + 1 degrees of freedom to resolve M sources are needed. The root-MUSIC algorithm discussed for the element space case may also be applied to this case, giving rise to beam space root-MUSIC [14, 15]. Computational savings for this method are the same as for beam space methods compared to element space methods in general [1].

N. Improved Music Algorithm

MUSIC algorithm has advantages over other estimation algorithms because of the sharp needle spectrum peaks which can efficiently estimate the independent source signals with high precisions unlike the other estimation processes which are limited with low precisions. It has many practical applications as it provides unbiased estimation results. The MUSIC algorithms to estimate the direction has even proved to have better performance in a multiple signal environment. MUSIC algorithms has better resolution, higher precision and accuracy with multiple signals. But this algorithm achieves high resolution in DOA estimation [17], only when the signals being incident on the sensor array are non-coherent. It losses efficiency when the signals are coherent. Keeping all the parameters same as those used for the conventional MUSIC in all the previous simulations and considering the coherent signals to be incident on the sensor array. As the peaks obtain are not sharp and narrow, they fail to estimate the arrival angle for coherent signals. So we need to move towards an improved MUSIC algorithm to meet the estimation requirements for coherent signals. To improve the results for MUSIC algorithm [18], we simply introduce an identity transition matrix 'T' so that the new received signal matrix. MUSIC algorithm fails to obtain narrow and sharp peaks. An Improved version of the MUSIC algorithm as discussed in this paper can be implemented for coherent signals as well. This improved algorithm achieves sharp peaks and makes the estimation process much accurate [7].

O. Propagator Method

Unlike the MUSIC algorithm, the propagator method is computationally low complex because it does not need Eigen decomposition of the covariance matrix, but it uses the whole of it, to obtain the propagation operator [13, 15]. Therefore, this algorithm is only suitable to the presence of white Gaussian noise and its performance will be degraded in spatial non-uniform coloured noise. In this way propagator is constructed [5]. A generalization propagator method (GPM) is presented. It is the extension of traditional propagator method(PM). In order to make full use of the received data, many propagators are structured according to different block structures of array manifold. By these propagators, a high order matrix is obtained in a symmetric mode, and it is orthogonal with array manifold. Based on this matrix, a generalization spectral function is obtained to solve the problem of direction-of-arrival (DOA) estimation by spectral peak searching. Moreover, in order to avoid spectral peak searching, a generalization root propagator method (GRPM) is also proposed, and shows excellent estimation precision. Numerical simulations demonstrate the performance of the proposed method [8].

P. Min Norm method

Norm method is applicable for ULA, and finds the DOA estimate by searching for peak locations in the spectrum [18, 19]. The minimum norm technique [1, 9] is generally considered to be a high-resolution method which assumes a ULA structure. The algorithm is described as the following. After estimating the cross correlation matrix R ˆ xx. [5]

Q. CLOSEST Method

The CLOSEST method is useful for locating sources in a selected sector. Contrary to beam space methods, which work by first forming beams in selected directions, CLOSEST operates in the element space and in that sense it is an alternative to beam space MUSIC [1]. In a way, it is a generalization of the minimum-norm method. It searches for array weights in the noise subspace that are close to the steering vectors corresponding to DOAs in the sector under consideration, and thus its name. Depending on the definition of closeness, it leads to various schemes. A method referred to as FINE (First Principal Vector) selects an array weight vector by minimizing the angle between the selected vector and the subspace spanned by the steering vectors corresponding to DOAs in the selected sector. In short, the method replaces the vector e1 used in the minimum-norm method by a suitable vector depending on the definition of closeness used. For details about the selection of these vectors and the relative merits of the CLOSEST method [1].

R. ESPRIT Method

Estimation of signal parameters via rotational invariance techniques (ESPRIT) is a computationally efficient and robust method of DOA estimation [20]. It uses two identical arrays in the sense that array elements need to form matched pairs with an identical displacement vector, that is, the second element of each pair ought to be displaced by the same distance and in the same direction relative to the first element. However, this does not mean that one has to have two separate arrays. The array geometry should be such that the elements could be selected to have this property. For example, a ULA of four identical elements with inter-element spacing d may be thought of as two arrays of three matched pairs, one with first three elements and the second with last three elements such that the first and the second elements form a pair, the second and the third elements form another pair, and so on. The two arrays are displaced by the distance d. The way ESPRIT exploits this subarray structure for DOA estimation is now briefly described [1].

S. Weighted Subspace Fitting Method

The weighted subspace fitting (WSF) method is a unified approach to schemes such as MLM, MUSIC, and ESPRIT [22, 23]. It requires that the number of directional sources be known. The method finds the DOA such that the weighted version of a matrix whose columns are the steering vectors associated with these directions is close to a data dependent matrix. The data-dependent matrix could be a Hermitian square root of the array correlation matrix or a matrix whose columns are the eigenvectors associated with the largest eigenvalues of the array correlation matrix. The framework proposed in the method can be used for deriving common numerical algorithms for various Eigen structure methods as well as for their performance studies [1].

III.

Property	Comparison
Bias	Biased, Bartlett > LP > MLM
Resolution	Depends on array aperture
Sensitivity	Robust to element position errors
Array	General array

Table .1. Bartlett Method [1]

Property	Comparison
Bias	Unbiased
Variance	Minimum
Resolution	MVDR > Bartlett Does not have best resolution of any method
Array	General array

Table .2. MVDR Method [1]

Property	Comparison
Bias	Biased
Resolution	ME > MVDR > Bartlett Can resolve at lower SNR than Bartlett

Table .3. Maximum Entropy Method [1]

Property	Comparison
Bias	Unbiased Less than LP, Bartlett, MUSIC

Variance	• Less than MUSIC for small samples Asymptotically efficient for random signals • Not efficient for finite samples • Less efficient for deterministic signals than random signals • Asymptotically efficient for deterministic signals using very large array
Computation	Intensive with large samples
Performance	• Same for deterministic and random signals for large arrays • Applicable for correlated arrivals Works with one sample

Table .4. ML Method [1]

Property	Comparison
Bias	Biased
Variance	• Less than ESPRIT and TAM for large samples, minimum norm Close to MLM, CLOSEST, FINE • Variance of weighted MUSIC is more than unweighted MUSIC • Asymptotically efficient for large array
Resolution	Limited by bias
Array	Applicable for general array Increasing aperture makes it robust
Performance	Fails to resolve correlated sources
Computation	Intensive
Sensitivity	• Array calibration is critical, sensitivity to phase error depends more on array aperture than number of elements, pre-processing can improve resolution Correct estimate of source number is important • MSE depends on both gain and phase errors and is lower than for ESPRIT Increase in gain and phase errors beyond certain value causes an abrupt deterioration in bias and variance

Table .5. Element Space MUSIC Method [6]

Property	Comparison
Bias	Less than element space MUSIC
Variance	Larger than element space MUSIC
RMS Error	Less than ESPRIT, minimum norm
Resolution	• Similar to beam space minimum norm, CLOSEST • Better than element space MUSIC, element space minimum norm • Threshold SNR decreases as the separation between the sources increases
Computation	Less than element space MUSIC
Sensitivity	More robust than element space MUSIC

Table .6. Beam Space MUSIC Method [6]

Property	Comparison
Variance	Less than root minimum norm, ESPRIT
RMS Error	Less than LS ESPRIT
Resolution	Beam space root-MUSIC has better probability of resolution than beam space MUSIC
Array	Equispaced linear array
Performance	• Better than spectral MUSIC • Similar to TLS ESPRIT at SNR lower than MUSIC threshold • Beam space root-MUSIC is similar to element space root MUSIC

Table .7. Root-MUSIC Method [6]

Property	Comparison
Bias	Less than MUSIC
Resolution	Better than CLOSEST, element space MUSIC
Method	Equivalent to TLS

Table .9. CLOSEST Method [1]

Property	Comparison
Bias	TLS ESPRIT unbiased LS ESPRIT biased
RMS Error	Less than minimum norm TLS similar to LS
Variance	Less than MUSIC for large samples and difference increases with number of elements in array

Computation	• Less than MUSIC • Beam space ESPRIT needs less computation than beam space root-MUSIC and ES ESPRIT
Method	LS ESPRIT is similar to TAM
Array	Needs doublets, no calibration needed
Performance	• Optimum-weighted ESPRIT is better than uniform-weighted ESPRIT • TLS ESPRIT is better than LS ESPRIT
Sensitivity	• More robust than MUSIC and cannot handle correlated sources • MSE robust for sensor gain errors • MSE is lowest for maximum overlapping subarrays under sensor perturbation

Table .10. ESPRIT Method [1]

Property	Comparison
Bias	Less than MUSIC
Resolution	Better than MUSIC and minimum norm
Variance	Less than minimum norm
Performance	Good at low SNR

Table .11. FINE Method [1]

IV.

In this paper, real life scenario is simulated by studying the performance of each method based on the noise environment by testing with SNR1 = 1 dB (high noise level) and SNR2 = 20 dB (low noise level).

It should be noted that all the methods are computed using MATLAB and the results are plotted in decibel using the formula

$$P(dB) = 10\log_{10}\left(\frac{spectrum}{Max[spectrum]}\right),$$

logical order as described in this paper, computed for two values of SNR:

$$RMSE(\hat{P}(\theta), P(\theta)) = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(\hat{P}(\theta_n) - P(\theta_n))^2}.$$

represent the RMSE between each method and the true spectrum for SNR = 1 dB and SNR = 20 dB respectively.

METHOD	RMSE
Bartlett spectrum	0.40
Capon beam former spectrum	0.27
Linear prediction spectrum	0.13
Maximum entropy spectrum.	0.075
Minimum norm spectrum	0.06
MUSIC spectrum	0.10
Propagator spectrum	0.075

Table .12. RMSE, SNR = 1 dB [5]

METHOD	RMSE
Bartlett spectrum	0.39
Capon beam former spectrum	0.10
Linear prediction spectrum	0.065
Maximum entropy spectrum.	0.05
Minimum norm spectrum	0.06
MUSIC spectrum	0.04
Propagator spectrum	0.045

Table .13. RMSE, SNR = 20 dB [5].

V. CONCLUSION

In this paper some algorithms for one dimensional and two dimensional direction of arrival (DOA) estimation in stationary case for smart antennas and spatially uniform AWGN was compared, starting with Bartlett method. In order to compare the performance of different algorithms and techniques of direction of arrival. SNR of 1dB and 20dB was considered. The results with high level and low level noise was displayed using bar graph. In the comparative study the authors have tried to make a comparison between the different types of one dimensional narrow band and two dimensional wide band sources with different algorithms and techniques.

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