

Fuzzy Number Analysis in Neural Networks

S. Jackson¹, S.Manicka Vinayagam², S.Sundar Raj³

¹ Assistant Professor, ^{2,3} PG student

^{1,2,3} P.G& .Research Department of Mathematics, V.O.Chidambaram College, Thoothukudi, India

Abstract- Fuzzy set theory, which was first proposed by the researcher Zadeh (1965), has become a very important tool to solve problems and it provides an appropriate framework for representing vague concepts by allowing partial membership. Neural Networks is an interconnected network that resembles human brain, used to be a key segment of mathematical education. Artificial neural network are widely used as an effective approach for handling non-linear and noisy data, especially in situations where the physical process relationships are not fully understood and they are also particularly well suited to modeling complex systems on a real time basis. Numerous models have been created in the literature for the description of the neural system. In this paper, we utilize pentagonal fuzzy number to pick the best machine for work by Feed-Forward Neural Network (FFNN).

Keywords: Fuzzy Number, Pentagonal Fuzzy Number, Feed-Forward Neural Network.

1. INTRODUCTION

Artificial neural networks (ANN) are powerful tools that can be used to manage knowledge and solve problems. They are information processing systems that reproduce by computer the function of a very simplified biological neural network, composed of a certain number of interconnected neurons. Intelligent behavior springs from appropriate interactions between interconnected units. Feed-forward networks involve connections that always move towards the networks output, i.e., there are no feedback loops. The nodes of a neural network are most often organized into layers where connections exist only between nodes in adjacent layers. A fully connected network contains links between all nodes in adjacent layers. A multi layer network contains at least one layer between the network input and output, i.e., inputs are not fed directly to output nodes. Figure 1 illustrates a fully connected two input, Single-output, feed-forward, multi layer network with a single hidden layer consisting of three nodes

There are three basically types of fuzzy neural networks depending on the type of fuzzification of inputs, outputs and weights (including biases): fuzzy weights and crisp inputs, crisp weights and fuzzy inputs and fuzzy weights and fuzzy inputs. In what follows we consider the most complete fuzzification of neural networks: fuzzy inputs, fuzzy weights and fuzzy outputs. According to Faqs. Organisation [2010] activations function are needed for hidden layer of the NN to introduce non-linearity. Without them NN would be same as plain perceptions. If linear functions were used, NN would not be as powerful as they are. Activation function can be linear, threshold or sigmoid function. Sigmoid activation function is usually used for hidden layer because it combines

nearly linear behaviour, curvilinear behaviour and nearly constant behaviour depending on the input value. Layer SUM is collection of the output nodes from hidden layer that have been multiplied by connection weights, added to get single number and put through sigmoid function (activation function). Inputs to sigmoid is any value between negative infinity and positive infinity number while the output can only be a number between 0 and 1.

2. PRELIMINARIES:

Definition 2.1 [3]: Fuzzy number

- (i) $\mu_A(x_0)$ is piecewise continuous
- (ii) There exist at least one $x_0 \in \mathbb{R}$ with $\mu_A(x_0) = \bar{1}$
- (iii) μ_A must be normal and convex

Definition 2.2 [5]: Pentagonal fuzzy number:

A pentagonal fuzzy number of a fuzzy set A is defined as $A = (a_1, a_2, a_3, a_4, a_5)$ and its membership is given by,

$$\mu_A(x) = \begin{cases} 0 & \text{for } x \leq a_1 \\ \frac{x-a_1}{a_2-a_1} & \text{for } a_1 \leq x \leq a_2 \\ \frac{x-a_2}{a_3-a_2} & \text{for } a_2 \leq x \leq a_3 \\ 1 & \text{for } x = a_3 \\ \frac{a_4-x}{a_4-a_3} & \text{for } a_3 \leq x \leq a_4 \\ \frac{a_5-x}{a_5-a_4} & \text{for } a_4 \leq x \leq a_5 \\ 0 & \text{for } x \geq a_5 \end{cases}$$

Definition 2.3 [5]: Pentagonal fuzzy number matrix

The elements of pentagonal fuzzy number matrix is $A = (a_{ij})_{n \times n}$ $a_{ij} = (a_{ijL}, a_{ijM}, a_{ijN}, a_{ijR}, a_{ijs})$ be the ij^{th} element of A .

Definition 2.4 [5]: Pentagonal fuzzy membership matrix

The membership function of $a_{ij} = (a_{ijL}, a_{ijM}, a_{ijN}, a_{ijR}, a_{ijs})$ is defined as

$$\left[\frac{a_i, a_{ijM}, a_{ijN}, a_{ijR}, a_{ijs}}{10 \quad 10 \quad 10 \quad 10 \quad 10} \right], \text{ If } 0 \leq a_{ijL} \leq a_{ijM} \leq a_{ijN} \leq a_{ijR} \leq a_{ijs} \leq 1 \text{ where,}$$

$$0 \leq \frac{a_{ijL}}{10} \leq \frac{a_{ijM}}{10} \leq \frac{a_{ijN}}{10} \leq \frac{a_{ijR}}{10} \leq \frac{a_{ijs}}{10} \leq 1$$

Definition 2.5 [3]: Addition

Let $A = (a_{ij})_{m \times n}$ and $B = (b_{ij})_{m \times n}$ are two pentagonal fuzzy number matrices of same order $m \times n$. $(A+B) = (a_{ij} + b_{ij})_{m \times n}$ where $(a_{ij} + b_{ij}) = (a_{ijL} + b_{ijL}, a_{ijM} + b_{ijM}, a_{ijN} + b_{ijN}, a_{ijR} + b_{ijR}, a_{ijs} + b_{ijs})$ is the ij^{th} element of $(A+B)$.

Definition 2.6 [3]: Subtraction

$(A-B) = (a_{ij} - b_{ij})_{m \times n}$ where $(a_{ij} - b_{ij}) = (a_{ijL} - b_{ijL}, a_{ijM} - b_{ijM}, a_{ijN} - b_{ijN}, a_{ijR} - b_{ijR}, a_{ijs} - b_{ijs})$ is the ij^{th} element of $(A-B)$.

Definition 2.7 [3]: (Multiplication of Two Pentagonal Fuzzy Numbers)

If $\tilde{A}_p = (a_1, b_1, c_1, d_1, e_1)$; $\tilde{O} = (a_2, b_2, c_2, d_2, e_2)$. Then $\tilde{A}_p * \tilde{O} = (a_1 * a_2, b_1 * b_2, c_1 * c_2, d_1 * d_2, e_1 * e_2)$.

Definition 2.8 [5]: (Construction of Pentagonal Fuzzy Number): The pentagonal fuzzy number is represented by the five parameters such as a, b, c, d and e , where a and b denote the smallest possible values, c the most promising value and d, e the largest possible value. Formula to generate fuzzy pentagonal number defined as follows

$$\tilde{A} = (a-2, a-1, a, a+1, a+2), \text{ for all } a = 3, 4, 5, 6, 7$$

Since fuzzy number scale is defined from 1 to 9.

3. PROCEDURE

STEP 1: Gather the imprecise estimation needed for the problem which is in the form of pentagonal fuzzy number.

STEP 2: Convert the element of pentagonal fuzzy number matrix into its membership function by using (Definition 2.3).

STEP 3: Set the pentagonal fuzzy number is weight of the FFN.

STEP 4: Assume the input value 0 and 1.

STEP 5: Calculate the weighted sums $m = \sum x_i w_i$

STEP 6: The output of a neuron (s) is a function of the weight sum $S = f(m)$

STEP 7: Calculated sigmoid function by $f(m) = 1.0 / (1.0 + \exp(-m))$

STEP 8: Determine the minimum value of $f(m)$.

4. FUZZY NUMBER ANALYSIS IN NEURAL NETWORKS:

Suppose there are three weaving machine m_1, m_2, m_3 . Let the possible attributes to the above machines $w = \{a, b, c, d, e\}$ as universal set, where a, b, c, d, e represents the time

period, power consumption, spinning & weaving, maintenances & servicing and dying & finishing respectively. Estimate the pentagonal fuzzy number in three machine m_1, m_2, m_3 by considering to complete work.

- STEP 1: $w_1 = (76, 60, 80, 74, 93)$
 $w_2 = (76, 73, 80, 83, 95)$
 $w_3 = (68, 70, 53, 68, 49)$
 $w_4 = (58, 73, 71, 62, 63)$
 $W_5 = (76, 70, 76, 71, 83)$
 $W_6 = (78, 73, 80, 80, 82)$
 $W_7 = (68, 63, 80, 74, 71)$
 $W_8 = (76, 73, 80, 80, 76)$
 $W_9 = (86, 83, 80, 83, 80)$
 $W_{10} = (74, 87, 70, 77, 87)$

STEP 2 : Convert the pentagonal fuzzy number into membership function.

- $w_1 = (0.76, 0.60, 0.80, 0.74, 0.93)$
 $w_2 = (0.76, 0.73, 0.80, 0.83, 0.95)$
 $w_3 = (0.68, 0.70, 0.53, 0.68, 0.49)$
 $w_4 = (0.58, 0.73, 0.71, 0.62, 0.63)$
 $W_5 = (0.76, 0.70, 0.76, 0.71, 0.83)$
 $W_6 = (0.78, 0.73, 0.80, 0.80, 0.82)$
 $W_7 = (0.68, 0.63, 0.80, 0.74, 0.71)$
 $W_8 = (0.76, 0.73, 0.80, 0.80, 0.76)$
 $W_9 = (0.86, 0.83, 0.80, 0.83, 0.80)$
 $W_{10} = (0.74, 0.87, 0.70, 0.77, 0.87)$

STEP 3: Considered the above pentagonal fuzzy number is fuzzy weight w_{ij}

STEP 4: Assume input value

- a) $x = (0, 0, 1, 1, 1)$, $x_1 = 0, x_2 = 0, x_3 = 1, x_4 = 1, x_5 = 1$.
 b) $x = (1, 1, 1, 0, 0)$, $x_1 = 1, x_2 = 1, x_3 = 1, x_4 = 0, x_5 = 0$.
 c) $x = (0, 1, 1, 1, 0)$, $x_1 = 0, x_2 = 1, x_3 = 1, x_4 = 1, x_5 = 0$.

STEP 5: Calculate the weighted sum $m_i = \sum w_{ij} x_j$

- $m_1 = 0 + 0 + 0.8 + 0.74 + 0.93 = 2.47$
 $m_2 = 0 + 0 + 0.8 + 0.83 + 0.95 = 2.58$
 $m_3 = 0 + 0 + 0.53 + 0.68 + 0.49 = 1.7$
 $m_4 = 0 + 0 + 0.71 + 0.62 + 0.63 = 1.96$
 $m_5 = 0 + 0 + 0.76 + 0.71 + 0.83 = 2.3$
 $m_6 = 0 + 0 + 0.8 + 0.8 + 0.82 = 2.42$
 $m_7 = 0 + 0 + 0.8 + 0.74 + 0.71 = 2.25$
 $m_8 = 0 + 0 + 0.8 + 0.8 + 0.76 = 2.36$
 $m_9 = 0 + 0 + 0.8 + 0.83 + 0.8 = 2.43$
 $m_{10} = 0 + 0 + 0.7 + 0.77 + 0.87 = 2.34$
 b) $m_1 = 0.76 + 0.60 + 0.80 + 0 + 0 = 2.16$
 $m_2 = 0.76 + 0.73 + 0.80 + 0 + 0 = 2.29$
 $m_3 = 0.68 + 0.70 + 0.53 + 0 + 0 = 1.91$
 $m_4 = 0.58 + 0.73 + 0.71 + 0 + 0 = 2.02$
 $m_5 = 0.76 + 0.70 + 0.76 + 0 + 0 = 2.22$
 $m_6 = 0.78 + 0.73 + 0.80 + 0 + 0 = 2.31$
 $m_7 = 0.68 + 0.63 + 0.80 + 0 + 0 = 2.11$
 $m_8 = 0.76 + 0.73 + 0.80 + 0 + 0 = 2.29$
 $m_9 = 0.86 + 0.83 + 0.80 + 0 + 0 = 2.49$
 $m_{10} = 0.74 + 0.87 + 0.70 + 0 + 0 = 2.31$
 c) $m_1 = 0 + 0.6 + 0.8 + 0.74 + 0 = 2.14$
 $m_2 = 0 + 0.73 + 0.80 + 0.83 + 0 = 2.36$
 $m_3 = 0 + 0.7 + 0.53 + 0.68 + 0 = 1.91$

$$\begin{aligned}
 m_4 &= 0 + 0.73 + 0.71 + 0.62 + 0 = 2.06 \\
 m_5 &= 0 + 0.7 + 0.76 + 0.71 + 0 = 2.17 \\
 m_6 &= 0 + 0.73 + 0.8 + 0.8 + 0 = 2.33 \\
 m_7 &= 0 + 0.63 + 0.8 + 0.74 + 0 = 2.17 \\
 m_8 &= 0 + 0.73 + 0.8 + 0.8 + 0 = 2.33 \\
 m_9 &= 0 + 0.83 + 0.8 + 0.83 + 0 = 2.46 \\
 m_{10} &= 0 + 0.87 + 0.7 + 0.77 + 0 = 2.34
 \end{aligned}$$

STEP 6: The output neuron is $s_i = f(m_i)$

STEP 7: Calculate the sigmoid function

6. REFERENCES:

- [1] Baruah.H.K, Towards forming a field of fuzzy sets, international Journal of energy, Information and communications, 2(1)(2011), 16-20.
- [2] Carling.A, Introducing Neural Network, SIGMA PRESS-Wilmslow, United Kingdom, (1992).
- [3] Dubois.dandPrade.H, Operations on Fuzzy numbers, International Journal of systems science, 9, 613-626.
- [4] Ishibuchi. H,Okada.H, Tanaka.H, "Fuzzy neural networks with fuzzy weights and fuzzy biases", Proc. of ICNN'93, pp. 1650-1655, 1993.
- [5] Pathinathan.T and Ponnivalaan.K, Pentagonal Fuzzy number, International Journal of Computing Algorithm, 03(2014).
- [6] Hagan.T Martin and Mohammad Menhaj.b, Training Feedforward Networks with the Marquardt Algorithm, IEEE Transactions on Neural Networks, 5(6)(1994).
- [7] Zadeh.L.A, Fuzzy sets, Information and control, (1965), 338-353

a	b	c
$f(m_1) = f(2.47) =$ 0.922	$f(m_1) = f(2.16) =$ 0.8966	$f(m_1) = f(2.14) =$ 0.8947
$f(m_2) = f(2.58) =$ 0.9296	$f(m_2) = f(2.29) =$ 0.9081	$f(m_2) = f(2.36) =$ 0.9137
$f(m_3) = f(1.7) =$ 0.8455	$f(m_3) = f(1.91) =$ 0.871	$f(m_3) = f(1.91) =$ 0.871
$f(m_4) = f(1.96) =$ 0.8765	$f(m_4) = f(2.02) =$ 0.8829	$f(m_4) = f(2.06) =$ 0.887
$f(m_5) = f(2.3) =$ 0.9089	$f(m_5) = f(2.22) =$ 0.902	$f(m_5) = f(2.17) =$ 0.8975
$f(m_6) = f(2.42) =$ 0.9183	$f(m_6) = f(2.31) =$ 0.9097	$f(m_6) = f(2.83) =$ 0.9113
$f(m_7) = f(2.25) =$ 0.9047	$f(m_7) = f(2.11) =$ 0.8919	$f(m_7) = f(2.17) =$ 0.8975
$f(m_8) = f(2.36) =$ 0.9137	$f(m_8) = f(2.29) =$ 0.9081	$f(m_8) = f(2.33) =$ 0.9113
$f(m_9) = f(2.43) =$ 0.9191	$f(m_9) = f(2.49) =$ 0.9234	$f(m_9) = f(2.46) =$ 0.9213
$f(m_{10}) = f(2.34) =$ 0.9121	$f(m_{10}) = f(2.31) =$ 0.9097	$f(m_{10}) = f(2.34) =$ 0.9121

STEP 8: Determine the minimum value of $f(m_i)$

The minimum value is 0.8455.

The minimum value is 0.871.

The minimum value is 0.871.

CONCLUSION :

In all 3 cases m_3 is minimum. So, m_3 is the best machine