On $\hat{P}g$ Closed Sets

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Abstract- **Dunham [3]explored the properties of T½spaces and defined a new closure operator cl* by using generalized closed sets which was introduced by famous topologist Levine. S.Pious Missier and S.Jackson [7]cleared another pathway by introducing a new notion of generalized closed sets called $\hat{P}g$ closed sets. This paper is devoted to $\hat{P}g$derived set,$\hat{P}g$exterior,$\hat{P}g$Frontier and $\hat{P}g$Border of a subset of a topological space. We investigate the fundamental properties of the above speculations and explore the inner relationship between them.**

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INTRODUCTION

The concept of generalized Closed sets introduced by Levine[4] plays a significant role in General Topology. This notion has been studied extensively in recent years by many topologists. The investigation of generalized Closed sets has led to several new and interesting concepts. Dunham [3] further investigated the Properties of $T_{½}$ spaces and defined a new Closure operator Cl* by using generalized Closed sets. In 1996, H.Maki, J. Umehara and T. Noiri [5] introduced the Class of Pre generalized Closed sets and used them to obtain Properties of Pre-$T_{1/2}$ spaces. The modified forms of generalized closed sets and generalized continuity were studied by K. Balachandran, P. Sundaram and H. Maki [1].

M.K.R.S.Veerakumar et.al[11] introduced a new Classes of Open sets namely g* - Closed sets .This characterization paved a new pathway. Dr.S.Pious missier and S.Jackson[7] introduced a new class of generailsed closed sets called $\hat{p}g$ closed sets. This paper is devoted to $\hat{P}g$derived set,$\hat{P}g$exterior,$\hat{P}g$Frontier and $\hat{P}g$Border of a subset of a topological space. We investigate the fundamental properties of the above speculations and explore the inner relationship between them. Throughout this paper, spaces (X , τ),(Y, σ), (Z,η) always mean topological space on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X. The Closure of A and the interior of A are denoted by Cl(A) and Int(A) respectively.

2.PRELIMINARIES:

Definition2.1: - A subset A of a topological space (X, τ) is called

(i) a **Pre-Open** set[5] if A ⊆ int(Cl(A)) and a **Pre-Closed** set if Cl(int(A)) ⊆ A.

(ii) a **g closed set** [4] if Cl(A) ⊆U whenever A ⊆ U , U is Open in (X, τ).The complement of g closed set is **g open**.

(iii) a **Pre*-Open** set[10] if A ⊆ int*(Cl(A)) and a **Pre*-Closed** set if Cl*(int(A)) ⊆ A

(iv) a **$\hat{p}g$ closed set** [7] P*Cl(A) ⊆ U whenever A ⊆ U , U is **Pre*-Open** in (X, τ).

Definition 2.2. Let A be a subset of X.

(i) The **interior** of A is defined as the union of all open subsets of A and is denoted by Int(A).

(ii) The **closure** of A is defined as the intersection of all closed sets containing A and is denoted by Cl(A) .

(iii) The **border of A** is defined as Bd(A)=A\Int(A).

(iv) The **frontier of A** is defined as Fr(A)=Cl(A)\Int(A).

(v) The **exterior of A** is defined as Ext(A)=Int(x\A).

Definition 2.3. Let A be a subset of X.

(i) The **pre-interior** (respectively generalized interior and pre*-interior) of A is defined as the union of all pre-open (respectively g-open and pre*-open) subsets of A and is denoted by pInt(A) (respectively Int*(A) and p*Int(A)).

(ii) The **pre-closure** (respectively generalized closure and pre*-closure) of A is defined as the intersection of all pre-closed (respectively g-closed and pre*-closed) sets containing A and is denoted by pCl(A) (respectively Cl*(A) and p*Cl(A)).

Defnition2.4:

The **pre generalized-border** pgBd(A), the **pre generalized-frontier** pgFr(A) and the **pregeneralied-exterior** pgExt(A) of a subset A are defined analogously by replacing Int(A) and Cl(A) respectively by pgInt(A) and pgcl(A) in the definitions of the border, the frontier and the exterior of A.

Definition 2.5:

Let A be a subset of X. A point x in X is a **$\hat{p}g$ -interior point** of A if A contains a pre*-open set containing x.

Definition 2.6:

Let A be a subset of X. A point x in X is a **$\hat{p}g$-limit point** of A if every $\hat{p}g$-open set containing x intersects A in a point different from x.

Definition 2.7: [7]

Let A be a subset of X.

i. The intersection of all $\hat{P}g$ Closed sets containing A is called **$\hat{P}g$ Closure** of A and denoted by P̂g Cl(A). P̂g Cl(A) = ∩{F: A ⊆ F and F∈P̂g C(X)}

ii. The union of all P̂g Open sets contained in A is called **P̂g interior** of A and denoted by P̂gInt(A).P̂g int(A) = ∪{V: V ⊆ A and V∈P̂g O(X)}.

Theorem 2.8. [7] Let A be a subset of X. Then

i. A is P̂g Closed if and only if P̂g Cl(A)=A.

ii. A is P̂g Open if and only if P̂g Int(A)=A.

Theorem 2.9:[7] Let A and B be subsets of (X, τ). Then the following results hold. (i) P̂g Cl(φ)=φ and P̂g Cl(X)=X.

(ii) A⊆P̂g Cl(A).

(iii) If A⊆B, then P̂g Cl(A)⊆ P̂g Cl(B).

(iv) A⊆P̂g Cl(A)⊆PCl(A)⊆Cl(A).

(v) P̂g Cl(P̂g Cl(A))= P̂g Cl(A)

Theorem 2.10:[7] Let A and B be subsets of (X, τ). Then the following results hold.

(i) P̂g int(φ)=φ and P̂g int(X)=X.

(ii) P̂g int(A) ⊆ A.

(iii) If A⊆B, thenP̂g int(A) ⊆ P̂g int(B).

(iv) A⊆int(A) ⊆ Pint(A) ⊆ P̂g int(A).

(v) P̂g Int(P̂g Int(A))= P̂g Int(A)

3. MAIN RESULTS

Definition 3.1. The set of all p̂g-limit points of A is called **p̂g-Derived set of A** and is denoted by $D_{\hat{p}g}$ [A].

Theorem 3.2. In any topological space (X,τ), The following hold.

If A and B are subsets of X,

(i) $D_{\hat{p}g}$ [A] ⊆ Dpg [A] ⊆ D[A].

(ii)A⊆B⇒ $D_{\hat{p}g}$ [A] ⊆ $D_{\hat{p}g}$ [B].

(iii) $D_{\hat{p}g}$ [$D_{\hat{p}g}$ [A]]\A⊆ $D_{\hat{p}g}$ [A].

(iv) $D_{\hat{p}g}$ [AUB]⊇ $D_{\hat{p}g}$ [A]U $D_{\hat{p}g}$ [B].

(v) $D_{\hat{p}g}$ [A∩B]⊆ $D_{\hat{p}g}$ [A] ∩ $D_{\hat{p}g}$ [B].

(vi) $D_{\hat{p}g}$ [AU $D_{\hat{p}g}$ [A]] ⊆AU $D_{\hat{p}g}$ [A].

(vii)p̂gInt(A)=A\ $D_{\hat{p}g}$ [X\A].

(viii) p̂gCl(A)= AU $D_{\hat{p}g}$ [A].

Proof:

(i) follows from the fact every open set(pre generalized open set) is p̂g open.

(ii) follows from Definition 3.1.

(iii) Let x∈ $D_{\hat{p}g}$ [$D_{\hat{p}g}$ [A]]\A and U be a p̂g − open set containing x. Then U∩ ($D_{\hat{p}g}$ [A]\{x})≠ϕ. Let y∈U∩ ($D_{\hat{p}g}$ [A]\{x}). Then x≠y∈U and y∈$D_{\hat{p}g}$ [A] and hence $D_{\hat{p}g}$ [$D_{\hat{p}g}$ [A]]\A⊆ $D_{\hat{p}g}$ [A].

(iv) and (v) follows from (ii) .

(vi) follows from (iii).

(vii) Let x∈ A\$D_{\hat{p}g}$ [X\A]. Then x ∉ $D_{\hat{p}g}$ [X\A]. which implies that there is a p̂g-open set U containing x such that U∩[(X\A)\{x}]≠ϕ. Since x∈A, U∩(X\A)=ϕ which implies U⊆A. Then x∈U⊆A and hence x∈p̂gInt(A). On the other hand if x∈p̂gInt(A) then p̂gInt(A) is a p̂g-open set containing x and contained in A. Hence p̂gInt(A) ∩(X\A)=ϕ which implies that x∉ $D_{\hat{p}g}$ [X\A]. Thus p̂gInt(A)⊆A\ $D_{\hat{p}g}$ [X\A].

(viii) follows from the definition 3.1 and the fact that every p̂g − open set U containing x intersects A if x∈ $D_{\hat{p}g}$ [A).

Definition 3.3. If A is a subset of X, then the **p̂g-border of A** is defined by p̂gBd(A)=A\p̂gInt(A).

Theorem 3.4: If A is a subset of X, then

i. $(\hat{p}gInt(A))^C$ = $\hat{p}gCl(A^C)$.

ii. p̂gInt(A) = $(\hat{p}gCl(A^C))^C$.

iii. p̂gCl(A) = $(\hat{p}gInt(A^C))^C$.

Proof:

i. Let x ∈ $(\hat{p}gInt(A))^C$,then x ∉ p̂gInt(A) thus every p̂g open set U containing x U∩ A^C≠ϕ.Then by the definition of p̂g Closure x ∈ $\hat{p}gCl(A^C)$.Therefore, $(\hat{p}gInt(A))^C$ ⊆ $\hat{p}gCl(A^C)$.Conversely, x ∈ $\hat{p}gCl(A^C)$. Then for every p̂g open set U containing x U∩ A^C≠ϕ.Thus by the definition of p̂g Interior , x ∉ p̂gInt(A) .Then, x ∈ $(\hat{p}gInt(A))^C$. Therefore, $\hat{p}gCl(A^C)$ ⊆ $(\hat{p}gInt(A))^C$.Then by combining the sufficient and necessity we get $(\hat{p}gInt(A))^C$ = $\hat{p}gCl(A^C)$.

ii. Follows by taking complements in (i).

iii. Follows by replacing A by A^C in (i).

Theorem 3.5: In any topological space (X, τ) the following hold.

(i)p̂gBd(ϕ)=ϕ.

(ii) p̂gBd(X)=ϕ.

If A is a subset of X,

(iii) p̂g Bd(A) ⊆A.

(iv)A=p̂gInt(A)Up̂g Bd(A).

(v) p̂gInt(A)∩ p̂g Bd(A)= ϕ.

(vi) p̂gInt(p̂g Bd(A)) =ϕ.

(vii) A is p̂g-open if and only if p̂gBd(A)=ϕ.

(viii) p̂gBd(p̂gInt(A))=ϕ.

(ix) p̂g Bd(p̂g Bd(A))= p̂g Bd(A).

(x) p̂g Bd(A)=A∩ p̂gCl(X\A).

(xi) p̂g Bd(A)=A∩ $D_{\hat{p}g}$ [X\A].

Proof:

(i), (ii), (iii), (iv) and (v) follow from Definition 3.3.

(vi) If possible, let x∈p̂gInt(p̂g Bd(A)). The x∈p̂g Bd(A). Since p̂g Bd(A) ⊆A, x∈p̂gInt(p̂g Bd(A))⊆ p̂g Int(A). This implies x∈p̂gInt(A) ∩ p̂g Bd(A) which contradicts (v).

(vii) follows from Definition 3.3 and Theorem 2.8.

(viii) follows from (vii) and the fact that ∩ p̂g Int(A) is p̂g-open.

(ix) follows from (vi).

(x) follows from Definition 3.3 and theorem 3.4.

(xi)from (x) and the fact p̂g Bd(A)=A∩ p̂gCl(X\A)=A∩((X\A)U$D_{\hat{p}g}$ [X\A])= A∩ $D_{\hat{p}g}$ [X\A].

Definition 3.6: If A is a subset of X then the **p̂g-frontier** of A is defined by p̂g Fr(A)=p̂gCl(A)\ p̂g Int(A).

Theorem 3.7: If A is a subset of a space X, the following hold.
(i) p̂g Fr(ϕ) = ϕ.
(ii) p̂g Fr(X)=ϕ.
(iii) p̂g Cl(A)=p̂gInt(A)U p̂g Fr(A).
(iv) p̂g Int(A) ∩ p̂g Fr(A)=ϕ.
(v) p̂g Bd(A) ⊆p̂g Fr(A)⊆ p̂gCl(A).
(vi) A is p̂g-closed if and only if A=p̂gInt(A)U p̂gFr(A).
(vii) p̂g Fr(A) ⊆ pgFr(A) ⊆Fr(A).
(viii) p̂g Fr(A)= p̂g Cl(A) U p̂gCl(X\A).
(ix) p̂gFr(A) is a p̂g-closed and hence p̂gCl(p̂gFr(A))= p̂gFr(A).
(x) p̂gFr(A)= p̂gFr(X\A).
(xi) p̂gFr(A)⊆ p̂gBd(A)U $D_{\hat{p}g}$ [A].
(xii) A is p̂g-closed if and only if p̂gFr(A) = p̂gBd(A). Hence A is p̂g-closed if and only if A contains its p̂g-frontier.
(xiii) A is p̂g-regular if and only if p̂gFr(A)=ϕ
(xiv) If A is p̂g-open, then p̂gFr(A) ⊆ $D_{\hat{p}g}$ [A].
(xv) p̂gFr(p̂gInt(A))⊆ p̂gFr(A).
(xvi) p̂gFr(p̂gCl(A)) ⊆ p̂gFr(A).
(xvii) p̂gFr(p̂gFr(A)) ⊆ p̂gFr(A).
(xviii) X=p̂gInt(A) U p̂gInt(X\A) U p̂gFr(A).

Proof:
(i), (ii), (iii), (iv) and (v) follow from Definition 3.6.
(vi) follows from (iii) and Theorem 2.8(i).
(vii). follows from the fact every open set(pre generalized open set) is p̂g open.
(viii) follows from Definition 3.6 and Theorem 3.4(i).
(ix) follows from the fact that p̂gCl(A) is p̂g-closed and (viii) .
(x) follows from (viii) .
(xi) follows from p̂gCl(A)=AU$D_{\hat{p}g}$ [A]= p̂gInt(A)U p̂gBd(A)U $D_{\hat{p}g}$ [A].
(xii) follows from the fact that A is p̂g-closed if and only if p̂gCl(A)=A and from p̂gBd(A) ⊆A. (xiii) follows from Definition 2.7 ,3.6 and the fact that A set is p̂g-regular if and only if it is both p̂g Closed and p̂g- Open.
(xiv) follows from Theorem 3.2 (viii).
(xv) follows from Definition 3.6 and Theorem 2.10(v).
(xvi) follows from Definition 3.6 and Theorem 2.9(v).
(xvii) follows from Definition 3.6 and the fact that p̂gFr(A) is p̂g-closed.
(xviii) follows from (iii) and Theorem 3.4(iii).

Theorem 3.8: If A is a subset of X, then p̂gFr(p̂g Fr(p̂g Fr(A)))= p̂g Fr(p̂g Fr(A)).

Proof. From Theorem 3.5((vii) and (ix)) we have
p̂g Fr(p̂gFr(p̂gFr(A))) = p̂gCl(p̂gFr(p̂gFr(A))) ∩ p̂gCl(X\p̂gFr(p̂gFr(A)))
=p̂gFr(p̂gFr(A)) ∩ p̂gCl(X\p̂gFr(p̂gFr(A))). --------------- (1)
Now, X\p̂gFr(p̂gFr(A)) =X\[p̂gCl(p̂gFr(A)) ∩ p̂gCl(X\p̂gFr(A))] (by Theorem 3.7(viii))
= X\[p̂gFr(A)∩ p̂gCl(X\p̂gFr(A))] (by Theorem 3.7(ix))
= [X\p̂gFr(A)]U[X\(p̂gCl(X\p̂gFr(A)))].
p̂g Cl(X\p̂gFr(p̂gFr(A)))= p̂gCl{[X\p̂gFr(A)]U[X\(p̂gCl(X\ p̂gFr(A)))]}
⊇ p̂gCl[X\p̂gFr(A)]U p̂gCl[X\(p̂gCl(X\p̂gFr(A)))]
=DU p̂gCl(X\D) where D= p̂gCl[X\p̂gFr(A)]
⊇DU(X\D)=X.
Hence from (1), we have p̂gFr(p̂gFr(p̂gFr(A)))= p̂gFr(p̂gFr(A)) ∩X=p̂gFr(p̂gFr(A)).

Theorem 3.9:. If a subset A is a p̂g − open or p̂g − closed in X, then p̂g Fr(p̂g Fr(A))= p̂g Fr(A).

Proof. p̂gFr(p̂gFr(A))= p̂gCl(p̂gFr(A))∩ p̂gCl(X\p̂gFr(A)) (by Theorem 3.7(viii))
=p̂gFr(A) p̂gCl(X\p̂gFr(A)) (by Theorem 3.5(vii))
=p̂gCl(A)∩ p̂gCl(X\A)∩ p̂gCl(X\p̂gFr(A)).
If A is p̂g-open in X and p̂gFr(A) ∩A=ϕ. This implies that A⊆X\p̂gFr(A)⟹ p̂gCl(A) ⊆p̂gCl(X\p̂gFr(A))⟹ p̂gCl(A)∩ p̂gCl(X\p̂gFr(A))= p̂gCl(A).
If A is p̂g-closed in X, p̂gFr(A) ⊆A.
Hence, X\A⊆X\p̂gFr(A) ⟹ p̂gCl(X\A) ⊆ p̂gCl(X\p̂gFr(A)) ⟹ p̂gCl(X\A) ⊆ p̂gCl(X\p̂gFr(A))= p̂gCl(X\A).
From the above, we get p̂gFr(p̂gFr(A))= p̂gCl(A)∩ p̂gCl(X\A)= p̂gFr(A).

Theorem3.10. If A and B are subsets of X such that A⊆B and B is p̂g − closed in X, then p̂g Fr(A) ⊆B.

Proof. p̂gFr(A)= p̂gCl(A)\ p̂gInt(A)⊆ p̂gCl(B)\ p̂gInt(A) ⊆B.

Lemma 3.11. If A and B are subsets of X such that A∩ B= ϕ and A is p̂g − open in X, then A∩ p̂gCl(B)= ϕ.

Proof. On the contrary, suppose that x∈ A∩ p̂gCl(B). Then A is a p̂g-open set containing x and x∈p̂gCl(B). This implies A∩B=ϕ, which is a contradiction to the assumption.

Theorem 3.12. If A and B are subsets of X such that A∩ B= ϕ and A is p̂g − open in X, then A∩ p̂gFr(B)= ϕ.

Proof. Follows from the fact is p̂gFr(B)⊆ p̂gCl(B) and Lemma 3.11.

Definition 3.13. If A is a subset of X then **p̂g-exterior** of A is defined by p̂gExt(A)= p̂gInt(X\A).

Theorem 3.14. If A,B are subsets of a topological space X, the following hold.
(i)p̂gBd(X)=ϕ.
(ii) p̂gExt(ϕ)=X.
(iii) A⊆B⇒p̂gExt(A) ⊇p̂gExt(B).
(iv) p̂gExt(A) is p̂g − open.
(v)Ext(A) ⊆p̂gExt(A) ⊆X\A.
(vi)A is p̂g − closed if and only if p̂gExt(A) =X\A.
(vii) p̂gExt(A) =X\[p̂gCl(A)].
(viii) p̂gExt(p̂gExt(A))= p̂gInt(p̂gCl(A)).
(ix) p̂gExt(A)= p̂gExt(X\p̂gExt(A)).
(x) p̂gInt(A)⊆ p̂gExt(X\p̂gExt(A)).
(xi)X=p̂gInt(A)Up̂gExt(A)Up̂g Fr(A).
(xii) p̂g Ext(AUB) ⊆p̂gExt(A)∩ p̂gExt(B).
(xiii) p̂g Ext(A∩B) ⊆p̂gExt(A)Up̂gExt(B).

Proof:
(i), (ii) and (iii) follow from Definition 3.13.

(iv) follows from Definition 3.13 and the fact that p̂gInt A is the largest p̂gopen set contained in A.
(v) follows from Theorem 2.10(iv).
(vi) follows from Theorem 2.8(ii).
(vii) follows from Theorem 3.48(ii)[11].
(viii) follows from Theorem 3.4(i).
(ix)follows from p̂gExt(X\p̂gExt(A))= p̂gExt(X\p̂gInt(X\A))= p̂gInt(X(X\ p̂gInt(X\A)))= p̂gInt(X\A)= p̂gExt(A).
(x) follows from (viii).
(xi) follows from Definition 3.13 and Theorem 3.7(xviii).
(xii)and (xiii) follow from (iii) above and set theoretic properties.

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