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On Intuitionistic Fuzzy Ideals In Boolean Like Semi Rings

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Abstract- **In this paper ,we introduce the concept of Intuitionistic fuzzy ideals in Boolean like semi rings and investigate some of their properties.**

Keywords: Boolean like semi ring , Intuitionistic fuzzy set , Intuitionistic fuzzy ideal.

INTRODUCTION

Boolean like semi rings were introduced in role by K.Venkatesawarlu, B.V.N.Murthy and N.Amaranth during 2011. Boolean like rings of A.L.Foster arise naturally from general ring dulity consideration and preserves many of the formal properties of Boolean ring .A Boolean like ring is commutative ring with unity and is of characteristics 2. It is clear that every Boolen ring is Boolean like ring but not conversely. The concept of a fuzzy subset of nonempty set was introduced by Zadeh. Fuzzy ideals of rings were introduced by Ziu , and it has been studied by several authors. The notion of fuzzy ideals and its properties were applied various areas: Semi groups ,Bck-algebras and semi rings. Y.B .Jun introduced the notion of fuzzy ideals in near rings. In this paper we introduced the of intuitionistic fuzzy ideals in Boolean like semi ring and study some of its properties

2.PRELIMINARIES

Definition 2.1

A non- empty set R with two binary operation '+' and '·' is called a near-ring if

i) $(R ,+)$ is a group, (not necessarily abelian)
ii) $(R ,\ \cdot)$ is a semigroup,
iii) $x.(y+z) = x\cdot y + x\cdot z$ for all $x,y,z \in R$.

Definition 2.2

A system $(R,+,\cdot)$ is a Boolean semi ring iff the foolowing property are hold

i) $(R,+)$is an additive (abelian) group (whose 'zero' will be denoted by'0')
ii) $(R,\cdot)$ is a semigroup of idempotents in sense $aa = a$ for all $a \in R$.
iii) $a\,(b+c) = ab + bc$ and
iv) $abc = bac$ and $a,b,c \in R$.

Definition 2.3

A non-empty set R together with two-binary operation $+$ and $\cdot$ statisfying the following condition is called Boolean like semi ring

i) $(R,+)$is an abelian group .
ii) $(R,\cdot)$is semi group.
iii) $a(b+c) = a\cdot b + b\cdot c$ for all $a,b,c \in R$.
iv) $a + a = 0$ for all $a \in R$
v) $ab\,(a+b+ab) = ab$ for all $a,b \in R$.

Definition 2.4

A non-empty subset I of R is said to be an ideal if

i) $(I,+)$ is a subgroup of $(R\ ,+)$ for $a,b \in I \implies a+b \in I$
ii) $ra \in I$for all $r \in R$ i.e, $RI \subseteq I$.
iii) $(r+a)s + rs \in I$for all $r,s \in R,\ a \in I$.

Definition 2.5

Let μ be fuzzy set defined on R , then μ is said to be fuzzy ideal of R if

i) $\mu\,(x-y) \geq \min\{\mu(x)\,,\mu(y)\}$
for all x,y∈R
ii) $\mu\,(ra) \geq \mu\,(a)\, r,a \in R$.
iii) $\mu\left((r+a)s + rs\right) \geq \mu\,(a)$for all $a,r,s \in R$.

Definition 2.6

Let μ be fuzzy set defined on R , then μ is said to be anti- fuzzy ideal of R if

i) $\mu\,(x-y) \leq max\{\mu(x),\mu(y)\}$
for all x,y∈R
ii) $\mu\,(ra) \leq \mu\,(a)$for $r,a \in R$.
iii) $\mu\left((r+a)s + rs\right) \leq \mu\,(a)$for all $a,r,s \in R$.

Definition 2.7

Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS) A in X is object having the from $A = \{\langle x\,,\mu_A(x),\nu_A(x)\rangle / x \in X\}$ where the function$\mu_A : X \to [0\,,1]$ and $\nu_A : X \to [0.1]$ denote the degree of membership and degree of non –membership of each element $x \in X$ to

theset A, respectively, and
$0 \leq \mu_A(x) + \nu_A\,(x) \leq 1$.

Notation :

For the sake of simplicity, we shall use the symbol $A = \langle \mu_A, \nu_A\rangle$ for IFS

$$A = \{\langle x, \mu_A(x)\,, \nu_A(x)\rangle / x \in X\}$$

Definition 2.8

Let X be a non-empty set and let $A = \langle \mu_A, \nu_A \rangle$ and $B = \langle \mu_B, \nu_B \rangle$ be IFSs in X. Then .

1) $A \subset B$ iff $\mu_A \le \mu_B$ and $\nu_A \ge \nu_B$.
2) $A = B$ iff $A \subset B$ and $B \subset A$.
3) $A^C = \langle \mu_A, \nu_A \rangle$.
4) $A \cap B = (\mu_A \wedge \mu_B, \nu_A \vee \nu_B)$.
5) $A \cup B = (\mu_A \vee \mu_B, \nu_A \wedge \nu_B)$.
6) $A = (\mu_A, 1-\mu_A)$,
   $A = (1-\nu_A, \nu_A)$.

Definition 2.9

If R and R' are Boolean like semi rings a mapping $f: R \to R'$ is said to be Boolean semi ring homomorphism (or simply homomorphism) of R into R' if

$f(a+b) = f(a) + f(b)$ and

$f(ab) = f(a)f(b)$ for all $a, b \in R$

Definition 2.10

A function $f: R \to R'$ is said to be Boolean like semi ring endomorphism if $R' \subseteq R$ where f is Boolean like semi ring homomorphism

Definition 2.11

Let $f: X \to Y$ be mapping of Boolean like semi ring and A be a intuitionistic fuzzy set of Y then the map $f^{-1}(A)$ is the pre-image of A under f if

$\mu_{f^{-1}(A)}(x) = \mu_A(f(x))$ and

$\nu_{f^{-1}(A)}(x) = \nu_A(f(x))$ for all $x \in X$.

3.INTUITIONISTIC FUZZY IDEAL

Definition 3.1

An IFS $A = \langle \mu_A, \nu_A \rangle$ in R is called an intuitionistic fuzzy left (resp. right) ideal of Boolean like semi ring R if

i) $\mu_A(x-y) \ge \{\mu_A(x) \wedge \mu_A(y)\}$.
ii) $\mu_A(ra) \ge \mu_A(a)$
iii) $\mu_A((r+a)s+rs) \ge \mu_A(a)$
iv) $\nu_A(x-y) \le \{\nu_A(x) \vee \nu_A(y)\}$
v) $\nu_A(ra) \le \nu_A(a)$
vi) $\nu_A((r+a)s+rs) \le \nu_A(a)$

$\forall\ r, s\, x, y \in R$.

Example 3.2

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	a	b	a	0

·	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	0	b	b
c	0	a	b	c

Clearly it is a Boolean –like –semi ring. Let $A = \langle \mu_A, \nu_A \rangle$ be an intuitionistic fuzzy ideal defined on Boolean like semi ring R by $\mu_A(0) = 0.7$, $\mu_A(a) = 0.6$, $\mu_A(b) = 0.5$, $\mu_A(c) = 0.5$ and $\nu_A(x) = 0.6$ for every $x \in R$. Then $A = \langle \mu_A, \nu_A \rangle$ is an intuitionistic fuzzy ideal of R.

Theorem 3.3

If A is an ideal of aBoolean like semiring R, then the IFS $\hat{A} = \langle \chi_A, \overline{\chi_A} \rangle$ is an intuitionitic fuzzy ideal of R.

Proof

Let $x, y, r, s \in R$.

If $x, y, u, v, a \in A$

Then $x - y \in A$, $ra \in A$, and $(r+a)s + rs \in A$ .Since $A$is an ideal of R

Hence $\chi_A(x-y) = 1 \ge \{\chi_A(x) \wedge \chi_A(y)\}$

$\chi_A(ra) = 1 \ge \chi_A(a)$

$\chi_A((r+a)s+rs) = 1 \ge \chi_A(a)$

Also we have ,

$0 = 1 - \chi_A(x-y) = \overline{\chi_A}(x-y)$

$\le \{\overline{\chi_A}(x) \vee \overline{\chi_A}(y)\}$

$0 = 1 - \chi_A(ra) = \overline{\chi_A}(ra) \le \overline{\chi_A}(a)$

$0 = 1 - \chi_A((r+a)s+rs)$

$= \overline{\chi_A}((r+a)s+rs)$

$= \overline{\chi_A}(a)$

If $x \notin A$ or $y \notin A$ then $\chi_A(x) = 0$ (or) $\chi_A(y) = 0$

Thus we have,

$\chi_A(x-y) \ge \{\chi_A(x) \wedge \chi_A(y)\}$

$\chi_A(ra) \ge \chi_A(a)$

$\chi_A((r+a)s+rs) \ge \chi_A(a)$

Also,

$\overline{\chi_A}(x-y) = 1 - \chi_A(x-y)$

$\le 1 - \{\chi_A(x) \wedge \chi_A(y)\}$

$= \overline{\{\chi_A(x) \wedge \chi_A(y)\}}$

$= \overline{\chi_A}(x) \vee \overline{\chi_A}(y)$

$\overline{\chi_A}(ra) = 1 - \chi_A(ra)$

$= 1 - \chi_A(a)$

$= \overline{\chi_A}(a)$

$\overline{\chi_A}((r+a)s+rs) = 1 - \chi_A((r+a)s+rs)$

$= 1 - \chi_A(a)$

$= \overline{\chi_A}(a)$.

This completes the proof.

Definition 3.4

An intuitionistic fuzzy left (resp. right) ideal in Boolean like semi ring R is said to be normal if $\mu_A(0) = 1$ and $\nu_A(0) = 0$.

Theorem 3.5

Let $A = \langle \mu_A, \nu_A \rangle$ be an intuitionistic fuzzy left(resp. right) ideal of Boolean like semi ring R and let ${\mu_A}^+(x) = \mu_{A(x)} + 1 - \mu_A(0)$ and ${\nu_A}^+(x) = \nu_A(x) - \nu_A(0)$ if

$\mu_A(x) + \nu_A(x) \leq 1$ for all $x \in M$. Then $A^+ = \langle \mu_A{}^+, \nu_A{}^+ \rangle$ is a intuitionistic fuzzy left (resp. right) ideal of R.

Proof:

We first observe that $\mu_A{}^+(0) = 1$ and $\nu_A{}^+(0) = 0$ and

$\mu_A{}^+(0), \nu_A{}^+(0) \in [0,1]$for all $x \in R$.

Hence,$A^+ = \langle \mu_A{}^+, \nu_A{}^+ \rangle$ is a normal intuitionistic fuzzy set .

To prove: it is intuitionistic fuzzy left (resp. right) ideal.

Let $x, ry, r, a, s \in R$, Then

$$\mu_A{}^+(x-y) = \mu_A(x-y) + 1 - \mu_A(0)$$
$$\geq \{\mu_A(x) \wedge \mu_A(y)\} + 1 - \mu_A(0)$$
$$\geq \{\mu_A(x) + 1 - \mu_A(0)\} \wedge \{\mu_A(y) + 1 - \mu_A(0)\}$$
$$\geq \mu_A{}^+(x) \wedge \mu_A{}^+(y)$$
$$\mu_A{}^+(ra) = \mu_A(ra) + 1 - \mu_A(0)$$
$$\geq \mu_A(a) + 1 - \mu_A(0)$$
$$= \mu_A{}^+(a)$$
$$\mu_A{}^+((r+a)s + rs)$$
$$= \mu_A((r+a)s + rs) + 1 - \mu_A(0)$$
$$\geq \mu_A(a) + 1 - \mu_A(0)$$
$$= \mu_A{}^+(a)$$
$$\nu_A{}^+(x-y) = \nu_A(x-y) - \nu_A(0)$$
$$\leq \{\nu_A(x) \vee \nu_A(y)\} - \nu_A(0) = \{\nu_A(x) - \nu_A(0)\} \vee \{\nu_A(y) - \nu_A(0)\}$$
$$= \nu_A{}^+(x) \vee \nu_A{}^+(y)$$
$$\nu_A{}^+(ra) = \nu_A(ra) - \nu_A(0) \leq \nu_A{}^+(a)$$
$$\nu_A{}^+((r+a)s + rs)$$
$$= \nu_A((r+a)s + rs) - \nu_A(0)$$
$$\leq \nu_A(a) - \nu_A(0)$$
$$= \nu_A{}^+(a)$$

This shows that A^+ is an intuitionistic fuzzy left ideal of R.

So A^+ is a normal intuitionistic fuzzy left(resp. right) ideal of R.

Theorem 3.6

If the IFS $A = \langle \mu_A, \nu_A \rangle$ is an intuitionistic fuzzy left(resp. right) ideal of R, then the set

$R_A = \{x \in R / \mu_A(x) = \mu_A(0) \text{ and } \nu_A(x) = \nu_A(0)\}$is an ideal of R.

Proof:

Let $, y, a \in R_A$, then

$\mu_A(x) = \mu_A(y) = \mu_A(a) = \mu_A(0)$and $\nu_A(x) = \nu_A(y) = \nu_A(a) = \nu_A(0)$.

Since A is an intuitionistic fuzzy ideal of R it follows.

$$\mu_A(x-y) \geq \{\mu_A(x) \wedge \mu_A(y)\}$$
$$= \{\mu_A(0) \wedge \mu_A(0)\}$$
$$= \mu_A(0)$$

Hence $\mu_A(x-y) = \mu_A(0)$

$$\mu_A(ra) \geq \mu_A(a)$$
$$= \mu_A(0)$$

Let $r, s \in R$

Therefore, $\mu_A((r+a)s + rs) \geq \mu_A(a)$

$$= \mu_A(0)$$

Similarly,

$$\nu_A(x-y) \leq \{\nu_A(x) \vee \nu_A(y)\}$$
$$= \{\nu_A(0) \vee \nu_A(0)\}$$
$$= \nu_A(0)$$

Hence $\nu_A(x-y) = \nu_A(0)$

So ,$x - y \in R_A$

$$\nu_A(ra) \leq \nu_A(a) = \nu_A(0)$$

Hence $\mu_A(ra) = \mu_A(0)$and $\nu_A(ra) = \nu_A(0)$

So ,$ra \in R_A$

Let $r, s \in R$

$$\nu_A((r+a)s + rs) \leq \nu_A(a)$$
$$= \nu_A(0)$$

Hence $\mu_A((r+a)s + ra) \geq \mu_A(0)$

and$\nu_A((r+a)s + rs) \leq \nu_A(0)$

So ,$(r+a)s + rs \in R_A$. Hence R_A is an intuitionistic fuzzy ideal of R.

Theorem 3.7

Let A be an intuitionistic fuzzyleft(resp. right) ideal of a Boolean like semi ring R. For each pair $\langle t, s \rangle \in [0,1]$, the level set $A_{\langle t,s \rangle}$ is an ideal of R.

Proof:

Let $x, y, a \in A_{\langle t,s \rangle}$, then

$\mu_A(x) \geq t$, $\mu_A(y) \geq t$and$\mu_A(a) \geq t$

$\nu_A(x) \leq s$, $\nu_A(y) \leq s$ and$\nu_A(a) \leq s$.

Since A is an intuitionistic fuzzy left(resp. right) ideal we have,

$$\mu_A(x-y) \geq \{\mu_A(x) \wedge \mu_A(y)\} \geq t$$
$$\mu_A(ra) \geq \mu_A(a) \geq t$$

Then $\mu_A((r+a)s + rs) \geq \mu_A(a) \geq t$

$$\nu_A(x-y) \leq \{\nu_A(x) \vee \nu_A(y)\} \leq s$$

So $x - y \in A_{\langle t,s \rangle}$

$$\nu_A(ra) \leq \nu_A(a) \leq s$$

So $ra \in A_{\langle t,s \rangle}$

$$\nu_A((r+a)s + rs) \leq \nu_A(a) \leq s$$

So $(r+a)s + rs \in A_{\langle t,s \rangle}$.

Hence $A_{\langle t,s \rangle}$ is an ideal of R.

Theorem 3.8

Let f be a Boolean semi-ring homomorphism of R if the IFS $A = \langle \mu_A, \nu_A \rangle$ is an intuitionistic fuzzy ideal of R, then $B = \langle \mu_{f^{-1}(A)}, \nu_{f^{-1}(A)} \rangle$ is an intuitionistic fuzzy left(resp.right) ideal of R.

Proof:

For any $x, y, r, a, s \in R$, we have

$$\mu_{f^{-1}(A)}(x-y) = \mu_A(f(x-y))$$
$$= \mu_A(f(x) - f(y))$$
$$\geq \{\mu_A(f(x)) \wedge \mu_A(f(y))\}$$
$$= \{\mu_{f^{-1}(A)}(x) \wedge \mu_{f^{-1}(A)}(y)\}$$
$$\mu_{f^{-1}(A)}(ra) = \mu_A(f(ra))$$
$$= \mu_A(f(r)f(a)) \geq \mu_A f(a)$$
$$= \mu_{f^{-1}(A)}(a)$$
$$\mu_{f^{-1}(A)}((r+a)s + rs)$$
$$= \mu_A(f(r+a)s + rs)$$
$$= \mu_A(f(r+a) + f(rs))$$
$$\geq \mu_A(f(a))$$
$$= \mu_{f^{-1}(A)}(a)$$

Similarly,

$$\nu_{f^{-1}(A)}(x-y) = \nu_A(f(x-y))$$
$$= \nu_A(f(x) - f(y))$$
$$\leq \{\nu_A(f(x)) \vee \nu_A(f(y))\}$$

$= \{\nu_{f^{-1}(A)}(x) \vee \nu_{f^{-1}(A)}(x)\}$

$\nu_{f^{-1}(A)}(ra) \quad = \nu_A(f(ra))$

$= \nu_A\,(f(r)f(a))$

$\leq \quad \nu_A f(a)$

$\leq \nu_{f^{-1}(A)}(a)$

$\nu_{f^{-1}(A)}((r+a)s+rs))$

$= \nu_A\,(f(r+a)s + rs)$

$= \nu_A\big(f(r+a)s + f(rs)\big)$

$\leq \nu_A f(a)$

$= \nu_{f^{-1}(A)}(a)$

Hence B is an intuitionistic fuzzy left(resp. right) ideal of R.

Theorem 3.9

Intersection of a non- empty collection of intuitionistic fuzzy left (resp. right) ideals of Boolean like semi ring R is an intuitionistic fuzzy left(resp. right) ideal of R.

Proof:

Let R be a Boolean like semi ring.

Let $A = \{\langle \mu_{A_i}, \nu_{A_i}\rangle / i \in I\}$ be the family of intuitionistic ideal of Rand let $x, y, r, a, s \in R$.

Then we have,

i) $(\bigcap_{i\in I} \mu_{A_i})(x-y) \quad = inf_{i\in I}\,\{\mu_{A_i}(x-y)\}$

$\geq inf_{i\in I}\{\mu_{A_i}(x) \wedge \mu_{A_i}(\mathrm{Ky})\}$

$= \{inf_{i\in I}\,\mu_{A_i}(x) \wedge \quad inf_{i\in I}\,\mu_{A_i}(y)\}$

$= \{\bigcap_{i\in I} \mu_{A_i}(x) \wedge \bigcap_{i\in I} \mu_{A_i}(y)\}$

ii) $(\bigcap_{i\in I} \mu_{A_i})\,(ra) \quad = \quad inf_{i\in I}\{\mu_{A_i}(ra)\}$

$\geq inf_{i\in I}\{\mu_{A_i}(a)\}$

. $= (\bigcap_{i\in I} \mu_{A_i})(a)$

iii) $(\bigcap_{i\in I} \mu_{A_i})((r+a)s+rs)$

$= \quad inf_{i\in I}\{\mu_{A_i}(r+a)s+rs)\}$

$\geq inf_{i\in I}\{\mu_{A_i}(a)\}$

$= \left(\bigcap_{i\in I} \mu_{A_i}\right)(a)$

Similarly,

i) $(\bigcap_{i\in I} \nu_{A_i})\;(x-y) = inf_{i\in I}\{\nu_{A_i}(x-y)\}$

$\leq inf_{i\in I}\,\{\nu_{A_i}(x) \vee \nu_{A_i}(x)\} \qquad = \{inf_{i\in I}\nu_{A_i}(x) \vee inf_{i\in I}\nu_{A_i}(y)\,\}$

$= \{\bigcap_{i\in I} \nu_{A_i}(x) \vee \bigcap_{i\in I} \nu_{A_i}(y)\,\}$

ii) $(\bigcap_{i\in I} \nu_{A_i})(ra) \;= inf_{i\in I}\,(\nu_{A_i}(ra))$

$\leq inf_{i\in I}\,\{\nu_{A_i}(a)\}$

$= (\bigcap_{i\in I} \nu_{A_i})(a)$

iii) $(\bigcap_{i\in I} \nu_{A_i})((r+a)s+rs\,)$

$= \quad inf_{i\in I}\,\{\nu_{A_i}((r+a)s+rs)\}$

$\leq \; inf_{i\in I}\,\{\nu_{A_i}(a)\}$

$= (\bigcap_{i\in I} \nu_{A_i})(a)$

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