Some Remarks on Soft JP Open Sets in Soft Topological Spaces

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Abstract- **When managing uncertainties, a few speculations such as Fuzzy set theory, Intuitionistic fuzzy set theory, Vague set theory and Rough set theory have innate challenges because of deficiency of parameterization. In 1999,Russian Mathematician D.Molodstov [2] stipulated soft set theory to annihilate the ambiguity that emerges in the most of the problem solving methods. The conception of soft sets became stable once Shabir and Naz introduced soft topological spaces in 2011, which are defined over an initial universe with a fixed set of parameters. In 2016, The authors of this paper cleared another pathway by presenting a new class of generalized closed set called soft JP closed sets in soft topological spaces. This paper is devoted to Soft JP Open and Soft JP closed maps. We also studied some of its basic properties and the interrelationship of the above maps with other Soft maps. Also its several characterizations and properties are obtained.**

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1. INTRODUCTION

When dealing with uncertainties, several theories such as Fuzzy set theory, Intuitionistic fuzzy set theory,Vague set theory and Rough set theory have inherent difficulties due to inadequacy of parameterization. In 1999, Russian Mathematician D.Molodstov [2]stipulated soft set theory to eradicate the vagueness that arises in the most of the problem solving methods.Shabir and Naz[5](2011) introduce the notion of soft topological spaces.In 2016,The authors of this paper introduced soft JP closed sets in soft topological spaces. This paper is devoted to Soft Totally JP Continuous and Soft contra JP Continuous functions which are the different forms of Soft JP continuous function and their relationship with other soft continuous functions are derived.

2.PRELIMINARIES

Throughout this work,X refers to an initial universe, E is a set of parameters, $P(X)$ is the powerset of X,and$A \subseteq E$.

Definition 2.1[2]:
A soft set (F,A) over X is said to be **Null Soft Set** denoted by F_{ϕ} or $\tilde{\Phi}$if for all e ∈A, F(e) = φ. A soft set (F,E) over X is said to be an **Absolute Soft Set** denoted by F_X or $\tilde{X}$if for all e∈A,F(e) = X.

Definition 2.2[2]:
The **Union** of two soft sets(F, A) and (G, B)over X is the soft set (H, C), where C = A ∪B, and for all e ∈C,H(e) = F(e), if e ∈A\B, H(e) = G(e) if e ∈B\A and H(e) = F(e) ∪G(e)if e ∈A ∩B and is denoted as(F, A) $\tilde{\cup}$(G, B) = (H, C).

Definition 2.3[2]:
The **Intersection** of two soft sets (F, A) and (G, B) over X is the soft set (H,C), where C = A ∩B and H(e) = F(e) ∩G(e)for all e ∈C and is denoted as (F, A) $\tilde{\cap}$(G,B) = (H, C).

Definition 2.6[2]:
Let (F,A) and (G, B) be soft sets over X, we say that (F,A) is a **Soft Subset** of (G,B) if A ⊆B and for all e ∈A,F(e)and G(e) are identical approximations. We write (F, A) $\tilde{\subseteq}$(G, B).

Definition 2.7[8]:
Let τ be a collection of soft sets over X with the fixed set E of parameters. Then τ is called a **Soft Topology** on X if **i.** $\tilde{\Phi}$,$\tilde{X}$belongs to τ. **ii.** The union of any number of soft sets in τ belongs to τ.**iii.** The intersection of any two soft sets in τ belongs to τ.The triplet (X, τ, E) is called **Soft Topological Spaces** over X. The members of τ are called **Soft Open** sets in X and complements of them are called **Soft Closed** sets in X.

Definition 2.8:
A Subset of a soft topological space (X, τ, E) is said to be

1. **SoftSemi-Open** set[1] if (A,E) $\tilde{\subseteq}$Cl(int(A,E)).
2. **Softα-Open** set[4] if (A,E) $\tilde{\subseteq}$Int(Cl(int(A,E))).
3. **Soft $\hat{g}$-Closed** [3] if Cl(A,E) $\tilde{\subseteq}$ (U ,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft semi Open in (X,τ,E).
4. **Soft JP closed set**[4] if Scl(F,E)$\tilde{\subseteq}$Int(U,E) whenever (F,E) $\tilde{\subseteq}$(U,E) and (U,E) is soft $\hat{g}$ open. The

complement of soft JP Closed set is called soft JP Open set.

5.

Definition 2.9[5]

Let (X, τ, E) be a Soft Topological Spaces over X. The **Soft JP Interior** of (F, E) denoted by SJPInt(F, E) is the union of all soft JP open subsets of (F,E).

i)SJPInt (F, E) =Ũ{(O.E): (O, E) is soft open and (O,E) ⊆̃(F,E)}.

The **SoftClosure** of (F, E) denoted by SJPCl (F,E) is the intersection of soft JP closed sets containing (F, E).

ii)SJPCl (F,E) =∩̃{(O. E): (O, E) is soft closed and (F, E) ⊆̃(O,E)}.

Proposition3.4[4]: Every soft semi closed set is soft JP closed.

Proposition3.6[4]: Every soft closed set is soft JP closed.

Proposition3.8[4]: Every soft α closed set is soft JP closed.

Proposition 4.3[4]:

1. Every soft Open is soft JP Open.

2. Every soft semi Open is soft JP Open.
3. Every soft α Open is soft JP Open.
4. Every soft JP Open is soft gs Open.

Definition 2.12:

A map f:(X,τ ,E)→(Y,σ,K)is said to be

1. **Soft JP continuous[6]** if inverse image of every Soft open set in (Y,σ,K) is Soft JP open in (X,τ ,E).
2. **Soft JP irresolute[6]** if inverse image of every Soft JPopen set in (Y,σ,K) is Soft JP open in (X,τ ,E).
3. **Soft strongly JP continuous[7]** if inverse image of every Soft JP open set in (Y,σ,K) is Soft open in (X,τ ,E).

Soft perfectly JP continuous[7] if inverse image of every Soft JP open

1. set in (Y,σ,K) is Soft clopen in (X,τ ,E).

3.**Soft JP Open Map:-**

Defnition: 3.1

A map f: (X,τ ,E)→(Y,σ,K)is said to be **Soft JP Open map** if image of every Soft open set in (X,τ,E) is Soft JP Open in (Y,σ,K).

Defnition: 3.2

A map f: (X,τ ,E)→(Y,σ,K)is said to be **Soft JP Closed map** if image of every Soft closed set in (X,τ,E) is Soft JP closed in (Y,σ,K).

Example:3.3

Let$X=\{x_1,x_2\}$,Y={a,b},$E=\{e_1,e_2\}$,$K=\{k_1,k_2\}$,Define u:X→Y and p:E→K as $u(x_1)$= a,$u(x_2)$= band $p(e_1)=k_2$, $p(e_2)=k_1$. Let us consider the topology τ={ $\tilde{\varphi},\tilde{X}$,($F_1$,E),(F_2,E)} such that $F_1(e_1)$= x_2,$F_1(e_2)$= x1, $F_2(e_1)$= x_2,$F_2(e_2)$=X and σ ={$\tilde{\varphi},\tilde{Y}$, (U,K),(V,K)} such that $U(k_1)$= ϕ,$U(k_2)$= a, $V(k_1)$=a,,$V(k_2)$= a ,Then f:(X,τ,E)→(Y,σ,K) is a Soft JP open (Closed) Map.

Theorem: 3.4

If f:(X,τ,E)→(Y,σ,K) is a Soft JP Closed map then SJPCl(f(A,E))⊆f(Cl (A,E)) for every soft subset ((A,E) in (X,τ,E).

Proof:

Suppose f is a soft JP closed map and (A,E) is a soft subset of (X,τ,E).Then f(Cl (A,E))is Soft JP closed in(Y,σ,K).Then,f(A,E)⊆f(Cl(A,E)) implies SJPCl (f(A,E))⊆ SJPCl (f(Cl(A,E))=f(Cl(A,E)).

Remark:3.5

The converse of the above theorem is not true and it can be seen from the following observation.

Let X={a,b,c},Y={a,b,c},E={e},K={k}, Define u:X→Y and p:E→K as u(a)= c, u(b)=a and u(c)=b p(e)=k,. Let us consider the topology τ={ $\tilde{\varphi},\tilde{X}$,(e,a)} and σ ={$\tilde{\varphi},\tilde{Y}$, (k,a),(k,{b,c}} ,let f:(X,τ,E)→(Y,σ,K) is a Soft mapping. Clearly here SJPCl(f(A,E))⊆f(Cl(A,E)) for every soft subset ((A,E) in (X,τ,E). But it is not a soft JP closed map because f(e,a)=(k,c) is not Soft JP closed in (Y,σ,K).

Theorem: 3.6

A soft surjective map f:(X,τ,E)→(Y,σ,K) is soft JP closed if and only if for each subset (A,K) of (Y,σ,K) and for each Soft open set (U,E) in (X,τ,E) containing f^{-1}(A,K) there is a Soft JP open set (V,K) in (Y,σ,K) such that (A,K) ⊆ (V,K) and f^{-1}(V,K) ⊆(U,E).

Proof:

Necessity: Suppose f:(X,τ,E)→(Y,σ,K) is soft JP closed .Let(A,K) be a soft subset of (Y,σ,K) and (U,E) an Soft open set of (X,τ,E) containing f^{-1}(A,K) .Put (V,K)=$(f(U,E))^C$ then (V,K) is Soft JP Open in (Y,σ,K) containing (A,K) and f^{-1}(V,K) ⊆(U,E).

Sufficiency: Let (B,E) be any Soft closed set of (X,τ,E), then $f^{-1}(f((B,E)^C) \subseteq (B,E)^C$ and $(B,E)^C$ is Soft open .Then by hypothesis, there exists a Soft JP Open set(V,K) of (Y,σ,K) such that $f((B,E)^C \subseteq (V,K)$ and $f^{-1}(V,K) \subseteq (B,E)^C$ and so $(B,E) \subseteq (f^{-1}(V,K))^C$ .Hence $(V,K)^C \subseteq f(B,E) \subseteq f(f^{-1}(V,K))^C \subseteq (V,K)^C$ which implies $f(B,E)=(V,K)^C$.Since $(V,K)^C$ is Soft JP Closed then f(B,E) is Soft JP Closed and therefore f is Soft JP Closed.

Defnition: 3.7

A map f: (X,τ ,E)→(Y,σ,K)is said to be **Soft Semi JP Closed map** if image of every Soft semi closed set in (X,τ,E) is Soft JP closed in (Y,σ,K).

Theorem: 3.8

If the map f:(X,τ,E)→(Y,σ,K) is Soft $\hat{g}$ irresolute ,Soft Semi JP Closed and (A,E) is Soft JP Closed in (X,τ,E),then f(A,E)is Soft JP Closed in (Y,σ,K).

Proof:

Let(A,E) be Soft JP closed Subset of (X,τ,E) and (U,K) be any soft $\hat{g}$ open set in (Y,σ,K) containing f(A,E).Then (A,E) ⊆ f^{-1}(U,K) and f^{-1}(U,K) soft $\hat{g}$ open in (X,τ,E) by the hypothesis. SSCl(A,E) ⊆ $Int(f^{-1}(U,K)) \subseteq f^{-1}(U,K))$ hence f(A,E) ⊆f(SSCl(A,E)) ⊆(U,K) .Since f is Soft Semi JP Closed and SSCl(A,E) is Soft Semi closed in (X,τ,E) then f(SSCl(A,E))is Soft JP Closed in (Y,σ,K).Then f(SSCl(A,E)) ⊆ (U,K) with f(SSCl(A,E)) as Soft JP Closed in and (U,K) an Soft $\hat{g}$ open.Hence SSCl(f(SSCl(A,E))) ⊆ Int((U,K)) but f(A,E) ⊆f(SSCl(A,E))). Then SScl (f(A,E))⊆ SSCl(f(SSCl(A,E))).Thus SScl (f(A,E))⊆ Int((U,K)).This Shows that f(A,E)is Soft JP Closed in (Y,σ,K).

Remark:3.9

The compositions of two Soft JP Open(closed) maps need not be Soft JP Open(closed) as seen from following observation.

Let X={x_1,x_2},Y={y_1,y_2},Z={z_1,z_2}, E={e_1,e_2}, K={k_1,k_2}, R={r_1,r_2},Define u:X→Y and p:E→K& q:K--> R as u(x_1)= y_1,u(x_2)= y_2 v(y_1)= z_1,v(y_2)= z_2 ,p(e_1)=k_1, p(e_2)=k_2 andq(k_1)= r_1,q(k_2)= r_2. Let us consider the topology τ={ $\tilde{\varphi}$, $\tilde{X}$,(F,E)} such that F(e_1)= x_2,F(e_2)= x_2 ,σ ={$\tilde{\varphi}$, $\tilde{Y}$, (G,K)}such that G(k_1)= y_2, H_1(k_2)= Y, and η={$\tilde{\varphi}$, $\tilde{Z}$,(H_1,R), (H_2,R)} such that H_1(r_1)= z_1, H_1(r_2)= z_1, then f:(X,τ,E)→(Y,σ,K)and g: (Y,σ,K)→(Z, η,R) are two Soft mappings.Clearly f and g are Soft JP closed mappings. but their composition is not Soft JP closed because gof(F,E)= {(r_1, z_2),(r_2, z_2)}is not soft JP closed in (Z, η,R).

Theorem: 3.10

Let f:(X,τ,E)→(Y,σ,K)and g: (Y,σ,K)→(Z, η,R) are two Soft mappings. Then their composition gof : (X,τ,E)→(Z, η,R) is Soft JP Closed if both f and g are Soft JP closed maps and (Y,σ,K) is Soft T_{JP} space.

Proof:

Let (F,E) be a Soft closed set in (X,τ,E),Then f(F,E) is Soft JP closed and by hypothesis it is Soft closed in (Y,σ,K) .Then gof(F,E)=g(f(F,E)) is Soft JP closed in (Z, η,R). Then gof : (X,τ,E)→ (Z, η,R) is Soft JP Closed .

Theorem: 3.11

For any Soft bijecrtive map f:(X,τ,E)→(Y,σ,K) the following are equivalent.

(1) f^{-1} : (Y,σ,K)→(X,τ,E) is Soft JP Continuous.
(2) f is Soft JP Open map
(3) f is Soft JP Closed map.

Proof:

(1) → 2: Let (U,E) be a Soft open set in (X,τ,E), By assumption($(f^{-1})^{-1}$(U,E)) =f(U,E) is Soft JP open in (Y,σ,K). Then f is Soft JP open map.
(2) →(3): Let (F,E) be a Soft closed set in (X,τ,E),$(F,E)^C$ is Soft open set in (X,τ,E).By assumption f($(F,E)^C$) is Soft JP open in (Y,σ,K).Then f((F,E)) is Soft JP closed in (Y,σ,K).Hence f is Soft JP closed map.
(3) →(1): Let (F,E) be a Soft closed set in (X,τ,E), f(F,E)= $((f^{-1})^{-1}$(F,E)) is Soft JP closed in (Y,σ,K).Then f^{-1} : (Y,σ,K)→(X,τ,E) is soft JP continuous.

Theorem3.12:

Let f:(X,τ,E)→(Y,σ,K) be soft open map ,Assume that SJPO(Y,σ,K) is closed under soft union then the following are equivalent.

(1) F is soft JP open map.
(2) For each soft subset (A,E) of (X,τ,E), f(Int(A,E)) ⊆ SJPInt(f(A,E)).
(3) For each soft point (x,e) $\tilde{\in}$ $\tilde{X}$ and each soft neighborhood (U,E) of (x,e) in $\tilde{X}$ there exists a soft neighborhood (V,K) of f(x,e) in (Y,σ,K) such that (V,K) ⊆ f(U,E).

Proof:

(i)→(ii): Suppose f is soft JP open and (A,E) be a soft subset of (X,τ,E), Then f(int(A,E)) is soft JP open in (Y,σ,K) **.** Then f(Int(A,E)) ⊆ SJPInt(f(A,E)) from the facts f(Int(A,E)) ⊆ f(A,E) and f(A,E) ⊆ SJPInt(f(A,E)).

(ii)→(iii): Let (x,e) $\tilde{\in}$ $\tilde{X}$ and (U,E) be a soft neighborhood of (x,e) in $\tilde{X}$. Then there exists a soft open set (G,E) in (X,τ,E) such that (x,e) $\tilde{\in}$ (G,E) ⊆ (U,E).Then (iii) can be proved by taking f(G,E) = (V,K) in (Y,σ,K).

(iii)→(i): Suppose For each soft point (x,e) $\tilde{\in}$ $\tilde{X}$ and each soft neighborhood (U,E) of (x,e) in $\tilde{X}$ there exists a soft neighborhood (V,K) of f(x,e) in (Y,σ,K) such that (V,K) ⊆ f(U,E).Since by hypothesis, SJPO(Y,σ,K) is closed under soft union then f(U,E)= $\tilde{\bigcup}$$\{(V,K)_{(y,k)}$: f(x,e)=(y,k) $\tilde{\in}$ f(U,E)} is soft JP open in (Y,σ,K). Hence f is soft JP open map.

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