Strongly, Perfectly and Contra $M_I{}^*$-Continuous In Ideal Topological Space

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Abstract- **In this paper we have to introduced the concept of strongly $M_I{}^*$-continuous,perfectly $M_I{}^*$-*-continuous and contra $M_I{}^*$-continuous maps in ideal topological spaces.Also we have discussed the relationship with other existing continuous maps,composition between these continuous maps and its equivalent properties.**

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1. INTRODUCTION

The concept of ideals in topological spaces is treated in the classic text by Kuratowski [9]and Vaidyanathaswamy [10]. Jankovic and Hamlett [4] investigated further properties ofideal spaces.An Ideal I on a topological space (X,τ) is a non-empty collection of subsetsof X which satisfies the following properties: (i) A∈I and B ⊂ A implies B∈I (ii)A∈I and B∈I implies A∪B∈ I .An ideal topological space(or an ideal space)is a topological space (X,τ) with an ideal I on X and is denoted by (X,τ,I).For a subset A ⊂X ,$A^*(I,\tau)$ = {x ∈X : A ∩U ∉I for every U∈ $\tau(x)$} is called the localfunction of A with respect to I and τ [9].We simply write A^* in case there is no chance for confusion. A Kuratowski closure operator $cl^*(.)$ for a topology $\tau^*(I,\tau)$ calledthe ∗-topology,finer than τ is de_ned by $cl^*(A) = A \cup A^*$ [10].

2.PRILIMINARIES

Definition 2.1. A map f : (X, τ) → (Y, σ) is called (i)contra continuous[2] if f −1 (V) is closed in (X, τ) for each open set V of (Y, σ). (ii) contra α -continuous [3] if f −1 (V) is α -closed in (X, τ) for each open subset V of (Y, σ). (iii) contra β -continuous [1]if f −1 (V) is β -closed in (X, τ) for each open subset V of (Y, σ). (iv) contra pre-continuous [4] if f −1 (V) is pre-closed in (X, τ) for each open subset V of (Y, σ). (v) contra semi-continuous if f −1 (V) is semi-closed in (X, τ) for each open subset V of (Y, σ).

Definition 2.2. A map f : (X, τ) → (Y, σ) is called (i) strongly continuous [29] if f −1 (V) is both open and closed in (X, τ) for each subset V of (Y, σ). (ii) perfectly continuous [40] if f −1 (V) is both open and closed in (X, τ) for each open subset V of (Y, σ).

Definition 2.3.[7]A subset A of an ideal topological space (X, τ, I) is called M_I^***-closed if** $spcl(A) \subset U$ **whenever** $A \subset U$ **and U is** I_ω**-open in** (X, τ, I).The classof all M_I^***-closed sets in** (X, τ, I) is denoted by $M_I^* - C(X)$.That is, $M_I^* - C(X)=\{A\subset X$:A is M_I^***-closed in** $(X, \tau, I)\}$.

Definition 2.4[7]*A function $f : (X,\tau,I) \to (Y,\sigma,J)$ is called M_I^*-irresolute if $f^{-1}(V)$ is M_I^*-closed in (X,τ,I) for every M_I^*-closed set V in (Y, σ,J).*

Theorem 2.5[7]. Every closed (resp.α-closed, preclosed, semi-closed,β-closed) set is M_I^*-closed but not conversely.

Lemma 2.6. The following are the properties for the subsets A,B of a space X.

(i)$x \in \ker(A)$ if and only if $A \cap F \neq \emptyset$ for any closed set F of X containing x.

(ii) $A \subset \ker(A)$ and $A = \ker(A)$ if A is open in X

(iii) If $A \subset B$ then $\ker(A) \subset \ker(B)$.

3.CONTRA $M_I{}^*$-CONTINUOUS

Deftnition 3.1. *A function $f : (X,\tau,I) \to (Y,\sigma)$ is called contra M_I^*-continuous if $f^{-1}(V)$ is M_I^*-open (resp. M_I^*-closed) in (X,τ,I) for every closed (resp.open) set V in (Y, σ).*

Example 3.2. *Let $X = Y = \{a,b,c\}, \tau = \{\emptyset,X,\{a\},\{a,b\},\{a,c\}\}, I = \{\emptyset,\{b\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$ .Define a* *function* $f : (X, \tau, I) \to (Y, \sigma)$ by $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then f is contra M_I^*-continuous.

Theorem 3.3. (i) Every contra cosntinuous function is a contra M_I^*-continuous function but not conversely.

(ii) Every contra α-continuous function is a contra M_I^*-continuous function but not con- versely.

(iii) Every contra pre-continuous function is a contra M_I^*-continuous function but not conversely.

(iv) Every contra semi-continuous function is a contra M_I^*-continuous function but not conversely.

(v) Every contra β-continuous function is a contra M_I^*-continuous function but not con- versely.

Proof. (i) Let $f : (X, \tau, I) \to (Y, \sigma)$ be a contra continuous function and V be an open set of (Y, σ). Since f is contra continuous, $f^{-1}(V)$ is closed in (X, τ). Hence by Proposition 2.0.31, $f^{-1}(V)$ is M_I^*-closed in (X, τ). Thus f is a contra M_I^*-continuous function. The proof of the other parts are similar.

Example 3.4. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$, $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$ and $I = \{\emptyset, \{c\}\}$. Define a function $f: (X, \tau, I) \to (Y, \sigma)$ by $f(a) = a$, $f(b) = b$ and $f(c)=a$. Then f is contra M_I^*-continuous but not contra continuous.for an open set $\{a\}$,we have, $f^{-1}(\{a\}) = \{a, c\}$,which is M_I^8-closed but not closed (resp.α-closed,preclosed,semi-closed,β-closed) in (X, τ, I).

Remark 3.5. The composition of two contra M_I^*-continuous functions need not be contra M_I^*-continuous as seen from the following example.

Example 3.6 Let $X = Y = Z = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$, $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$, $\gamma = \{\emptyset, Z, \{a\}, \{a, b\}\}$, $I = \{\emptyset, \{b\}\}$ and $J = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$. Define a function $f: (X, \tau, I) \to (Y, \sigma, J)$ by $f(a) = c$, $f(b) = a$ and $f(c) = b$ and $g : (Y, \sigma, J) \to (Z, \gamma)$ by $g(a) = c$, $g(b) = a$ and $g(c) = b$. Then f and g both are contra MI* -continuous but their composition is not contra MI* -continuous.For an open set $\{a, b\}$ in (Z, γ) ,we have $(g \circ f)^{-1}(\{a, b\}) = f^{-1}(g^{-1}(\{a, b\}) = f^{-1}(\{b, c\}) = \{a, c\}$ which is not M_I^*-closed in (X, τ, I).

Theorem3.7. The following are equivalent for a function $f : (X,\tau,I) \to (Y,\sigma)$. Assume that $M_I^* - O(X)$ (resp. $M_I^* - C(X)$) is closed under any union.(resp.intersection)

(i) f is contra M_I^*-continuous.

(ii) The inverse image of a closed set F of (Y,σ) is M_I^*-open in (X,τ,I).

(iii) For each $x \in X$ and each closed set F of Y containing $f(x)$, there exists $U \in M_I^* - O(X)$ and $x \in U$ such that $f(U) \subseteq F$.

(iv) $f(M_I^* cl(A)) \subseteq Ker(f(A))$ for every subset A of X.

(v) $M_I^* cl(f^{-1}(B)) \subseteq f^{-1}(Ker(B))$ for every subset B of Y.

Proof. The implication (i)⇒(ii) is obvious.

(ii)⇒ (iii) :Let x be any point of X and F be any closed set of Y containing f (x) .By (ii), $f^{-1}(F)$ is M_I^*-closed in (X, τ, I) and $x \in f^{-1}(F)$ .Put $U = f^{-1}(F)$,then $U \in M_I^* - O(X)$ and $f(U) \subseteq F$.

(iii)⇒ (ii) Let F be any closed set of Y and $x \in f^{-1}(F)$. Then $f(x) \in F$ and there exists $U_x \in M_I^* - O(X)$ such that $f(U_x) \subset F$ .Hence $f^{-1}(F) = \{U_x : x \in f^{-1}(F)\}$ is obtained and by assumption $f^{-1}(F)$ is M_I^**-open.*

(ii)⇒(iv) Let A be any subset of X .Suppose that $y \notin Ker(f(A))$ .Then by Lemma 2.6,there exists $F \in C(Y, f(x))$ such that $f(A) \cap F = \emptyset$ .Thus $A \cap f^{-1}(F) = \emptyset$ and $M_I^* - cl(A) \cap f^{-1}(F) = \emptyset$ .Hence $f(M_I^* - cl(A)) \cap F = \emptyset$ and $y \notin f(M_I^* - cl(A))$ are obtained. Thus $f(M_I^* - cl(A)) \subseteq Ker(f(A))$.

***(iv)* ⇒ (v)** Let B be any subset of Y .By **(iv)** and Lemma 2.6, $f(MI^*cl(f^{-1}(B))) \subset Ker(f(f^{-1}(B)) \subset Ker(B)$ and $MI^*cl(f^{-1}(B) \subset f^{-1}(Ker(B))$.

(v) ⇒ (i) Let U be an open set of Y .Then by Lemma 2.6, M_I^* $cl(f^{-1}(U)) \subset f^{-1}(Ker(U)) = f^{-1}(U)$ and M_I^* $cl(f^{-1}(U)) = f^{-1}(U)$ .By assumption, $f^{-1}(U)$ is M_I^*-closed in (X, τ, I) .Hence f is contra M_I^* -continuous.

Theorem 3.8. Let (X, τ, I), (Y, σ, J) be any two ideal topological spaces and (Y, σ, J) be $T_{M_I^*}$-space.Then the composition $g \circ f : (X, \tau, I) \to (Z, \gamma)$ is contra M_I^*-continuous if $f : (X, \tau, I) \to (Y, \sigma, J)$ is contra M_I^*-continuous and $g : (Y, \sigma, J) \to (Z, \gamma)$ is M_I^*- continuous.

Proof. Let F be any closed set of (Z, γ).Since g is M_I^*-continuous, $g^{-1}(F)$ is M_I^*- closed in (Y, σ, J) and (Y, σ, J) is T_M*-space, hence $g^{-1}(F)$ is closed in (Y, σ, J) .Since f is contra M_I^*-continuous, $f^{-1}(g^{-1}(F))$ is M_I^*-open in (X, τ, I) .Hence $g \circ f$ is contra M_I^*-continuous.

Theorem 3.9.. Let (X, τ, I), (Y, σ, J) be any two ideal topological spaces and (Y, σ, J) be $T_{M_I^*}$-space.Then the composition $g \circ f : (X,\tau,I) \to (Z,\gamma)$ is M_I^*-continuous if $f : (X,\tau,I) \to (Y,\sigma,J)$ is contra M_I^*-continuous and

$g:(Y,\sigma,J)\rightarrow(Z,\gamma)$ is contra M_I^* continuous.

Proof. Let F be an open set of (Z,γ).Since g is contra M_I^* -continuous, $g^{-1}(F)$ is M_I^* - closed in (Y,σ,J) and (Y,σ,J) is T_M*-space, hence $g^{-1}(F)$ is closed in (Y,σ,J) .Since **f** is contra M_I^* -continuous, $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is M_I^* -open in (X,τ,I).Hence $g\circ f$ is M_I^* -continuous.

Theorem 3.10.. Let (X,τ) be a topological space and (Y,σ,J) be an $\alpha T_{M_I}^*$ -space.Then the composition $g\circ f:(X,\tau)\rightarrow(Z,\gamma)$ is contra α -continuous if $f:(X,\tau)\rightarrow(Y,\sigma,J)$ is α -irresolute and $g:(Y,\sigma,J)\rightarrow(Z,\gamma)$ is contra M_I^* -continuous.

Proof. Let F be an open set of (Z,γ).Since g is contra M_I^* -continuous, $g^{-1}(F)$ is M_I^* -closed in (Y,σ,J) and (Y,σ,J) is an αT_M* =-space,n hence $g^{-1}(F)$ is α -closed in (Y,σ,J).Since f is α-irresolute, $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is α-closed in (X,τ).Hence $g\circ f$ is contra α -continuous.

Theorem 3.11. Let (X,τ) be a topological space and (Y,σ,J) be a pTMI* -space.Then the composition $g\circ f:(X,\tau)\rightarrow(Z,\gamma)$ is contra pre-continuous if $f:(X,\tau)\rightarrow(Y,\sigma,J)$ is pre-irresolute and $g:(Y,\sigma,J)\rightarrow(Z,\gamma)$ is contra M_I^* -continuous.

Proof. Let F be an open set of (Z,γ).Since g is contra M_I^* -continuous, $g^{-1}(F)$ is M_I^* - closed in (Y,σ,J) and (Y,σ,J) is pT_M* -space, hence $g^{-1}(F)$ is pre-closed in (Y,σ,J).Since f is pre-irresolute, $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is pre-closed in (X,τ).Hence $g\circ f$ is con- tra pre-continuous.

Theorem 3.12. Let (X,τ) be a topological space and (Y,σ,J) be a $sT_{M_I}^*$ -space.Then the composition $g\circ f:(X,\tau)\rightarrow(Z,\gamma)$ is contra semi-continuous if $f:(X,\tau)\rightarrow(Y,\sigma,J)$ is semi-irresolute and $g:(Y,\sigma,J)\rightarrow(Z,\gamma)$ is contra M_I^* -continuous.

Proof. The proof is similar

Theorem 3.13.. Let (X,τ) be a topological space and (Y,σ,J) be a $\beta T_{M_I}^*$ -space.Then the composition $g\circ f:(X,\tau)\rightarrow(Z,\gamma)$ is contra β -continuous if $f:(X,\tau)\rightarrow(Y,\sigma,J)$ is β -irresolute and $g:(Y,\sigma,J)\rightarrow(Z,\gamma)$ is contra M_I^* -continuous.

Proof. The proof is similar

Theorem 3.14.. If $f:(X,\tau,I)\rightarrow(Y,\sigma,J)$ is M_I^* -irresolute and $g:(Y,\sigma,J)\rightarrow(Z,\gamma)$ is contra M_I^* -continuous, then their composition $g\circ f:(X,\tau,I)\rightarrow(Z,\gamma)$ is contra M_I^* - continuous.

Proof. Let U be any open set of (Z,γ).Since g is contra M_I^* -continuous, then $g^{-1}(U)$ is M_I^* -closed in (Y,σ,J) and since **f** is M_I^*-rresolute, then $f^{-1}(g^{-1}(U))$ is M_I^* -closed in (X,τ,I).Therefore $g\circ f$ is contra M_I^* -continuous.

Theorem 3.15.. If $f:(X,\tau,I)\rightarrow(Y,\sigma,J)$ is M_I^* -irresolute and $g:(Y,\sigma,J)\rightarrow(Z,\gamma)$ is contra α -continuous, then their composition $g\circ f:(X,\tau,I)\rightarrow(Z,\gamma)$ is contra M_I^* - continuous.

Proof. Let U be any open set of (Z,γ) .Since g is contra α -continuous, then $g^{-1}(U)$ is α -closed in (Y,σ,J) .By Theorem 2.5., $g^{-1}(U)$ is M_I^* -closed in (Y,σ,J) and since **f** is M_I^* -irresolute,then $f^{-1}(g^{-1}(U))$ is M_I^* -closed in (X,τ,I).Therefore $g\circ f$ is contra M_I^* -continuous.

Theorem 3.16. If $f:(X,\tau,I)\rightarrow(Y,\sigma,J)$ is M_I^* -irresolute and $g:(Y,\sigma,J)\rightarrow(Z,\gamma)$ is contra pre-continuous, then their composition $g\circ f:(X,\tau,I)\rightarrow(Z,\gamma)$ is contra M_I^* - continuous.

Proof. The proof is similar.

Theorem 3.17.. If $f:(X,\tau,I)\rightarrow(Y,\sigma,J)$ is M_I^* -irresolute and $g:(Y,\sigma,J)\rightarrow(Z,\gamma)$ is contra semi-continuous ,then their composition $g\circ f:(X,\tau,I)\rightarrow(Z,\gamma)$ is contra M_I^* - continuous.

Proof. The proof is similar.

Theorem 3.18.: If $f:(X,\tau,I)\rightarrow(Y,\sigma,J)$ is M_I^* -irresolute and $g:(Y,\sigma,J)\rightarrow(Z,\gamma)$ is contra β -continuous, then their composition $g\circ f:(X,\tau,I)\rightarrow(Z,\gamma)$ is contra M_I^* - continuous.

Proof. The proof is similar.

Theorem 3.19. If $f:(X,\tau,I)\rightarrow(Y,\sigma)$ is contra M_I^* -continuous and $g:(Y,\sigma)\rightarrow(Z,\gamma)$ is continuous, then their composition $g\circ f:(X,\tau,I)\rightarrow(Z,\gamma)$ is contra M_I^* - continuous.

Proof. Let U be any open set of (Z,γ) .Since g is continuous,then $g^{-1}(U)$ is open in (Y,σ) and since **f** is contra M_I^* -continuous, then $f^{-1}(g^{-1}(U)) = (g\circ f)^{-1}(U)$ is M_I^* - closed in (X,τ,I) .Therefore $g\circ f$ is contra M_I^* -continuous.

4. Strongly and perfectly M_I^*-continuous maps

Deftnition 4.1.. A map $f:(X,\tau)\rightarrow(Y,\sigma,J)$ is said to be strongly M_I^*-continuous if the inverse image of every M_I^* -open set of (Y,σ,J) is open in (X,τ).

Proposition 4.2.. If a map $f:(X,\tau)\rightarrow(Y,\sigma,J)$ is strongly

M_I^*-continuous, then f is continuous but not conversely.

Proof. Since every open set is M_I^*-open, we get the proof.

Example 4.3.. Let X = Y = {a,b,c},τ = {∅,Y,{a},{b},{a, b}} and σ = {∅,X,{a},{a,b}}J = {∅} .Define a function f : (X,τ) →(Y,σ,J) by $f(a) = b, f(b) = a$ and f (c) = c .Then the function f is continuous but not strongly M_I^*-continuous. For a M_I^*- closed set {b},we have $f^{-1}(\{b\}) = \{a\}$ which is not closed in (X, τ) .

Proposition 4.4.. Let (X, τ) be a topological space, (Y, σ, J) be a $T_{M_I^*}$ -space and f : (X, τ) → (Y, σ, J) be a map.Then the following are equivalent:

(i) f is strongly M_I^*-continuous,

(ii) f is continuous.

Proof. (i) ⇒ (ii) : Let V ba closed set in (Y, σ, J) .By Proposition, V is M_I^*-closed in (Y, σ, J) .Since f is strongly M_I^*-continuous, then $f^{-1}(V)$ is closed in (X,τ).Hence, f is continuous.

(ii)⇒(i) : Let V be any M_I^*-open set in (Y,σ,J) .Since (Y,σ,J) is a $T_{M_I^*}$ -space,V is open in (Y,σ,J) .By(ii),$f^{-1}(V)$ is open in (X,τ) .Therefore, f is strongly M_I^*-continuous.

Proposition 4.5.. Every strongly M_I^*-continuous map is SMPC(strongly M-pre continuous) but not conversely.

Proof. The proof follows from the fact that every pre-open set is M_I^*-open.

Example 4.6. Let X = Y = {a, b, c}, τ = {∅, Y, {a}, {a, b}, {a, c}} and σ = {∅, X, {a},{a, b}}J = {∅} .Define a function f : (X, τ) → (Y, σ, J) by f (a) = a, f (b) = c and f(c) = b.Then the function f is SMPC continuous but not strongly M_I^*-continuous.For a M_I^*-closed set {a,c} in (Y,σ,J) ,we have $f^{-1}(\{a,c\}) = \{a,b\}$ which is not closed in (X,τ) .

Proposition 4.7.. If a map f : (X, τ) → (Y, σ, J) is strongly continuous,then f is strongly M_I^*-continuous but not conversely.

Example 4.8. Let X = Y = {a, b, c}, τ = {∅, {a}, {b}, {a, b}, {a, c}}, σ = {∅, {a}, {a, b}}and J = {∅} .Define a f : (X, τ) →(Y, σ, J) be an identity function.Then f is strongly M_I^*-continuous but not strongly continuous.For a subset {a} of (Y,σ,J) ,we have $f^{-1}(\{a\})$ ={a},which is not closed in (X, τ) .

Proposition 4.9. Let f : (X, τ, I) → (Y, σ, J) be a map.Both (X, τ, I) and (Y, σ, J) are $T_{M_I^*}$ space.Then the following are equivalent

(i) f is M_I^*-irresolute,

(ii) f is strongly M_I^*-continuous,

(iii) f is continuous,

(iv) f is M_I^*-continuous.

Proof. (i) ⇒(ii). Let F be a M_I^*-closed set in (Y,σ,J) .Since f is M_I^*-irresolute, $f^{-1}(F)$ is M_I^*-closed set in (X,τ,I) .Therefore $f^{-1}(F)$ is closed in (X,τ,I) ,since (X,τ,I) is a $T_{M_I^*}$ space. Hence f is strongly M_I^*-continuous. The other implications are proved from Definitions.

Theorem 4.10. The composition of two strongly M_I^*-continuous maps is strongly M_I^*- continuous.

Proof. Let O be a M_I^*-open set in (Z,γ,K) .Since g is strongly M_I^*-continuous, we get $g^{-1}(O)$ is open in (Y, σ,J) .By Theorem 2.5,, $g^{-1}(O)$ is M_I^*-open in (Y,σ,J) .As f is strongly M_I^*-continuous, $f^{-1}(g^{-1}(O)) = (g\circ f)^{-1}(O)$ is open in (X,τ,I) .Hence g∘f is strongly M_I^*-continuous.

Deftnition 4.11. A map f : (X,τ) →(Y,σ,J) called perfectly M_I^*-continuous if the inverse image of every M_I^*-open set in (Y,σ,J) is both open and closed in (X, τ) .

Proposition 4.12.. If a map f : (X,τ) →(Y,σ,J) is perfectly M_I^*-continuous, then f is perfectly continuous (resp. continuous) but not conversely.

Proof. Let V be an open set in (Y,σ,J) .Then V is M_I^*-open in (Y,σ,J) .Since f is perfectly M_I^*-continuous, $f^{-1}(V)$ is both open and closed in (X,τ) .Thus f is perfectly continuous and also continuous.

Example 4.13. Let X = Y {a,b,c},τ = {∅,X,{a},{b,c}},σ = {∅,X,{a},{a,b}} and J = {∅}.Define a map f : (X,τ) → (Y,σ,J) by f(a) = a, f(b) = a, f(c) = b .Here the inverse image of every closed set is clopen but the inverse image of a M_I^*-closet set a, c},we have $f^{-1}(\{a, c\} = \{a, b\}$ which is neither open nor closed. Thus f is perfectly continuous but not perfectly M_I^*-continuous.

Proposition 4.14. If f : (X,τ) → (Y,σ,J) is perfectly M_I^*-continuous, then it is strongly M_I^*-continuous but not conversely.

Proof. Similar to the proof of Proposition 4.12.

Example 4.15 Let X = {a,b,c}, τ = {∅,X,{a},{b},{a,b}}, Y = {a,b,c}, σ = {∅,X,{a},{a,b},{a,c}} and J = {∅,{b}}. Define a map f : (X,τ) →(Y,σ,J) by f(a) = a, f(b) = c, f(c) = c. For a M_I^*-closed set {b,c} in (Y,σ,J), we have f^{-1}({b,c}) = {b,c} is closed but not open in (X,τ). Thus f is strongly M_I^*-continuous but not perfectly M_I^*-continuous.

Theorem 4.16. A map f : (X,τ) →(Y,σ,J) from a topological space (X,τ) into an ideal topological space (Y,σ,J) is perfectly M_I^*-continuous iff $f^{-1}(O)$ is both open and closed in (X,τ) for every M_I^*-closed set in (Y,σ,J).

Proof. Let O be any M_I^*-closed set in (Y,σ,J). Then O^c is M_I^*-open in (Y,σ,J). Since f is perfectly M_I^*-continuous, $f^{-1}(O^c)$ is both open and closed in (X,τ). But $f^{-1}(O^c)$=X/$f^{-1}(O)$ and so $f^{-1}(O)$ is both open and closed in (X, τ).

Conversely, assume that the inverse image of every M_I^*-closed set in (Y,σ,J) is both open and closed in (X,τ). Let O be any M_I^*-open set in (Y,σ,J). Then O^c is M_I^*-closed in (Y,σ,J). By assumption, $f^{-1}(O^c)$ is both open and closed in (X,τ). But $f^{-1}(O^c)$ = X/$f^{-1}(O)$ and so $f^{-1}(O)$ is both open and closed in (X, τ). Therefore, f is perfectly M_I^*-continuous.

Proposition 4.17. Let (X, τ) be a discrete topological space , (Y, σ, J) be an ideal topological space and f : (X,τ) → (Y, σ, J) be a map. Then the following are equivalent:

(i) f is perfectly M_I^*-continuous,

(ii) f is strongly M_I^*-continuous.

Proof. (i) ⇒ (ii) Follows from Proposition 4.14.

(ii) ⇒ (i): Let U be any M_I^*-open set in (Y,σ,J). By hypothesis, $f^{-1}(U)$ is open in (X, τ). Since (X, τ) is a discrete space, $f^{-1}(U)$ is also closed in (X, τ) and so f is perfectly M_I^*-continuous.

Theorem 4.18. If f : (X,τ,I) → (Y,σ,J) is strongly M_I^*-continuous and g :(Y,σ,J) → (Z,γ) is contra M_I^*-continuous then g ∘ f : (X, τ, I) →(Z, γ) is contra continuous.

Proof. Let U be any open set of (Z,γ). Since g is contra M_I^*-continuous , then $g^{-1}(U)$ is M_I^*- closed in (Y,σ, J) .Since f is strongly M_I^*-continuous, then $f^{-1}(g^{-1}(U))$ =$(g∘f)^{-1}(U)$ is closed in (X, τ) . Therefore g∘f is contra continuous .

Theorem 4.19. If f : (X,τ,I) → (Y,σ,J) is M_I^*-irresolute and g : (Y,σ,J) → (Z,γ,K) is strongly M_I^*-continuous then g∘f : (X,τ,I) →(Z,γ,K) is M_I^*-irresolute.

Proof. Let U be any M_I^*-open set in (Z,γ,K). Since g is strongly M_I^*-continuous, then $g^{-1}(U)$ is closed in (Y, σ,J). By Theorem 2.5, $g^{-1}(U)$ is M_I^*-closed in (Y,σ,J). Since f is M_I^*-irresolute, then $f^{-1}(g^{-1}(U)) = (g∘f)^{-1}(U)$ is M_I^*-closed in (X,τ,I). There fore g∘f is M_I^*-irresolute.

Theorem 4.20. If f : (X,τ) →(Y,σ,J) is perfectly M_I^*-continuous and g : (Y,σ,J) → (Z,γ) is contra M_I^*-continuous then g∘f is contra M_I^*-continuous.

Proof. Let U be any open set in (Z,γ) . Since g is contra M_I^*-continuous , then $g^{-1}(U)$ is M_I^*- closed in (Y,σ) and since f is perfectly M_I^*-continuous, then $f^{-1}(g^{-1}(U)) = (g∘f)^{-1}(U)$ is both open and closed in (X, τ) and so $(g∘f)^{-1}(U)$ is both open and preclosed in (X, τ) . By Theorem 2.5, $(g∘f)^{-1}(U)$ is M_I^*-closed in (X, τ). Therefore g∘f is contra M_I^*-continuous .

Proposition 4.21. If f : (X,τ) →(Y,σ,J) and g : (Y,σ,J) → (Z, γ, K) are perfectly M_I^*-continuous, then their composition g∘f : (X,τ) →(Z,γ,K) is also perfectly M_I^*- continuous.

Proof. Let V be an M_I^*-open set in (Z,γ,K). Since g is perfectly M_I^*-continuous, $g^{-1}(V)$ is both open and closed in (Y,σ,J). As f is perfectly M_I^*-continuous, $f^{-1}(g^{-1}(V)) = (g∘f)^{-1}(V)$ is both open and closed in (X,τ). Thus g∘f is perfectly M_I^*-continuous.

Proposition 4.22. Let f : (X,τ,I) →(Y,σ,J) and g : (Y, σ,J) →(Z,γ,K) be any two maps. Then their composition g∘f : (X,τ,I) →(Z,γ,K) is

(i) M_I^*-irresolute if g is perfectly M_I^*-continuous and f is M_I^*-continuous.

(ii) strongly M_I^*-continuous if g is perfectly M_I^*-continuous and f is continuous.

(iii) perfectly M_I^*-continuous if g is strongly continuous and f is perfectly M_I^*-continuous.

(iv) perfectly M_I^*-continuous if g is strongly M_I^*-continuous and f is perfectly M_I^*- continuous.

Proof. Similar to the proof of the Proposition 4.20 .

Theorem 4.23. Let f : (X,τ) →(Y,σ,J) and g : (Y,σ,J) →(Z,γ,K) be any two maps.Then their composition g ∘ f : (X,τ) →(Z,γ,K) is strongly M_I^*-continuous if g is M_I^*-irresolute and f is strongly M_I^*-continuous.

Proof. Let F be a M_I^*-closed subset of (Z,γ,K) .Since g is M_I^*-irresolute, $g^{-1}(F)$ is M_I^*-closed in (Y,σ,J) .Also since f is strongly M_I^*-continuous, $f^{-1}(g^{-1}(F))$ is closed in (X,τ) .Hence g∘f is strongly M_I^*-continuous.

Theorem 4.24.. Let f : (X,τ) →(Y,σ,J) and g : (Y,σ, J) →(Z,γ) be any two maps.Then their composition g∘ f : (X,τ) →(Z,γ) is continuous if g is M_I^*-continuous and f is strongly M_I^*-continuous.

Proof. Let F be any closed subset of (Z,γ) .Since g is M_I^*-continuous, $g^{-1}(F)$ is M_I^*- closed in (Y,σ,J) .Also since f is strongly M_I^*-continuous, $f^{-1}(g^{-1}(F))$ is closed in (X, τ) .Hence g ∘f is continuous.

Theorem 4.25.. Let f : (X, τ) → (Y, σ, J) and g : (Y, σ, J) → (Z, γ) be any two maps.Then their composition g ∘ f : (X, τ) →(Z, γ) is continuous if g is α -continuous and f is strongly M_I^*-continuous.

Proof. Let F be any closed subset of (Z, γ) .Since g is α -continuous, $g^{-1}(F)$ is α - closed in (Y, σ, J) .ByTheorem 2.5, $g^{-1}(F)$ is M_I^*-closed in (Y,σ,J) .Also since f is strongly M_I^*-continuous, $f^{-1}(g^{-1}(F))$ is closed in (X,τ) .Hence g∘f is continuous.

Theorem 4.26. Let f : (X,τ) →(Y,σ,J) and g : (Y,σ, J) →(Z,γ) be any two maps.Then their composition g∘ f : (X,τ)→(Z,γ) is *continuous if g is pre-continuous and f is strongly M_I^*-continuous.*

Proof. The proof is similar

Theorem 4.27. *Let f : (X,τ) →(Y,σ,J) and g : (Y,σ,J) →(Z,γ) be any two maps.Then their composition g∘f : (X,τ)→(Z,γ) is continuous if g is semi-continuous and f is strongly M_I^*-continuous.*

Proof. The proof is similar

Theorem 4.28. *Let f : (X, τ) → (Y, σ, J) and g : (Y, σ, J) → (Z, γ) be any two maps.Then their composition g ∘f : (X, τ) →(Z, γ) is continuous if g is β -continuous and f is strongly M_I^*-continuous.*

Proof. The proof is similar

Theorem 4.29. *Let f : (X,τ) →(Y,σ,J) and g : (Y,σ,J) →(Z,γ) be any two maps.Then their composition g∘f : (X,τ) →(Z,γ) is continuous if g is α−I−continuous and f is strongly M_I^*-continuous.*

Proof. The proof is similar

Theorem 4.30. *Let f : (X, τ) → (Y, σ, J) and g : (Y, σ, J) → (Z, γ) be any two maps.Then their composition g∘f : (X,τ) →(Z,γ) is continuous if g is pre−I−continuous and f is strongly M_I^*-continuous.*

Proof. The proof is similar

Theorem 4.31. *Let f : (X, τ) → (Y, σ, J) and g : (Y, σ, J) → (Z, γ) be any two maps.Then their composition g∘f : (X,τ) →(Z,γ) is continuous if g is semi −I−continuous and f is strongly M_I^*-continuous.*

Proof. The proof is similar

Theorem 4.32. *Let f : (X,τ) →(Y,σ,J) and g : (Y,σ,J) →(Z,γ) be any two maps.Then their composition g∘f : (X,τ) →(Z,γ) is continuous if g is β−I−continuous and f is strongly M_I^*-continuous.*

Proof. The proof is similar

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