# A Fitted Operator and Fitted Mesh Method for Singularly Perturbed Convection Diffusion Problem

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*Abstract*- **A new numerical method for a singularly perturbed boundary value problem using fitted operator and fitted mesh methods is presented in this paper. The specialty of this problem is, it is a problem with a boundary layer at left end of the domain. The method is stable, uniform and optimal with respect to the parameter (singularly perturbation parameter) in the problem. This method is computationally faster and takes less storage space in modern digital computer. Experimental results are presented to view the applicability of the method with the help of real time problems, using MAT-LAB.**

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## I. INTRODUCTION

Singular perturbation problems(SPP) occurring in aircraft trajectory guidance, satellite orbit, control system electro magnetic wave propagation, semi-conductor device, fluid dynamics, etc[1-19]. The traditional standard numerical methods will not solve the SPPs due to instability of the numerical solution. And so, explicit exponential fitted schemes have been designed based on fitted operator methods.[1,3-18] To view the initial/ boundary/ interior layers computational methods have been designed using uniform and variable meshes[7-11, 13-15]. In particular, in [7], a two point boundary value problem have been solved using a computational method, in which at the terminal point the solution of the SPP is approximated by the solution of the reduced problem. The region of domain is partitioned into the smooth and transient region. Both the regions are solved by a single exponential fitted operator method with one mesh in the smooth region and another mesh in the transient region. In the transient region an iterative procedure is applied. After the introduction of Shishkin fitted mesh, lot of changes[1,19] in the field of SPPs. Few draw backs are there in Shishkin fitted mesh methods and in fitted operator method, in the sense that, a method designed for a linear SPP cannot be directly extended to non-linear. Similarly cannot be extended from one-space dimension to higher dimensions [1, 19]. In [1], a direction is given to select either fitted operator or fitted mesh methods for a SPP with respect to the real time situation. Both the fitted operator and fitted mesh methods have to be further developed [1].

In [3], using fitted operator, explicit exponentially fitted operator schemes have been designed for linear and non-linear ordinary and partial SPPs. In [4] fitted operator higher order (two) explicit, uniform and optimal methods for first order linear SPPs are designed. In [5], fitted operator method of order one is designed for nonlinear SPP. In [6], using fitted operator method and shooting method a computational procedure is given for second order SPPs with mixed boundary conditions and with left boundary layer. In [7], using fitted operator method and boundary value technique using two different meshes, one mesh for smooth and another mesh for left boundary layer a computational procedure is given. In [8], using fitted operator method a chemical reactor problem is solved. In [9], using fitted operator method and boundary value technique using two different meshes, one mesh(h1 ) for smooth and another mesh but same mesh( h2 ). for both left and right boundary layers a computational procedure is given., In [10], using fitted operator method one-space dimensional heat equation is solved. In [11], using fitted operator method and initial value technique a computational procedure is given. In [12], using fitted operator method and shooting method a computational procedure is given for SPPs with Dirichlet's

conditions with left boundary layer. In [13], a fitted operator method is presented for a non-linear SPP with initial layer. In [14], using fitted operator method and boundary value technique a computational procedure is given as in [9], but with a change in evaluation of solution at terminal points. In [15], using fitted operator method and boundary value technique a computational procedure is given for linear first order SPPs. In [16], using fitted operator method an uniform and optimal method is designed for non-linear SPPs. In [17], a finite difference scheme is presented for non-liner problems. In [18], using fitted operator method an uniform and optimal scheme is given for first order linear SPPs. In [19], a stable numerical method is designed for ball bearing problems using fitted operator methods in [3]. In [3, 20], a full literature survey is given and the 30 years of war in designing numerical methods for SPPs is narrated. In [2], a fitted operator fourth order Numerov method is given for a SPP with multi scale behaviour. Some numerical results with absolute error for three test problems are provided. The error estimations are not provided for the continuous problem and numerical convergence is not provided. The graphical performance to view is also not provided.

In this paper, a new uniformly convergent numerical method for a SPP with multi scale nature is designed using fitted operator Numerov method and to view twin boundary layers fitted mesh method is applied.

Mathematical modeling for the transient situation in the ascend of the aircraft during the take off is given in section 2. Domain decomposition is also given in section 3. In Section 4 a fitted operator method is presented for the SPP. Fitted mesh method for the SPP is given in section 5. An algorithm is given in section 6. Final section 7 gives the experimental results using modern digital computer.
Throughout this paper, ρ=h/ε and C will be used to denote a generic constant independent of i,h and ε. Error stands for absolute error.

## II. MATHEMATICAL MODELING

The mathematical modeling for the transient situation in the ascend of the aircraft during the take off is given by the SPP,

$$L(u(t)) \equiv \varepsilon u''(t) + a(t)u' - b(t)u(t) = f(t), \quad 0 < t < 1, \quad (2.1)$$

$$B_0\, u(0) \equiv u(0) = \varphi_1, \quad B_1\, u(1) \equiv u(1) = \varphi_2, \quad (2.2)$$

where 1>>ε>0 is a small parameter, $\varphi_1$and $\varphi_2$ are constants, a, b and f are smooth functions satisfying $a(t) \geq \alpha > 0$, $b(t) \geq \beta > 0$ for all $t \in [0,1]$. The operator L admits maximum principle which is stated in the following theorem [3]

**Theorem 2.1**. Suppose $v$ is a smooth function satisfying $B_0\, v(0) \geq 0$ , $B_1\, v(1) \geq 0$ and $Lv(t) \leq 0$ for all $t$ in [0,1]. Then, v(t)≥0 or all t in [0,1].
**Proof**. Refer[3].
The stability result is given in the following theorem.[3]
**Theorem.2.2.** Let L be the operator in (2.1) and v be any smooth function then for all t in [0,1],

$$|v(t)| \leq C\,(|v(0)| + |v(1)| + \sup |Lu(s)|\,), s \in [0,1]$$

where C is independent of ε.
**Proof**. Refer[3].
Using Theorem.2.1 we can show that (2.1)-(2.2) has a unique solution and this solution has a boundary layer at the left end point of the domain. Using Theorem 2.2, the solution of (2.1)-(2.2) is stable. The reduced problem in this sense is

$$a(t)u'_0(t) - b(t)u_0(t) = f(t), 0 < t < 1, u_0(1) = \varphi_2 \quad (2.3)$$

and we see that, in general, $u_0$(t) will not satisfy the left boundary condition at $t = 0$.
An asymptotic expansion of order zero, we propose[2,3]

$$U(t) = u_0(t) + v_0(\tau) + O(\varepsilon) \quad (2.4)$$

where $u_0$(t) satisties equation (2.3) and the boundary layer function $v_0(\tau)$ satisfy the following differential equation :

$$\left[\frac{d^2}{d\tau^2}\right] v_0(\tau) + a(0)\left[\frac{d}{d\tau}\right] v_0(\tau) = 0, \tau \in (0,\omega) \quad (2.5)$$

$$v_0(\tau = 0) = \varphi_1 - u_0(0) \quad (2.6)$$

$$v_0(\tau = \infty) - 0 \quad (2.7)$$

where $\tau = t/\varepsilon$. The above equation is obtained by taking Taylors series expansion of a(t), b(t) about t=0, making change of variabkes t to τ and then equating powers of ε. The error estimation between solution of (2.1)-(2.2) and asymptotic expansion for the solution of u(t) is given in the following theorem.

**Theorem 2.3**. If $u$ is the solution $of$ (2.1) - (2.2) and $U$ is the solution given in (2.4) then for sufficiently smooth $a, b$ and $f$

$$|u(t) - U(t)| \leq C\varepsilon \quad (2.8)$$

Where $C$ is independent of $\varepsilon$.
**Proof.** Refer[3].

## III. DOMAIN DECOMPOSITION

Deccompose the domain [0,1] of the original problem into two equal subdomains as $[0,1] = [0,1/2] \cup [1/2,1]$ and define the original problem (2.1)-(2.2) into two problems as follows:

$$Lv(t) \equiv \varepsilon v''(t) + a(t)v' - b(t)v(t) = f(t), \quad 0 < t < 1/2 \quad (3.1)$$

$$B_0\, v(0) \equiv v(0) = \varphi_1, \quad B_1\, v(1/2) \equiv v(1/2) = u_0(1/2) \quad (3.2)$$

and

$$Lw(t) \equiv \varepsilon w''(t) + a(t)w' - b(t)w(t) = f(t), \quad 1/2 < t < 1 \quad (3.3)$$

$$B_0\, w(1/2) \equiv w(1/2) = u_0(1/2), \quad B_1\, w(1) \equiv w(1) = \varphi_2 \quad (3.4)$$

where $u_0(t)$ is as defined in (2.3). Now the solution of the problem (2.1)-(2.2) is defined as

$$u(t) = \begin{cases} v(t), 0 \leq t \leq \frac{1}{2} \\ w(t), 1/2 \leq t \leq 1 \end{cases}$$

In the following the error estimation between the solution of the problem (2.1)-(2.2) and (3.1)-(3.2) and (3.3)-(3.4). respectively are derived using maximum principle.

**Theorm 3.1**. If u is the solution of (2.1)-(2.2) and v is the solution of (3.1)-(3.2) then for sufficiently smooth a, b and f

$$|u(t) - v(t)| \leq C\varepsilon \quad (3.5)$$

where C is independent of ε.

**Proof.** For $t = 0, u(0) - v(0) = \varphi_1, - \varphi_1 = 0,$

For $0 < t < \frac{1}{2}, L[u(t) - v(t)] = Lu(t) - Lv(t)$
$= f(t) - f(t) = 0.$

For $t = \frac{1}{2}, u\left(\frac{1}{2}\right) - v\left(\frac{1}{2}\right) = u\left(\frac{1}{2}\right) - u_0\left(\frac{1}{2}\right)$
$= v_0\,(1/2) + O(\varepsilon) = O(\varepsilon)$

since $v_0\,(\tau) = O(\varepsilon)$.

Using maximum principle,

$$|u(t) - v(t)| \le C\,(|u(0) - v(0)| + |u(1/2) - v(1/2)| + sup\,|L\,[u(s) - v(s)]\,|\,)\; for\; s\; in\; [0,1/2] \le C\;\varepsilon$$

Hence the desired result.

**Theorm 3.2**. If u is the solution of (2.1)-(2.2) and w is the solution of (3.3)-(3.4) then for sufficiently smooth b and f

$$|u(t) - w(t)| \le C\varepsilon \qquad (3.6)$$

Where C is independent of $\varepsilon$.

**Proof.** For

$t = 1/2, u(1/2) - w(1/2) = u(1/2) - u_0(1/2)$
$= v_0\,(\tau) + O(\varepsilon) = + O(\varepsilon)$

$For\; 1/2 < t < 1, L[u(t) - w(t)] = Lu(t) - Lw(t)$
$= f(t) - f(t) = 0.$

$For\; t = 1, u(1) - w(1) = \varphi_2 - \varphi_2 = 0.$

Using maximum principle,

$$|u(t) - w(t)| \le C\,(|u(1/2) - w(1/2) + |u(1) - w(1)| + sup\,|L\,u(s) - w(s)|\,)\; for\; s\; in\; [1/2,1] \le C\;\varepsilon.$$

Hence the desired result.

## IV. FITTED OPERATOR METHOD

We apply central difference method and Bernoulli's generating function for the numerical solution of the problem (2.1)-(2.2) in the interval [0, ½]

$$L^h v_i \equiv \epsilon\sigma_1(\rho)\;\delta^2 v_i + a(t_i)\,D_0 v_i - b(t_i)\,v_i = t(t_i),\; i=1(1)\,N1-1, \qquad (4.1)$$

$$v_0 = \phi_1, \quad v_{N1} = u_0(1/2) \qquad (4.2)$$

where $\sigma_1(\rho) = [\,\rho a(0)/2]\;\coth(\rho\, a(0)/2)$, $\rho = h/\varepsilon$

and for (2.1)-(2.2) in the interval [1/2,1]

$$L^h w_i \equiv \varepsilon\,\sigma_2(\rho)\;\delta^2 w_i + a(t_i)\;D_0 w_i - b(t_i)\,w_i = t(t_i),\; i=1(1)N1-1, \qquad (4.3)$$

$$w_0 = u_0(1/2), \quad w_{N2} = \phi_2 \qquad (4.4)$$

where $\sigma_1(\rho) = [\,\rho a(t_i)/1]\;\coth(\rho\, a(t_i)/2)$, $\rho = h/\varepsilon$

The schemes (4.1)-(4.2) and (4.3)-(4.4) are consistent with (2.1)-(2.2) as the step size h approaches zero.

The numerical solution $u_i^h$ of (2.1)-(2.2) is defined as

$$u_i^h = v_i,\; 0 \le t \le 1/2,$$

and

$$u_i^h = w_i,\; 1/2 \le t \le 1.$$

The solutions $v_i$ and $w_i$ satisfy the stability result which is stated in the following theorem.

**Theorem.4.1.** Let $L^h$ be the operator in (4.1) and $v_i$ be any smooth function then for all t in [0,1/2],

$|\,v_i\,| \le C\,(\,|v_0| + |\,v_{N1}\,| + \sup|\,L^h v_i\,|\,)$, i=1(1) N1– 1,

where C is independent of ε.

**Proof.** Refer]3].

**Theorem.4.2.** Let $L^h$ be the operator in (4.3) and $w_i$ be any smooth function then for all t in [1/2,1],

$|\,w_i\,| \le C\,(\,|w_0| + |\,w_{N2}\,| + \sup|\,L^h w_i\,|\,)$, i=1(1) N2– 1,

where C is independent of ε.

**Proof.** Refer[3].

We have the error estimate $|\,u\,(t_i) - u_i^h\,|$ in [0, 1] as follows:

$|\,u\,(t_i) - u_i^h\,| \le |\,u\,(t_i) - v(t_i)| + |\,v(t_i) - v_i\,|$ in [0,1/2]

$|\,u\,(t_i) - u_i^h\,| \le |\,u\,(t_i) - w(t_i)\,| + |\,w(t_i) - w_i\,|$ in [1/2,1]

**Theorem.4.3.** Let v and $v_i$ be the solutions of (3.1)-(3.2) and (4.1)-(4.2) in [0,1/2] then

$|\,v(t_i) - v_i\,| \le C\min(\varepsilon, h^2)$ in [0,1/2]

where C is independent of i, h and ε.

**Proof.**:Refer[3].

**Theorem.4.4..** Let w and $w_i$ be the solutions of (3.3)-(3.4) and (4.3)-(4.4 in [ /2,1] then

$|\,w(t_i) - w_i\,| \le C\min(\varepsilon, h^2)$ in [ /2,1]

where C is independent of i, h and ε.

**Proof**:Refer[3].

Following theorm gives the uniform and optimal convergence result which is the main result of this section.

**Theorem.4.5..** Let u(t) and $u_i^h$ be the solutions of (2.1)-(2.2) and (4.1)-(4.4) in [ 0,1] then

$\|\,u(t_i) - u_i^h\,\| \le C\min(\varepsilon, h^2)$

where C is independent of i, h and ε.

**Proof.** Refer[3].

We have from the above Theorems 3.1, 3.2, 4.3 and 4.4 in [0,1],

$|\,u\,(t_i) - u_i^h\,| \le |\,u\,(t_i) - v(t_i)\,| + |\,v(t_i) - v_i\,|$
$\le C\,[\varepsilon + \min(\varepsilon, h^2)\,]$ in [0,1/2],

$|\,u\,(t_i) - u_i^h\,| \le |\,u\,(t_i) - w(t_i)\,| + |\,w(t_i) - w_i|$
$\le C\,[\varepsilon + \min(\varepsilon, h^2)\,]$ in [1/2,1] .

To find the error estimation, define in [0, 1]

$\|\,u\,(t_i) - u_i^h\,\| = \max(\,\max\{\,|\,u\,(t_i) - u_i^h\,|$ in [0,1/2] },
$\max\{|\,u(t_i) - u_i^h\,|$ in [1/2,1] } ).

Using all the results derived above, we have

$\|\,u\,(t_i) - u_i^h\,\| \le C\,[\varepsilon + \min(\varepsilon, h^2)\,] \le C\,\min(\varepsilon, h^2)$

which is the desired result.

## V. FITTED MESH METHOD

 

In this section, the mashes are no longer uniform it is necessary to extend the fitted operator method from the uniform meshes in section 4 to non-uniform meshes[1]. To introduce the method of fitted mesh the problems discussed in the previous section is considered again here. In all cases a piecewise uniform mesh turns out to be sufficient for the construction of ε-uniform method. Of course more complicated meshes may also be used but the simplicity of the piecewise uniform meshes is considered. Furthermore piecewise uniform meshes turns out to be adequate for handling a surprisingly a wide variety of singularly perturbed problems, For linear convection diffusion problem the following piecewise uniform mesh is constructed on the interval Ω=(0,1) Because there are boundary layers at left boundary points t=0 the mesh should be condensing in a neighbourhood of the left boundary point t=0.. The transition point is therefore required and mesh comprises two uniform pieces.

Perhaps a simplest example of a piecewise uniform mesh is constructed on the interval Ω=(0,1) as follows. Choose a point τ satisfying $0<\tau \le 1/2$and assume that $N=2^r$, for some r≥2. The point τ is called a transition point and it divides Ω into the two intervals (0,τ) and (τ,1). The corresponding piecewise uniform mesh is constructed by dividing both (0,τ) and (τ,1) into N/2 equal subintervals. Piecewise uniform meshes with N subintervals and a single parameter ι are denoted by $\Omega^r_N$ .

**Figure 1. The piecewise uniform mesh ΩrN condensing at the point t=0**

The piecewise uniform mesh $\Omega^r_N$ is used with the following location of the transition point

τ=min {1/2,2εln (N)} (5.1)

Depend on ε and N. This means location of the mesh points changes whenever ε or N changes. The transition point ï takes the value ½ if N is exponentially large and so $\Omega^r_N$ will be a uniform mesh with N subintervals. This will happen rarely in practice. We are interested in real time situation in which for all other values of ï, 0<ï<1/2. The subinterval (0,τ) is smaller than the subinterval and (τ,1). The fitted operator method in the previous section can be applied to the piecewise uniform mesh $\Omega^r_N$ . This leads to the following fitted mesh method

Find $\{u_\varepsilon\}_0^N \in R^{N+1}$, defined on $\Omega^r_N$ , such that $u_0 = \phi_1$ , $u_N = \phi_2$ and for all 1≤i≤N-1,

$$\varepsilon\sigma_1(\rho)\ \delta^2 u_i + a(t_i)\ D_0 u_i - b(t_i)\ u_i = t(t_i),\ i=1(1)\ N-1,$$

The fitted mesh method discussed above is ε-uniform and the solution satisfies the ε-uniform error estimate, for all $N \ge N_0$, $0<\varepsilon\le 1$, $\sup \| u_{\varepsilon,N} - u_\varepsilon \|_\omega \le C\, N^{-1} \ln(N)$ , where N and C are independent of ε.

## VI. ALGORITHM

An algorithm is presented so that a user can perform experiment without any difficulty in steps.

Step 1: Subdivide the interval (0, 1) into N intervals and generate a sequence x0, x1,...,xN.

Step 2: Subdivide the interval (0,1) into two subintervals (0,1/2) and (1/2,1)

Step 3: Subdivide the subintervals (0,1/2) and (1/2,1) into N/2 intervals of each.

Step 4: Rewrite the scheme (4.1)-(4.4) in tri-diagonal form

Step 5: Using sweep method rewrite the tri-diagonal form into a single step equation and solve for ui.

Step 6:Apply the scheme (4.1)-(4.2) in the subintervals (0,1/2).

Step 7: Apply the scheme (4.3)-(4.4) in the subintervals (1/2,1).

Step 8: Subdivide the subinterval (0,1) into (0,τ) and (τ,1).

Step 9: Subdivide the interval (τ,1) into (τ,1/2) and (1/2,1).

Step 10: Subdivide subintervals (τ,1/2) and (1/2,1) into N/4 intervals of each.

Step 11: Apply the scheme (4.1)-(4.2) in the subintervals (0,τ).

Step 12: Apply the scheme (4.3)-(4.4) in the subintervals (τ,1/2) and (1/2,1)

Using the steps 1-12, the numerical solution of u can be evaluated.. Bernouli's function with constant coefficient involved in the scheme (4.1)-(4.4) reduces both computation time and storage space in modern digital computers. The scheme is solved as a single step method. So the method is computationally faster.

Note: If a (t) =0 then the SPP reduces to the form of reaction diffusion type. A test problem of this type is given in the next section to show the application of the convection diffusion problem.

## VII. EXPERIMENTAL RESULTS

To show the performance of the fitted operator and fitted mesh method and to view the uniform and optimal convergence experiments were performed with the help of two test problems using modern digital computers

**Test Problem.1**

we consider a convection diffusion problem:

$\acute{\varepsilon}\, u'' + u' - u = 0$ , $t \in [0,1]$, u(0)=1, u(1)=0.05

whose exact solution is given by u(t) = $[C\, e^{tm_1} + D e^{tm_2}] / [e^{m_1} - e^{m_2}]$ where $m_{1,2} = [\,1 \pm \sqrt{1+4\varepsilon}\,]/2\varepsilon$, C=0.05-$e^{m_2}$ and D=$e^{m_1}$-0.05.

**Test Problem.2**

we consider a reaction diffusion problem:

$-\acute{\varepsilon}\, u'' + u = -[\,\cos^2(\pi t) + 2\pi^2 \cos(2\pi t)\,]$, $t \in [0,1]$, u(0)=0, u(1)=0

\whose exact solution is given by

$u(t) = [\,(e^{-(1-t)/\sqrt{\varepsilon}} + e^{-t/\sqrt{\varepsilon}})/(1 + e^{-1/\sqrt{\varepsilon}})\,] - \cos^2(\pi t)$.

The graphical results are given in Figure 2 for convection diffusion Test Problem 1 for $\varepsilon=\frac{1}{16}$ and h=$\frac{1}{32}$ It shows the numerical solution solves the problem exactly. This problem have only one boundary layer on the left boundary point t=0 of the domain [0,1].

The numerical method proposed in this paper is applied to reaction diffusion problem and it is tested with the Test Problem 2. The graphical results are given in Figure 3 $\varepsilon=\frac{1}{16}$ and h=$\frac{1}{32}$. It shows the numerical solution solves the problem exactly. This problem have two boundary layers, one at the left end of the boundary point t=0 and the other boundary layer at the right end of the boundary point t=1 of the domain [0,1].

Using the sweep method, [21] the scheme is converted into a single step method. Because of it the computation time and storage space for the execution of the computer program get reduced considerably. No need for inversion of matrix for the evaluation of the numerical solution.

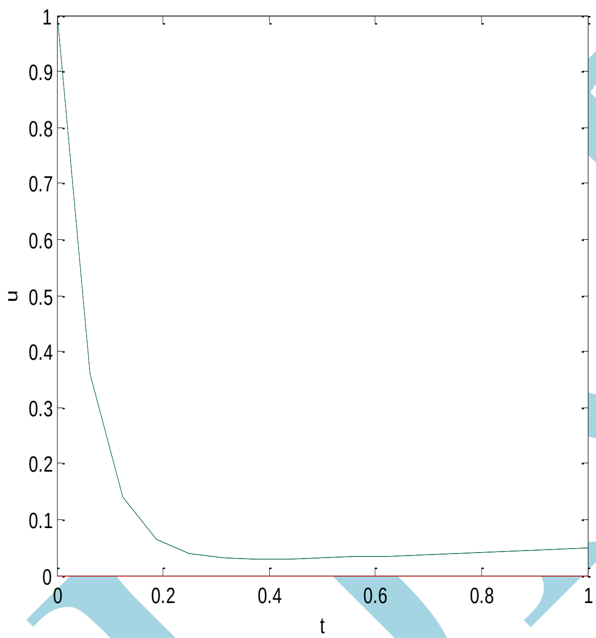


**Figure 2. The solution of the Test Problem 1**

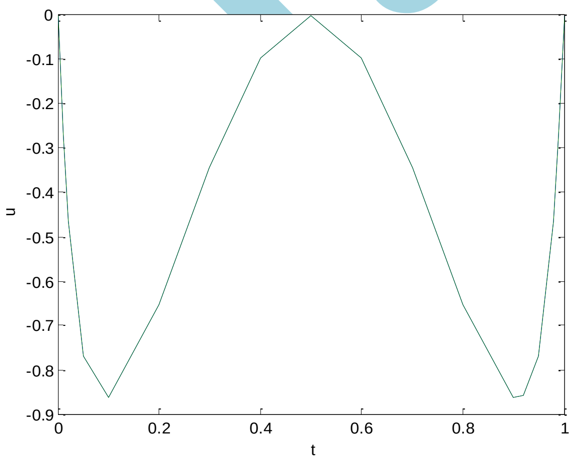


**Figure 3. The solution of the Test Problem 2**

The numerical results are given in Tables 1 and 2 using the Test problem 2.. It is observed that for $\varepsilon = 10^{-3}$ in Table 1. both absolute and relative errors are of order $10^{-6}$ shows the method is optimal and uniform. Similar results are obtained for small values of $\varepsilon = 10^{-4}$ in Table2.

**Table 1. Absolute and relative errors for ε= $10^{-3}$, h= $10^{-2}$**

| $t_i$ | $u(t_i)$ | $u_i$ | Absolute error | Relative error |
|---|---|---|---|---|
| **0.01** | -0.2701199 | -0.2701197 | *0.0000002* | 7.40E-07 |
| **0.02** | -0.4647717 | -0.4647713 | 0.0000004 | 8.61E-07 |
| **0.05** | -0.7697875 | -0.7697869 | 0.0000006 | 7.79E-07 |
| **0.1** | -0.8621789 | -0.8621783 | 0.0000001 | 1.16E-07 |
| **0.2** | -0.6527154 | -0.6527151 | 0.0000003 | 4.60E-07 |
| **0.3** | -0.3454137 | -0.3454138 | 0.0000001 | 2.90E-07 |
| **0.4** | -0.0954866 | -0.0954872 | 0.0000006 | 6.28E-06 |
| **0.5** | -0.0000003 | -0.0000005 | 0.0000006 | 6.28E-06 |
| **0.6** | -0.0954908 | -0.0954914 | 0.0000002 | 5.79E-07 |
| **0.7** | -0.3454204 | -0.3454206 | 0.0000003 | 4.60E-07 |
| **0.8** | -0.6527221 | -0.6527218 | 0.0000008 | 9.28E-07 |
| **0.9** | -0.8621831 | -0.8621823 | 0.0000009 | 1.05E-06 |
| **0.92** | -0.8584834 | -0.8584825 | 0.0000008 | 1.04E-06 |
| **0.95** | -0.7697898 | -0.769789 | 0.0000011 | 2.37E-06 |
| **0.98** | -0.4647733 | -0.4647722 | 0.0000013 | 4.81E-06 |
| **0.99** | -0.2701215 | -0.2701202 | 0 | - |
| **1** | 0 | 0 | 0 | - |
| | | Maximum | 1.30E-06 | 6.28E-06 |

Table 2. Absolute and relative errors for ε = $10^{-4}$, h= $10^{-2}$

| $t_i$ | $u(t_i)$ | $u_i$ | Absolute error | Relative error |
|---|---|---|---|---|
| **0** | 0 | 0 | 0 | 0 |
| **0.01** | -0.63113 | -0.63113 | 4.9E-06 | 7.76E-06 |
| **0.02** | -0.86072 | -0.86072 | 8.7E-06 | 1.01E-06 |
| **0.05** | -0.96879 | -0.96878 | 7.2E-06 | 7.43E-06 |
| **0.1** | -0.90446 | -0.90446 | 6.3E-06 | 6.97E-06 |
| **0.2** | -0.65451 | -0.6545 | 2.4E-06 | 3.67E-06 |
| **0.3** | -0.34549 | -0.34549 | 2.4E-06 | 6.95E-06 |
| **0.4** | -0.09549 | -0.0955 | 6.4E-06 | 6.70E-05 |
| **0.5** | 0 | -7.9E-06 | 7.9E-06 | - |
| **0.6** | -0.09549 | -0.0955 | 6.4E-06 | 6.70E-06 |
| **0.7** | -0.3455 | -0.3455 | 2.5E-06 | 7.24E-06 |
| **0.8** | -0.65451 | -0.65451 | 2.2E-06 | 3.36E-06 |
| **0.9** | -0.90447 | -0.90446 | 6.3E-06 | 6.97E-06 |
| **0.92** | -0.93782 | -0.93781 | 6.8E-06 | 7.25E-06 |

 

| | | | | |
|---|---|---|---|---|
| **0.95** | -0.96879 | -0.96879 | 7.2E-06 | 7.43E-06 |
| **0.98** | -0.86072 | -0.86072 | 7.2E-06 | 8.37E-06 |
| **0.99** | -0.63114 | -0.63113 | 6.8E-06 | 1.08E-05 |
| **1** | 0 | 0 | 0 | -- |
| | | Maximum | 8.70E-06 | 6.70E-05 |

## VIII. CONCLUSION

In this paper a convection diffusion problem with a boundary layer at left end of the boundary is considered for the numerical solution. A fitted operator and a fitted mesh method are designed which take less time and storage space for the computation in modern digital computers. The methods involved need no iteration, no matrix inversion for the numerical convergence. It works as a single step method. The method is uniform and optimal and so the numerical solution reflects the properties of the exact solution of the problem to be solved for small values of the singular perturbation parameter. This method can be applied to reaction diffusion problem which is proved experimentally in this paper.

## REFERENCES

[1] J.J.H.Miller, "Numerical solution of singularly perturbed differential equations", Lecture notes, Institute for Numerical Computation and Analysis, Dublin, Ireland, Feb 1994

[2] K.Phaneendral, P.Premond Chakravarthy and Y.N.Reddy, "A fitted Numerov method for singular perturbation Problems exhibiting twin layers", Applied Math & Inform. Sciences, 4(3), pp341-352, 2010.

[3] K.Selvakumar, "Uniformly Convergent Difference Schemes for Differential Equations with a Parameter", Ph.D.Thesis, Bharathidasan University, Tiruchirapalli, India, 1992.

[4] K.Selvakumar , "Optimal uniform finite difference schemes of order two for stiff initial value Problems", Communications in Numerical Methods in Engineering vol 10, pp. 611- 622,1994.

[5] K Selvakumar, "Optimal uniform finite difference schemes of order one for singularly. perturbed Riccati equation", Communications in Numerical Methods in Engineering, vol .13, pp 1-1 2, 1997

[6] K.Selvakumar, "A computational procedure for solving a chemical flow-reactor problem using shooting method", Applied Mathematics and Computation, vol 68,,pp.27-40, 1995.

[7] K.Selvakumar, "A computational method for solving singularly perturbation problems using exponentially fitted finite difference schemes" , Applied Mathematics and Computation, v66,pp 277-292, 1994

[8] K.Selvakumar, "Uniformly convergent finite difference schemes for singular perturbation problem arising in chemical reactor theory", International Journal of Computational Science and Mathematics, vol 2, pp. 77-90, 2010 .

[9] K.Selvakumar, "A computational method for solving singular Perturbed two point boundary value Problems without first derivative term", International e-Journal of Mathematics and Engineering, vol. 70, pp. 694-707.

[10] K.Selvakumar, "An exponentially fitted finite difference scheme For Heat equation", International e-Journal of Mathematics and Engineering, vol 79, pp. 776-787, 2010.

[11] K.Selvakumar, "Initial value method for solving second order singularly perturbed two point boundary value problem", International e-Journal of Mathematics and Engineering Vol 99, pp.920-931, 2010

[12] K Selvakumar, "A computational procedure for solving singular perturbation problem arising in Control system using shooting method", International Journal of computational Science and Mathematics, vol 1, pp 1-10, 2011.

[13] K.Selvakumar, "Optimal and uniform finite difference scheme for singularly perturbed Riccati Equation", International Journal of Computational Science and Mathematics, vol 3, pp 11-18, 2011.

[14] K.Selvakumar, "A computational method for solving singular perturbation problems without first derivative term" , International Journal of Computational Science and Mathematics, vol 3, No 1, pp 19-34, 2011.

[15] K.Selvakumar, " A computational method for solving singularly perturbed initial value problems", International Journal of Mathematics & Engineering, vol 161, pp.1487- 1501, 2012

[16] K.Selva kumar, "Optimal and uniform finite difference scheme for the scalar singularly perturbed Riccati equation", International Journ al of Mathematical Science, Technology & Humanities, vol.68, pp. 370- 382, 2012.

[17] K.Selvakumar,"A finite difference method for the numerical solution of First order nonlinear Differential equation", International e-Journal Mathematics & Engineering, vol. 184, pp1702- 1709, 2012

[18] K.Selvakumar, "Explicit but not fully implicit optimal and uniform finite difference schemes of order one for stiff initial value problems", Inl Journal of Mathematical Science, Technology & Humanities, vol. 53,pp.561-576, 2012.

[19] M.S.Starvin, K.Manisekar, A.Simon Christopher, K.Selvakumar, "A New Stable Numerical Method to find the Change in Angle of Contact in Angular contact

Ball bearing Subject to Thrust Load", Journal Balkan Tribiological Association, Journal of the Balkan Tribiological Association, Vol.21, No. 2, pp. 359-350, 2015.

[20] M.Stynes, "Numerical methods for convection-diffusion problems of The 30 years war, 20th Biennial"Numerical Analysis, University Dundee, Dundee, Scotland, June 24-27, 2003.

[21] Volkov, E.A, "Numerical Methods", Mir Publishers, Moscow, 1986.