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Strongly Ĝ*-Closed Sets in Topological Spaces

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Abstract: - In this paper, we introduce the concept of strongly g^*-closed sets in Topological spaces and investigate the relation between other closed sets. Also we characterize the properties. Mathematics Subject Classification: 54A05

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g^*-closed set, g*-closed set, strongly g^*-closed set.

1. INTRODUCTION

Norman Levine introduced and studied generalized closed (briefly g-closed) sets [1] in 1970. Njastad [3] introduced the concepts of α-sets for topological spaces. Palaniappan and Rao [4] introduced regular generalized closed sets (briefly rg-closed) sets in 1993.Pauline Mary Helen and Gayathri [5] introduced ĝ*-closed sets in topological spaces.In this paper, we introduce strongly ĝ*-closed sets together with the relationship of these sets with some sets. Throughout this paper (X,τ) and (Y, σ) represent non-empty topological spaces on which no separation axioms are assumed unless or otherwise mentioned. For a subset A of a space (X,τ) , cl(A) and int(A) denote the closure and the interior of A respectively.

Let us recall the following definitions, which are useful in the sequel.

DEFINITION 1.1- A subset A of a topological space (X, τ) a **pre-closed set** [2] if cl(int(A)) $\subseteq$ A.

DEFINITION 1.2- A subset A of a topological space (X, τ) an **α-closed set** [3] if cl(int(cl(A))) $\subseteq$ A.

DEFINITION 1.3- A subset A of a topological space (X, τ) a **regular closed set** [2] if A = cl(int(A)).

DEFINITION 1.4 - A subset A of a topological space (X,τ) is called a generalized closed set (briefly **g-closed**) [1] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ). The complement of a g-closed set is called a g-open set.

DEFINITION 1.5- A subset A of a topological space (X,τ) is called a regular generalized closed set(briefly **rg-closed**)[4] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular-open in (X, τ).

DEFINITION 1.6 - A subset A of a topological space (X,τ) is called a **g*-closed set** [6] if cl(A)$\subseteq$ U whenever A $\subseteq$ U and U is g-open in (X, τ).

DEFINITION 1.7 - A subset A of a topological space (X,τ) is called a **ĝ*-closed set** [5] if cl(A)$\subseteq$ U whenever A $\subseteq$ U and U is ĝ-open in (X, τ).

2. MAIN RESULTS

DEFINITION 2.1:

A subset A of a topological space (X,τ) is said to be a **strongly ĝ*-closed set** if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is ĝ-open in X.

THEOREM 2.2:
Every closed set is strongly ĝ*- closed set
PROOF:
Let A be a closed set
Let A $\subseteq$ U and U be ĝ-open in X
Since A is a closed set, cl(int(A)) $\subseteq$ cl(A) = A $\subseteq$ U and U is ĝ-open in X.
Hence, A is a strongly ĝ*-closed set.

REMARK 2.3:
The converse of the above theorem need not be true as can be seen from the following example.

EXAMPLE 2.4:
Let X = {a, b, c} and τ = **{**ϕ, X, {a}, {a, b}**}**. Consider A = {a, c}. A is not a closed set. However A is a strongly ĝ*-closed set.

THEOREM 2.5:
Every pre-closed set is strongly ĝ*- closed set
PROOF:
Let A be a pre-closed set
Let A $\subseteq$ U and U be ĝ-open in X
Since A is a pre-closed set, cl(int(A)) $\subseteq$ A $\subseteq$ U and U is ĝ-open in X.
Hence, A is a strongly ĝ*-closed set.

REMARK 2.6:
The converse of the above theorem need not be true as can

be seen from the following example.

EXAMPLE 2.7:
Let X = {a, b, c} and τ = **{**ϕ, X, {a}, {a, b}**}**. Consider A = {a, c}. A is not a preclosed set. However A is a strongly ĝ*-closed set.

THEOREM 2.8:
Every α-closed set is strongly ĝ*- closed set
PROOF:
Let A be a α-closed set
Let A ⊆ U and U be ĝ-open in X
Since A is a α-closed set, cl(int(A)) ⊆ cl(int(cl(A))) ⊆ A ⊆ U and U is ĝ-open in X.
Hence, A is a strongly ĝ*-closed set.

REMARK 2.9:
The converse of the above theorem need not be true as can be seen from the following example.

EXAMPLE 2.10:
Let X = {a, b, c} and τ = **{**ϕ, X, {b}**}**. Consider A = {a, b}. A is not a α-closed set. However A is a strongly ĝ*-closed set.

THEOREM 2.11:
Every regular-closed set is strongly ĝ*- closed set
PROOF:
Let A be a regular-closed set
Let A ⊆ U and U be ĝ-open in X
Since A is a regular-closed set, cl(int(A)) = A ⊆ U and U is ĝ-open in X.
Hence, A is a strongly ĝ*-closed set.

REMARK 2.12:
The converse of the above theorem need not be true as can be seen from the following example.

EXAMPLE 2.13:
Let X = {a, b, c} and τ = **{**ϕ, X, {a}, {b,c}**}**. Consider A = {a, c}. A is not a regular-closed set. However A is a strongly ĝ*-closed set.

THEOREM 2.14:
Every g-closed set is strongly ĝ*- closed set
PROOF:
Let A be a g-closed set
Let A ⊆ U and U be open in X.
Since "Every open is ĝ-open", U is ĝ-open in X.
Since A is a g-closed set, cl(int(A)) ⊆ cl(A) ⊆ U
Thus we get, cl(int(A)) ⊆ U and U is ĝ-open in X.
Hence, A is a strongly ĝ*-closed set.

REMARK 2.15:
The converse of the above theorem need not be true as can be seen from the following example.

EXAMPLE 2.16:
Let X = {a, b, c} and τ = **{**ϕ, X, {a}, {b}, {a,b}**}**. Consider A = {a, b}. A is not a g-closed set. However A is a strongly ĝ*-closed set.

THEOREM 2.17:
Every g*-closed set is strongly ĝ*- closed set
PROOF:
Let A be a g*-closed set
Let A ⊆ U and U be ĝ-open in X.
Since "Every ĝ-open is open", U is open in X.
Since A is a g*-closed set, cl(int(A)) ⊆ cl(A) ⊆ U
Thus we get, cl(int(A)) ⊆ U and U is ĝ-open in X.
Hence, A is a strongly ĝ*-closed set.

REMARK 2.18:
The converse of the above theorem need not be true as can be seen from the following example.

EXAMPLE 2.19:
Let X = {a, b, c} and τ = **{**ϕ, X, {a}, {b}, {a,b}**}**. Consider A = {a, b}. A is not a g*-closed set. However A is a strongly ĝ*-closed set.

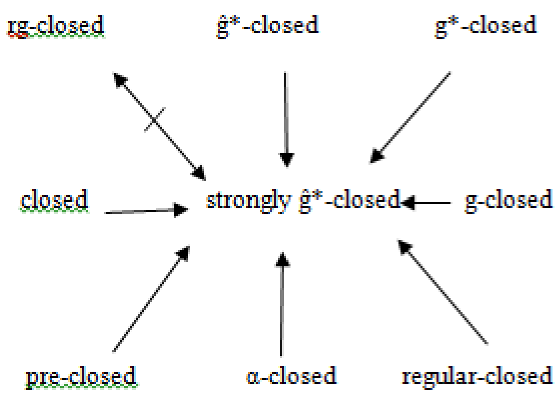
THEOREM 2.20:
Every ĝ*-closed set is strongly ĝ*- closed set
PROOF:
Let A be a ĝ *-closed set
Let A ⊆ U and U be ĝ-open in X.
Since A is a ĝ *-closed set, cl(int(A)) ⊆ cl(A) ⊆ U
Thus we get, cl(int(A)) ⊆ U and U is ĝ-open in X.
Hence, A is a strongly ĝ*-closed set.

REMARK 2.21:
The converse of the above theorem need not be true as can be seen from the following example.

EXAMPLE 2.22:
Let X = {a, b, c} and τ = **{**ϕ, X, {a}, {b}, {a,b}**}**. Consider A = {b}. A is not a ĝ *-closed set. However A is a strongly ĝ*-closed set.

REMARK 2.23:
The concepts of rg-closed sets and strongly ĝ*-closed sets are independent of each other.

REMARK 2.24:
The following diagram shows that the relationships between strongly ĝ * -closed sets and other existing generalized closed sets.



where A B (resp A B) represents A implies B (resp A and B are independent)

REMARK 2.25:
If A and B are strongly ĝ*-closed sets, then A∪B need not be strongly ĝ*- closed set as seen in the following example.

EXAMPLE 2.26:
Let X = {a, b, c} and τ = **{**ϕ, X, {a,c}**}**. Consider A = {a} and B = {c}.A and B are strongly ĝ*-closed sets. But A∪B is not strongly ĝ*-closed sets.

REMARK 2.27:
If A and B are strongly ĝ*-closed sets, then $A \cap B$ need not be strongly ĝ*- closed set as seen in the following example.

EXAMPLE 2.28:
Let X = {a, b, c, d} and τ = **{**ϕ, X, {a}, {a,c}, {a,b,d}**}**. Consider A = {a, b, c} and B = {a, c, d}
Here, A and B are strongly ĝ*-closed sets.
But $A \cap B$ is not strongly ĝ*-closed sets.

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