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New Class of Continuous Functions in Bitopological Spaces

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Abstract: - The concept of bitopological space was first introduced by J.C.Kelly in 1963 (i.e) a non empty set X equipped with two arbitrary topologies τ_1 and τ_2.The concept of generalized closed sets plays a significant role in general topology and these are the research topics of many Topologists worldwide.In 1970 Norman Levine introduced the concept of generalization of closed sets in topological spaces and he defined the semi-open sets and semi-continuity in bitopological spaces.The concept of continuity in topological spaces was extended to bitopological spaces by Pervin (1967).we have already introduced (i,j)-g^(##) closed sets (i.e)a subset A of a bitopological space (X,τ_1,τ_2) is called (i,j)-g^(##)-closed if τ_j-cl(A)⊆U,whenever A⊆U,U is τ_i-g^#-open in (X,τ_1 〖,τ〗 _2) and some of the properties were discussed. In this paper we introduce (i,j)-g^(##) continuous functions in bitopological spaces and discuss the relation with other continuous functions and obtained their characteristics

Keywords
Bitopological space,(i,j)-g^(##) closed set,(i,j)-g^(##) continuous map.

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I. INTRODUCTION

A triple (X,τ_1,τ_2) where X is a non-empty set τ_1 and τ_2 are topologies on X is called a bitopological space and Kelly[7] initiated the study of such spaces.In 1985,Fukutake[4] introduced the concepts of g-closed sets in bitopological spaces.In this paper we introduce new type of continuous map called (i,j)-$g^{\#\#}$ continuous map by applying (i,j)-$g^{\#\#}$ closed sets[12] in bitopological spaces and investigated their properties.

II. PRELIMINARIES

Definition 2.1 A subset A of a topological space (X,τ) is called

1. regular-open set[10] if $A = int(cl(A))$
2. generalized closed set[8](g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).
3. generalized star closed set[16](briefly g^*-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ U is g-open in (X,τ).
4. $g^\#$-closed set[18] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is αg-open in (X,τ).
5. α-closed[9] if $cl\left(int(cl(A))\right) \subseteq A$
6. a α-generalized closed[2] (briefly αg-closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).
7. g^*p-closed[17] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X,τ).

Definition 2.2 A subset A of a bitopological space (X,τ_i,τ_j) is called

1. a (i,j)-g-closed[4] if τ_j-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ_i.
2. a (i,j)-g^*-closed[14] if τ_j-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in τ_i.
3. a (i,j)-gs-closed[15] if τ_j-$scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ_i.
4. a (i,j)-gsp-closed[3] if τ_j-$spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ_i.
5. a (i,j)-gpr-closed[6] if τ_j-$pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is r-open in τ_i
6. a (i,j)-$(g^*p)^*$-closed[17] τ_j-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^*p-open in τ_i.
7. a (i,j)-$g^\#$-closed[16] τ_j-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is αg-open in τ_i.
8. a (i,j)-g^{**}-closed[11] if τ_j-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^*-open in τ_i.
9. a (i, j)-$g^{\#\#}$-closed[12] if τ_j-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $g^\#$-open in τ_i.
10. a (i,j)-wg-closed[5] if τ_j-$cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in τ_i.
11. a (i,j)-rg-closed[1] if τ_j-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is r-open in τ_i.
12. a (i,j)-αg-closed[13] if τ_j-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ_i.

Definition 2.3 A function $f:(X,\tau_1,\tau_2) \longrightarrow (Y,\sigma_1,\sigma_2)$ is called

1. (i,j)-g-continuous[4] if $f^{-1}(V)$ is a (i,j)-g-closed in (X,τ_1,τ_2) for every closed set V of (Y,σ_1,σ_2).
2. (i,j)-$g^{\#}$-continuous[17] if $f^{-1}(V)$ is a (i,j)-$g^{\#}$-closed in (X,τ_1,τ_2) for every closed set V of (Y,σ_1,σ_2).
3. (i,j)-gsp-continuous[3] if $f^{-1}(V)$ is a (i,j)-gsp-closed in (X,τ_1,τ_2) for every closed set V of (Y,σ_1,σ_2).
4. (i,j)-rg-continuous[1] if $f^{-1}(V)$ is a (i,j)-rg-closed in (X,τ_1,τ_2) for every closed set V of (Y,σ_1,σ_2).
5. (i,j)-wg-continuous[5] if $f^{-1}(V)$ is a (i,j)-wg-closed in (X,τ_1,τ_2) for every closed set V of (Y,σ_1,σ_2).
6. (i,j)-gpr-continuous[6] if $f^{-1}(V)$ is a (i,j)-gpr-closed in (X,τ_1,τ_2) for every closed set V of (Y,σ_1,σ_2).
7. (i,j)-g^{**}-continuous[11] if $f^{-1}(V)$ is a (i,j)-g^{**}-closed in (X,τ_1,τ_2) for every closed set V of (Y,σ_1,σ_2).
8. (i,j)-αg-continuous[13] if $f^{-1}(V)$ is a (i,j)-αg-closed in (X,τ_1,τ_2) for every closed set V of (Y,σ_1,σ_2).
9. (i,j)-$((g^*p)^*)$ continuous[17]if $f^{-1}(V)$ is a (i,j)-$((g^*p)^*)$-closed in (X,τ_1,τ_2) for every closed set V of (Y,σ_1,σ_2).
10. τ_j-σ_k continuous[14] if $f^{-1}(V) \in \tau_j$ for every $V \in \sigma_k$.

We introduce the following definition:

Definition 3.1:
A map $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ from a topological space (X,τ_1,τ_2) to a topological space (Y,σ_1,σ_2) is called a (i,j)-$g^{\#\#}$ continuous if the inverse image of every closed set in (Y,σ_1,σ_2) in (i,j)-$g^{\#\#}$ closed set in (X,τ_1,τ_2).

Preposition 3.2: Every continuous map is (i,j)-$g^{\#\#}$ continuous.

Proof : Let $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be a continuous map.
To prove: f is (i,j)-$g^{\#\#}$ continuous.
Let V be a closed set in (Y,σ_1,σ_2).
Since f is continuous,f^{-1} (V) is closed in (X,τ_1,τ_2).
Since f^{-1} (V) is (i,j)-$g^{\#\#}$ closed.
Hence f is (i,j)-$g^{\#\#}$ continuous.

Remark 3.3 Converse of the above proposition is not true in general as seen from the following example.

Example 3.4: Let $X = \{a,b,c\}$ τ_1=$\{\varphi,X,\{a\},\{a,b\}\}$,$\tau_2$=$\{\varphi,X,\{a\},\{b,c\}\}$
And let $Y = \{p,q\}$ σ_1=$\{\varphi,Y,\{p\}\}$,σ_2=$\{\varphi,Y,\{q\}\}$
Let $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be defined by $f(b)$=$f(c)$=$\{p\}$ and $f(a)$=$\{q\}$
To prove : f is (i,j)-$g^{\#\#}$ continuous but not continuous.
f^{-1} (p)=$\{b,c\}$ and f^{-1} (q)=$\{a\}$ are (i,j)-$g^{\#\#}$ closed in (X,τ_1,τ_2)
Therefore f is (i,j)-$g^{\#\#}$ continuous.
But f^{-1} (p)=$\{b,c\}$ and f^{-1} (q)=$\{a\}$ are not closed in (X,τ_1,τ_2).
Therefore f is not continuous.

Proposition 3.5: If $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is τ_j-σ_k continuous,then it is (i,j)-$g^{\#\#}$ continuous.

Proof: Let V be a closed set in (Y,σ_1,σ_2)
Let V be σ_k-closed.Then f^{-1} (V) is τ_j-closed and every τ_j-closed set is (i,j)-$g^{\#\#}$ closed.
Therefore f^{-1} (V) is (i,j)-$g^{\#\#}$ closed.
Hence f is (i,j)-$g^{\#\#}$ continuous.

Remark 3.6: Converse of the above proposition is not true as seen from the following example.

Example 3.7: Let X=Y=$\{a,b,c\}$,τ_1=$\{\varphi,X,\{b\}\}$,τ_2=$\{\varphi,X,\{c\}\}$ and
σ_1 =$\{\varphi,Y,\{a\}\}$,σ_2=$\{\varphi,Y,\{b\},\{c\}\}$
Define a mapping $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ by $f(a)$=$f(b)$=c;$f(c)$=a
$f^{-1}(c)$=$\{a,b\}$ and $f^{-1}(a)$=c are (i,j)-$g^{\#\#}$ closed in (X,τ_1,τ_2)
Hence f is a (i,j)-$g^{\#\#}$ continuous map
$\{c\} \in \sigma_2$ but $f^{-1}(c)$=$\{a,b\}$ is not a τ_2 closed.
Therefore f is not τ_2-σ_2 continuous.

Proposition 3.8: Every (i,j)-g-continuous map is (i,j)-$g^{\#\#}$ continuous.

Proof: Let V be a closed set in $(\mathrm{Y},\sigma_1,\sigma_2)$
Then $f^{-1}(V)$ is (i,j)-g- closed and every (i,j)-g-closed set is (i,j)-$g^{\#\#}$ closed.
Therefore $f^{-1}(V)$ is (i,j)-$g^{\#\#}$ closed.
Hence f is (i,j)-$g^{\#\#}$ continuous.

Proposition 3.9: Every (i,j)-$g^{\#\#}$ continuous map is (i,j)-gpr continuous.

Proof: Let $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be a (i,j)-$g^{\#\#}$ continuous map.
To prove : f is (i,j)-gpr continuous.
Let V be a closed set in (Y,σ_1,σ_2).
Since f is (i,j)-$g^{\#\#}$ continuous,f^{-1} (V) is (i,j)-$g^{\#\#}$ closed in (X,τ_1,τ_2).
f^{-1} (V) is (i,j)-gpr closed.
Hence f is (i,j)-gpr continuous.

Remark 3.10: Converse of the above proposition is not true as seen from the following example.

Example 3.11: Let X=Y=$\{a,b,c\}$,τ_1=$\{\varphi,X,\{b\},\{a,b\}\}$,$\tau_2$=$\{\varphi,X,\{a,c\}\}$ and
σ_1 =$\{\varphi,Y,\{b,c\}\}$,σ_2=$\{\varphi,Y,\{a,b\}\}$
Define a mapping $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ by $f(a)$=$f(b)$=c;$f(c)$=a
$f^{-1}(c)$=$\{a,b\}$ and $f^{-1}(a)$=c are (i,j)-gpr closed set in (X,τ_1,τ_2)
Hence f is a (i,j)-gpr continuous map.

$f^{-1}(c)$={a,b} is not a (i,j)-$g^{\#\#}$ closed.
Therefore f is not (i,j)-$g^{\#\#}$ continuous.
Hence f is (i,j)-gpr continuous but not (i,j)-$g^{\#\#}$ continuous.

Proposition 3.12: Every (i,j)-$((g^*p)^*)$ continuous map is (i,j)-$g^{\#\#}$ continuous.

Proof: Let **:** Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a (i,j)-$((g^*p)^*)$ continuous map.
To prove: f is (i,j)-$g^{\#\#}$ continuous.
Let V be closed set in (Y,σ_1,σ_2).
Since f is (i,j)-$((g^*p)^*)$ continuous, $f^{-1}(V)$ is (i,j)-$((g^*p)^*)$ closed in (X,τ_1,τ_2).
Since Every (i,j)-$((g^*p)^*)$ closed set is (i,j)-$g^{\#\#}$ closed.
So $f^{-1}(V)$ is (i,j)-$g^{\#\#}$ closed.
Hence f is (i,j)-$g^{\#\#}$ continuous.

Remark 3.13: Converse of the above proposition is not true as seen from the following example.

Example 3.14: Let X={a,b,c} ,τ_1={$\varphi,X,\{a\}$},τ_2={$\varphi,X,\{a\},\{a,b\}$} and Y={p,q}
σ_1 ={$\varphi,Y,\{p\}$},σ_2={$\varphi,Y,\{q\}$}
Define a mapping $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ by $f(a)$=$f(c)$=p;$f(b)$=q
To prove: f is (i,j)-$g^{\#\#}$ continuous but not (i,j)-$((g^*p)^*)$ continuous.
$f^{-1}(p)$={a,c} and $f^{-1}(q)$=b are (i,j)-$g^{\#\#}$ closed set in (X,τ_1,τ_2) but not (i,j)-$((g^*p)^*)$ closed in (X,τ_1,τ_2).
Hence f is (i,j)-$g^{\#\#}$ continuous but not (i,j)-$((g^*p)^*)$ continuous.

Proposition 3.15: Every (i,j)-g^* continuous map is (i,j)-$g^{\#\#}$ continuous.

Proof : Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be (i,j)-g^* continuous.
Let V be a closed set in (Y,σ_1,σ_2).
Let us prove that $f^{-1}(V)$ is (i,j)-$g^{\#\#}$ closed in (X,τ_1,τ_2).
Since f is (i,j)-g^* continuous, $f^{-1}(V)$ is (i,j)-g^* closed.
$f^{-1}(V)$ is (i,j)-$g^{\#\#}$ closed,since Every (i,j)-g^* closed set is (i,j)-$g^{\#\#}$ closed.
Therefore f is (i,j)-$g^{\#\#}$ continuous.

Remark 3.16: Converse of the above proposition is not true as seen from the following example:

Example 3.17: Let X=Y={a,b,c} ,τ_1={$\varphi,X,\{b\}$},τ_2={$\varphi,X,\{c\}$} and
σ_1={$\varphi,Y,\{b,c\}$},σ_2={$\varphi,Y,\{a\}$}
Define $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ by $f(a)$={b,c},$f(b)$=$f(c)$={a}
$f^{-1}\{b,c\}$={a} and $f^{-1}\{a\}$={b,c}
$f^{-1}\{a\}$={b,c} and $f^{-1}\{b,c\}$={a} are (i,j)-$g^{\#\#}$ closed sets in (X,τ_1,τ_2)
Hence f is (i,j)-$g^{\#\#}$ continuous map.
$f^{-1}\{a\}$={b,c} and $f^{-1}\{b,c\}$={a} are not a (i,j)-g^* closed set.
Therefore f is not (i,j)-g^* continuous map.
Hence f is (i,j)-$g^{\#\#}$ continuous but not (i,j)-g^* continuous.

Proposition 3.18: Every (i,j)-$g^{\#\#}$ continuous map (i,j)-wg continuous.

Proof : Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be (i,j)-$g^{\#\#}$ continuous.
Let V be a closed set in (Y,σ_1,σ_2).
Let us prove that $f^{-1}(V)$ is (i,j)-wg closed in (X,τ_1,τ_2).
Since f is (i,j)-$g^{\#\#}$ continuous, $f^{-1}(V)$ is (i,j)-$g^{\#\#}$ closed.
Since every (i,j)-$g^{\#\#}$ closed is (i,j)-wg closed, $f^{-1}(V)$ is (i,j)-wg closed.
Therefore f is (i,j)-wg continuous.

Remark 3.19: The converse of the above proposition is not true as seen from the following example:

Example 3.20:
Let$X$=Y={a,b,c} τ_1={$\varphi,X,\{a\},\{a,c\}$},τ_2={$\varphi,X,\{a\}$} and σ_1={$\varphi,Y,\{a,c\}$},σ_2={$\varphi,Y,\{c\}$}
Define $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ by $f(c)$={b}, $f(a)$=$f(b)$={c}
$f^{-1}\{b\}$={c} and $f^{-1}\{c\}$={a,b} are (i,j)-wg closed sets in (X,τ_1,τ_2)
Hence f is (i,j)-wg continuous map.
$f^{-1}\{b\}$={c} is not (i,j)-$g^{\#\#}$ closed sets in (X,τ_1,τ_2).
Therefore f is not (i,j)-$g^{\#\#}$ continuous.
Hence f is (i,j)-wg continuous but not (i,j)-$g^{\#\#}$ continuous.

Proposition 3.21: Every (i,j)-$g^{\#\#}$-continuous map is (i,j)-rg continuous.

Proof: Let f: $(X,\tau_1,\tau_2)\longrightarrow(\mathrm{Y},\sigma_1,\sigma_2)$ be a (i,j)-$g^{\#\#}$-continuous map.
Let us prove that f is (i,j)-rg continuous
Let V be a closed set in $(\mathrm{Y},\sigma_1,\sigma_2)$
Since f is (i,j)-$g^{\#\#}$-continuous, $f^{-1}(V)$ is (i,j)-$g^{\#\#}$-closed.
Every (i,j)-rg-closed set is (i,j)-$g^{\#\#}$ closed.
Therefore $f^{-1}(V)$ is (i,j)-rg closed.
Hence f is (i,j)-rg continuous.

Proposition 3.22: Every (i,j)-$g^{\#\#}$ continuous map (i,j)-gsp continuous.

Proof: Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be (i,j)-$g^{\#\#}$ continuous.
Let V be a closed set in (Y,σ_1,σ_2).
Let us prove that $f^{-1}(V)$ is (i,j)-gsp closed in (X,τ_1,τ_2).
Since f is (i,j)-$g^{\#\#}$ continuous, $f^{-1}(V)$ is (i,j)-$g^{\#\#}$ closed.
$f^{-1}(V)$ is (i,j)-gsp closed.since every (i,j)-$g^{\#\#}$ closed is (i,j)-gsp closed.
Therefore f is (i,j)-gsp continuous.

Remark 3.23: The converse of the above proposition is not true as seen from the following example:

Example 3.24:
Let $X=Y=$ {a,b,c},τ_1={$\varphi,X,\{b\},\{a,b\}$},τ_2={$\varphi,X,\{a,c\}$} and σ_1={$\varphi,Y,\{a,b\}$},σ_2={$\varphi,Y,\{c\}$}
Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ the identity map.

Let us prove that f is (i,j)-gsp continuous but not (i,j)-$g^{\#\#}$ continuous.
$f^{-1}(c)$=$\{c\}$ and $f^{-1}(a,b)$=$\{a,b\}$ is (i,j)-gsp closed in (X,τ_1,τ_2).
Hence f is (i,j)-gsp continuous.
$f^{-1}(a,b)$=$\{a,b\}$ is not (i,j)-$g^{\#\#}$ closed in (X,τ_1,τ_2)
Therefore f is not (i,j)-$g^{\#\#}$ continuous.
Hence f is (i,j)-gsp continuous but not (i,j)-$g^{\#\#}$ continuous.

Proposition 3.25: Every (i,j)-$g^{\#\#}$ continuous map (i,j)-αg continuous.

Proof: : Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be (i,j)-$g^{\#\#}$ continuous.
Let V be a closed set in (Y,σ_1,σ_2).
Let us prove that $f^{-1}(V)$ is (i,j)-αg closed in (X,τ_1,τ_2).
Since f is (i,j)-$g^{\#\#}$ continuous, $f^{-1}(V)$ is (i,j)-$g^{\#\#}$ closed.
Since $f^{-1}(V)$ is (i,j)-αg closed.
Therefore f is (i,j)-αg continuous.

Remark 3.26: The converse of the above proposition is not true as seen from the following example.

Example 3.27:
Let $X=Y=\{a,b,c\}$,τ_1=$\{\varphi,X,\{a\},\{b,c\}\}$,$\tau_2$=$\{\varphi,X,\{a\},\{b\},\{a,b\},\{a,c\}\}$ and σ_1=$\{\varphi,Y,\{b,c\}\}$,σ_2=$\{\varphi,Y,\{a\}\}$
Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ the identity map.
Let us prove that f is (i,j)-αg continuous but not (i,j)-$g^{\#\#}$ continuous.
$f^{-1}(a)$=$\{a\}$ and $f^{-1}(b,c)$=$\{b,c\}$ is (i,j)-αg closed sets in (X,τ_1,τ_2).
Hence f is (i,j)-αg continuous.
$f^{-1}(a)$=$\{a\}$ is not (i,j)-$g^{\#\#}$ closed in (X,τ_1,τ_2).
Therefore f is not (i,j)-$g^{\#\#}$ continuous.
Hence f is (i,j)-αg continuous but not (i,j)-$g^{\#\#}$ continuous.

Proposition 3.28: Every (i,j)-$g^{\#\#}$-continuous map (i,j)-g^{**} continuous.

Proof : Let f: $(X,\tau_1,\tau_2)\longrightarrow$ (Y, σ_1,σ_2) be a (i,j)-$g^{\#\#}$-continuous map.
Let us prove that f is (i,j)-g^{**} continuous
Let V be a closed set in (Y, σ_1,σ_2)
Since f is (i,j)-$g^{\#\#}$-continuous, $f^{-1}(V)$ is (i,j)-$g^{\#\#}$-closed.
Every (i,j)-g^{**}-closed set is (i,j)-$g^{\#\#}$ closed.
Therefore $f^{-1}(V)$ is (i,j)-g^{**} closed.
Hence f is (i,j)-g^{**} continuous.

Proposition 3.29: Every (i,j)-$g^{\#\#}$ continuous map (i,j)-gs continuous.

Proof: Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be (i,j)-$g^{\#\#}$ continuous.
Let V be a closed set in (Y,σ_1,σ_2).
Let us prove that $f^{-1}(V)$ is (i,j)-gs closed in (X,τ_1,τ_2).
Since f is (i,j)-$g^{\#\#}$ continuous, $f^{-1}(V)$ is (i,j)-$g^{\#\#}$ closed.
$f^{-1}(V)$ is (i,j)-gs closed.
Therefore f is (i,j)-gs continuous.

Remark 3.30: The converse of the above proposition is not true as seen from the following example:

Example 3.31:
Let $X=Y\{a,b,c\}$,τ_1=$\{\varphi,X,\{a\},\{a,b\}\}$,$\tau_2$=$\{\varphi,X,\{a\},\{b\},\{a,b\}\}$ and σ_1=$\{\varphi,Y,\{a,c\}\}$,σ_2=$\{\varphi,Y,\{b,c\}\}$
Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be the identity map.
To prove: f is (i,j)-gs continuous but not (i,j)-$g^{\#\#}$ continuous.
$f^{-1}(a)$=$\{a\}$ and $f^{-1}(b)$=$\{b\}$ is (i,j)-gs closed sets in (X,τ_1,τ_2).
Hence f is (i,j)-gs continuous.
$f^{-1}(a)$=$\{a\}$ and $f^{-1}(b)$=$\{b\}$ is not (i,j)-$g^{\#\#}$ closed in (X,τ_1,τ_2).
Therefore f is not (i,j)-$g^{\#\#}$ continuous.
Hence f is (i,j)-gs continuous but not (i,j)-$g^{\#\#}$ continuous.

Proposition 3.32: Every (i,j)-$g^{\#}$ continuous map is (i,j)-$g^{\#\#}$ continuous.

Proof : Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be (i,j)-$g^{\#}$ continuous.
Let V be a closed set in (Y,σ_1,σ_2).
Let us prove that $f^{-1}(V)$ is (i,j)-$g^{\#\#}$ closed in (X,τ_1,τ_2).
Since f is (i,j)-$g^{\#}$ continuous, $f^{-1}(V)$ is (i,j)-g^{*} closed.
$f^{-1}(V)$ is (i,j)-$g^{\#\#}$ closed.since Every (i,j)-$g^{\#}$ closed set is (i,j)-$g^{\#\#}$ closed.
Therefore f is (i,j)-$g^{\#\#}$ continuous.

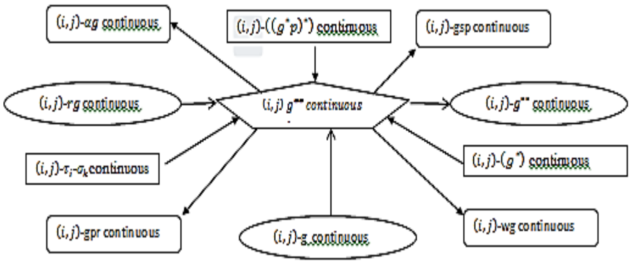
Remark 3.33: The converse of the above proposition is not true as seen from the following example:

Example3.34: Let X=$\{a,b,c\}$,τ_1=$\{\varphi,X,\{b\}\}$,τ_2=$\{\varphi,X,\{a,c\}\}$ and Y=$\{p,q\}$
σ_1 =$\{\varphi,Y,\{p\}\}$,σ_2=$\{\varphi,Y,\{q\}\}$
Define a mapping $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ by $f(b)$=$f(c)$=p;$f(a)$=q
$f^{-1}(p)$=$\{b,c\}$ and $f^{-1}(q)$=$\{a\}$ are (i,j)-$g^{\#\#}$ closed set in (X,τ_1,τ_2) but not (i,j)-$g^{\#}$ closed in (X,τ_1,τ_2)
Hence f is (i,j)-$g^{\#\#}$ continuous but not (i,j)-$g^{\#}$ continuous.

Proposition 3.35: Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ and $g:(Y,\sigma_1,\sigma_2)\to(Z,\eta_1,\eta_2)$ be two maps.Then $(g\circ f)$ is (i,j)-$g^{\#\#}$ continuous if g is continuous and f is (i,j)-$g^{\#\#}$ continuous.

Proof: Let V be closed set in (Z,η_1,η_2).
Let us prove that $(g\circ f)^{-1}(V)$ is (i,j)-$g^{\#\#}$ closed in (Y,σ_1,σ_2).
Since g is continuous,$g^{-1}(V)$ is closed in (Y,σ_1,σ_2).
Since f is (i,j)-$g^{\#\#}$ continuous,$f^{-1}(g^{-1}(V))$ is (i,j)-$g^{\#\#}$ closed in (X,τ_1,τ_2).
Therefore $(g\circ f)^{-1}(V)$ is (i,j)-$g^{\#\#}$ closed in (X,τ_1,τ_2).

Therefore $(g \circ f)$ is (i,j)-$g^{\#\#}$ continuous
The above results can be represented in the following figure:



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