On Intuitionistic Fuzzy Bi-Ideals in Boolean Like Semi Ring

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Abstract: - In this paper, we extend the notion of Intuitionistic fuzzy bi-ideals in Boolean like semi rings and obtain some of their related properties.

Keywords

Boolean like semi ring, Intuitionistic fuzzy set, Intuitionistic fuzzy bi-ideal.

1. INTRODUCTION

The notion of fuzzy sets and fuzzy logic was introduced by Lotfi A.Zadeh in 1965. Fuzziness occurs when the knowledge is not precise. A Fuzzy set can be defined by assigning to each individual of the universe under consideration, a value of membership. Fuzzy theory is associated with information theory and uncertainty. Fuzzy ideals of rings were introduced by Ziu, and it has been studied by several authors. Boolean like semi rings were introduced in roll by K.Venkatesawarlu, B.V.N. Murthy and N.Amaranth during 2011. A Boolean like ring is a commutative ring with unity and is of characteristic 2. It is clear that every Boolean ring is a Boolean like ring but not conversely. The idea of "Intuitionistic Fuzzy Set (IFS)" was first published by Atanassov as a generalization of the notion of fuzzy set. In this paper, we introduce the concept of On intuitionistic Fuzzy Bi-Ideals in Boolean like semi rings.

II. PRELIMINARIES

Definition : 2.1

A non empty set R with two binary operations '+' and '.' is called a near-ring if

(R,+) is a group

(R,.) is a semigroup

$x.(y+z) = x.y + x.z$, v x,y,z ϵ R

Definition : 2.2

A system (R,+, .) a Boolean semi ring iff the following properties hold

(R,+) is an additive(abelian)group(whose 'zero' will be denoted by'0')

(R,.) is a semigroup of idempotents in the sende aa=a, v a ϵ R

$a(b+c) = ab+ac$ &

$abc = bac$, v a,b,c ϵ R

Definition: 2.3

A non-empty set R together with two binary operations + and. satisfying the following conditions is called a Boolean like semi ring.

(R,+) is an abelian group

(R,.) is a semigroup

$a.(b+c) = a.b+a.c$, v a,b,c ϵ R

$a+a = 0$, v a ϵ R

$ab(a+b+ab) = ab$, v a,b ϵ R

Definition: 2.4

Let μ be a fuzzy set defined on R. Then μ is said to be a fuzzy ideal of R if

$\mu(x-y) \geq \min\{\mu(x),\mu(y)\}$,for all x,yϵR

$\mu(ra) \geq \mu(a)$,for all r,aϵR

$\mu((r+a)s+rs) \geq \mu(a)$, for all r,a,sϵR

Definition: 2.5

A subgroup B of (N,+) is said to be a bi-ideal of N if BNB ∩ (BN) * B ⊆ B. In case of zero symmetric near-ring a subgroup B of (N,+) is bi-ideal if BNB ⊆ B.

Definition: 2.6

Let μ be a fuzzy set defined on R. Then μ is said to be a fuzzy bi-ideal of R if

$\mu(x-y) \geq \min\{\mu(x),\mu(y)\}$,for all x,yϵR

$\mu(xyz) \geq \min\{\mu(x),\mu(y)\}$,for all x,y,zϵR

Definition: 2.7

A fuzzy set μ in a Boolean like semi ring R is called an anti-fuzzy left (respectively right) ideal of M, if

$\mu(x-y) \leq \max\{\mu(x),\mu(y)\}$,for all x,yϵR

$\mu(ra) \leq \mu(a)$,for all r,aϵR

$\mu((r + a)s + rs) \leq \mu(a)$,for all r,a,sϵR

Definition: 2.8

Let X be a non-empty set. An intuitionistic fuzzy set (IFS) A of X is defined by $A = \{(\mu A, \upsilon A)\}$, where $\mu A : X \to [0,1]$ and $\upsilon A : X \to [0,1]$ with the condition $0 \leq \mu A(x) + \upsilon A(x) \leq 1$, for all x ϵ X. The numbers $\mu A(x)$, $\upsilon A(x)$ ϵ [0,1] denote the degree of membership and non-membership of x to lie in A respectively.

Definition: 2.9

If ϑ is a fuzzy set in f(M), then the fuzzy set $\mu = \vartheta \circ f$ in M (ie)., the fuzzy set defined by
$\mu(x) = \vartheta(f(x))$ for all x in M is called the pre image of ϑ under f.

Definition: 2.10

Let X be a non-empty set and let $A = < \mu A, \upsilon A >$ and $B = < \mu B, \upsilon B >$ be IFS in X.Then,

$A \subset B$ if $\mu A \leq \mu B$ and $\upsilon A \geq \upsilon B$

$A = B$ if $A \subset B$ and $B \subset A$

$Ac = < \upsilon A, \mu A >$

$A \cap B = (\mu A \wedge \mu B , \upsilon A \vee \upsilon B)$

$A \cup B = (\mu A \vee \mu B , \upsilon A \wedge \upsilon B)$.

3. MAIN RESULTS :

Definition: 3.1

An Intuitionistic fuzzy set $A = < \mu A,\upsilon A >$ of M is called an intuitionistic fuzzy bi-ideal of R if,

$\mu A (x - y) \geq \{ \mu A (x) \wedge \mu A (y) \}$

$\mu A ((xyz) \wedge (x (y + z) - xz)) \geq \{ \mu A (x) \wedge \mu A (z) \}$, $\forall$ x,y,z $\in$ R

$\upsilon A (x - y) \leq \{ \upsilon A (x) \vee \upsilon A (y) \}$

$\upsilon A ((xyz) \vee (x (y + z) - xz)) \leq \{ \upsilon A (x) \vee \upsilon A (z) \}$, $\forall$ x,y,z $\in$ R

Lemma: 3.2

If B is a Bi-ideal of a Boolean like semi ring R, then for any $0 < t , s < 1$, there exists an intuitionistic fuzzy bi-ideal $C = < \mu c,\upsilon c >$ of R such that $C< t , s > = B$.

Proof:

Let $C \to [0, 1]$ be a function defined by ,

$$\mu B = \begin{cases} t , \text{if } x \in B \\ 0 , \text{if } x \notin B \end{cases} \quad \&$$

$$\upsilon B = \begin{cases} s , \text{if } y \in B \\ 1 , \text{if } y \notin B \end{cases}$$

For all x $\in$ R & s,t $\in$ [0 ,1]. Then,

$C< t , s > = B$ is an intuitionistic fuzzy bi-ideal of R with $t+s \leq 1$

Now,

Suppose that B is a bi-ideal of R. For all x, y $\in$ B, such that $x - y \in B$. We have,

$\mu c (x - y) \geq t = \{ \mu c (x) \wedge \mu c (y) \}$

$\upsilon c (x - y) \leq s = \{ \upsilon c (x) \vee \upsilon c (y) \}$

Also, for all x, y, z $\in$ B we have,

$\mu c ((xyz) \wedge (x (y + z) - xz)) \geq t = \{ \mu c (x) \wedge \mu c (z) \}$

$\upsilon c ((xyz) \vee (x (y + z) - xz)) \leq s = \{ \upsilon c (x) \vee \upsilon c (z) \}$

Thus, $C< t , s >$ is an intuitionistic fuzzy bi-ideal of R.

Theorem: 3.3

Let R be a Boolean like semi ring & $\{Ai\}_{i\in\Lambda}$ is a family of intuitionistic fuzzy bi-ideals of R , then $\cap$ Ai is an intuitionistic fuzzy bi-ideals of R ,

where $\cap Ai = \{ \wedge\mu Ai , \vee \upsilon Ai \}$

$\wedge\mu Ai (x) = \inf \{ \mu Ai (x) / i\in\Lambda , x\in R \}$ &

$\vee \upsilon Ai (x) = \sup \{ \upsilon Ai (x) / i\in V , x\in R \}$

Proof:

Let x,y,z $\in$R.

Then we have,

i) $\wedge\mu Ai (x - y) = \inf \{ \{ \mu Ai (x) \wedge \mu Ai (y) \} / i\in\Lambda , x,y\in R \}$

$\geq \{\{ \inf(\mu Ai (x)) \wedge \inf(\mu Ai (y)) \} / i\in\Lambda , x,y\in R \}$

$= \{\{ \inf(\mu Ai (x)) / i\in\Lambda , x\in R \} \wedge \{ \inf(\mu Ai (y)) / i\in\Lambda , y\in R \} \}$

$= \{\wedge\mu Ai (x) \wedge \wedge\mu Ai (y)\}$

ii) $\wedge\mu Ai ((xyz) \wedge (x(y + z) - xz)$

$= \inf \{ \{ \mu Ai (x) \wedge \mu Ai (z) \} / i\in\Lambda , x,z\in R \}$

$\geq \{\{ \inf(\mu Ai (x)) \wedge \inf(\mu Ai (z)) \} / i\in\Lambda , x,z\in R \}$

$= \{\{ \inf(\mu Ai (x)) / i\in\Lambda , x\in R \} \wedge \{ \inf(\mu Ai (y)) / i\in\Lambda , y\in R \} \}$

$= \{\wedge\mu Ai (x) \wedge \wedge\mu Ai (z)\}$

iii) $\vee \upsilon Ai (x - y) = \sup \{ \upsilon Ai (x) \vee \upsilon Ai (y) / i\in V , x,y\in R \}$

$\leq \{\{\sup (\upsilon Ai (x)) \vee \sup (\upsilon Ai (y)) / i\in V , x,y\in R \}$

$= \{\{\sup (\upsilon Ai (x)) / i\in V , x\in R \} \vee \{ \sup (\upsilon Ai (y)) / i\in V , y\in R \}$

$= \{ \vee \upsilon Ai (x) \vee \vee \upsilon Ai (y) \}$

iv) $\vee \upsilon Ai ((xyz) \vee (x(y + z) -xz)) = \sup \{ \upsilon Ai (x) \vee \upsilon Ai (z) / i\in V , x,z\in R \}$

$\leq \{\{\sup (\upsilon Ai (x)) \vee \sup (\upsilon Ai (z)) / i\in V , x,z\in R \}$

$= \{\{\sup (\upsilon Ai (x)) / i\in V , x\in R \} \vee \{ \sup (\upsilon Ai (z)) / i\in V , z\in R \}$

$= \{\vee \upsilon Ai (x) \vee \vee \upsilon Ai (z) \}$

Hence, $\cap Ai = \{\wedge\mu Ai, \vee \upsilon Ai \}$ is an intuitionistic fuzzy bi-ideal of R.

Theorem: 3.4

Let A be an intuitionistic fuzzy bi-ideal of Boolean like semi ring R , then A′ is also an intuitionistic fuzzy bi-ideal of R.

Proof :

Let x,y,z $\in$ R

$\mu A'(x - y) = 1 - \mu A (x - y)$

$\geq 1 - \{ \mu A (x) \wedge \mu A (y) \}$

$= \{\{1 - \mu A (x) \} \wedge \{ 1 - \mu A (y)\}\}$

$= \{ \mu A'(x) \wedge \mu A'(y) \}$

$\mu A'((xyz) \wedge (x (y + z) - xz))$

$= 1- \mu A ((xyz) \wedge (x (y + z) - xz))$

$\geq 1 - \{ \mu A (x) \wedge \mu A (z) \}$

$= \{\{1 - \mu A (x) \} \wedge \{ 1 - \mu A (z)\}\}$

$= \{ \mu A'(x) \wedge \mu A'(z) \}$

$\upsilon A'(x - y) = 1 - \upsilon A (x - y)$

$\leq 1 - \upsilon A (x - y)$

= {{1 - υA (x)} V{1 - υA (y)}}
= { υA′(x) V υA′(y) }

υA′((xyz) V (x (y + z) – xz))
= 1 - υA((xyz) V (x (y + z) – xz))
≤ 1 - { υA (x) V υA (z) }
= {{ 1 - υA (x)} V{ 1 - υA (z)}}
= { υA′(x) V υA′(z) }
Thus, A′ is also an intuitionistic fuzzy bi-ideal of R.

Theorem : 3.5
Let R be an Boolean like semi ring. An IFS A of R is an intuitionistic fuzzy bi-ideal of R iff the level sets
U (μA; t) = { xϵR / μ(x) ≥ t } & L(υA ; t) = { x ϵR / υA (x) ≤ t } are a bi-ideals of R.

Proof :
Let A be an intuitionistic fuzzy bi-ideal of R.
Then
μA (x - y) ≥ { μA (x) Λ μA (y) }.
Let x , y ϵ U(μA ; t)
⇒ μ(x) ≥ t , μ(y) ≥ t
... μA (x - y) ≥ { μA (x) Λ μA (y) } ≥ t
⇒ x - y ϵ U(μA ; t)

Also,
Let μA ((xyz) Λ (x (y + z) – xz)) ≥ {μA (x) Λ μA (z) }
Let x,y,z ϵ U(μA ;
⇒ μ(x) ≥ t , μ(y) ≥ t , μ(z) ≥ t
... μA ((xyz) Λ (x (y + z) – xz)) ≥ { μA (x) Λ μA (z) } ≥ t
⇒ (xyz) , (x(y + z) – xz) ϵ U(μA ; t)
Thus,
U(μA ; t) is a bi-ideal of R.
Also,
Let υA (x - y) ≤ { υA (x) V υA (y) }
Let x ,y ϵ L(υA ; t)
⇒ υA (x) ≤ t , υA (y) ≤ t
... υA (x - y) ≤ { υA (x) V υA (y) } ≤ t
⇒ x - y ϵ L(υA ; t)

Now,
Let υA ((xyz) V (x (y + z) – xz)) ≤ { υA (x) V υA (z) }
Let x ,y,z ϵ L(υA ; t)
⇒ υA (x) ≤ t , υA (y) ≤ t , υA (z) ≤ t
... υA ((xyz) V (x (y + z) – xz)) ≤ { υA (x) V υA (z) } ≤ t
⇒ (xyz) , (x(y + z) – xz) ϵ L(υA ; t)
Thus,
L(υA ; t) is a bi-ideal of R.
Conversely,

If U(μA ; t) is a bi-ideal of R. Let t = { μA (x) Λ μA (y) }
Then, x , y ϵ U(μA ; t)
⇒ x - y ϵ U(μA ; t)
⇒ μA (x - y) ≥ t
⇒ μA (x - y) ≥ { μA (x) Λ μA (y) }

If L(υA ; t) is a bi-ideal of R.
Let t = { υA (x) V υA (y) }
Then, x ,y ϵ L(υA ; t)
⇒ x - y ϵ L(υA ; t)
⇒ υA (x - y) ≤ t
⇒ υA (x - y) ≤ { υA (x) V υA (y) }

Define t = { μA (x) Λ μA (y) }.
Then, x,y,z ϵ U(μA ; t)
⇒ (xyz) , (x(y + z) – xz) ϵ U(μA ; t)
⇒ μA ((xyz) Λ (x (y + z) – xz)) ≥ t
⇒ μA ((xyz) Λ (x (y + z) – xz)) ≥ { μA (x) Λ μA (z) }

Define t = { υA (x) V υA (y) }.
Then, x ,y,z ϵ L(υA ; t)
⇒ (xyz) , (x(y + z) – xz) ϵ L(υA ; t)
⇒ υA ((xyz) V (x (y + z) – xz)) ≤ t
⇒ υA ((xyz) V (x (y + z) – xz)) ≤ { υA (x) V υA (z) }
Hence, A is an intuitionistic fuzzy bi-ideal of R.

Theorem : 3.6
If A is an intuitionistic fuzzy bi-ideal of Boolean like semi ring R, then the Boolean like semi ring homomorphic pre-image of a intuitionistic fuzzy bi-ideal is again a intuitionistic fuzzy bi-ideal.

Proof :
Let f: R → R’ be a Boolean like semi ring homomorphism and A be IFS on R’. Then the pre-image of μ under f, denoted by f-1(A) is defined by,
f -1(μA (x)) = μA(f(x)) &
f -1(υA(x)) = υA(f(x)) , V x ϵ R

f -1(μA (x - y)) = μA(f(x - y))
= μA(f(x) – f(y))
≥ {μA(f (x)) Λ μA(f (y))}
= f -1(μA (x)) Λ f -1(μA (y))

f -1(μA ((xyz) Λ (x (y + z) – xz))
= μA (f(xyz) Λ (x (y + z) – xz))
= μA ((f(xyz) Λ (f(x (y + z) – xz))
≥ {μA(f (x)) Λ μA(f (z))}
= f -1(μA (x)) Λ f -1(μA (z))

f -1(υA(x - y)) = υA (f(x - y))
= υA (f(x) – f(y))
≤ {υA(f(x)) V υA(f(y)) }

= f -1(υA(x)) V f -1(υA(y))

f -1((υA(xyz) V (x(y + z) – xz)))
= υA (f((xyz) V (x(y + z) – xz)))
= υA ((f(xyz)) V (f(x(y + z) – xz)))
≤ {υA(f(x)) V υA(f(z)) }
= f -1(υA(x)) V f -1(υA(z))

Hence, f -1(A) is an intuitionistic fuzzy bi-ideal of R.

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