New Forms of P̂g Continuous Functions

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Abstract- **The idea of generalized closed sets presented by Levine [3] assumes a critical part in General Topology. This thought has been examined widely as of late by numerous topologists. The investigation of generalized closed sets has prompted several new and fascinating ideas. Dunham further investigated the Properties of T½ spaces and defined a new Closure operator Cl* by using generalized closed sets. S.Pious Missier and S.Jackson[8] introduced a new notion of generalized closed sets called P̂g closed sets. The purpose of this paper is to define strongly P̂g Continuous and Perfectly P̂g Continuous functions which are the different forms of P̂g continuous function. In this paper we derived some important results and establish its relationship with other forms of P̂g continuous functions.**

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KEYWARDS: p̂g closed sets, p̂g open sets, p̂g continuous, strongly p̂g continuous, perfectly p̂g continuous functions

INTRODUCTION

The concept of generalized closed sets introduced by Levine[3] plays a significant role in General Topology. This notion has been studied extensively in recent years by many topologists. The investigation of generalized closed sets has led to several new and interesting concepts. Dunham [4] further investigated the Properties of $T_{½}$ spaces and defined a new Closure operator Cl* by using generalized Closed sets. In 1996, H.Maki, J.Umehara and T.Noiri [5] introduced the Class of Pre generalized Closed sets and used them to obtain Properties of Pre-$T_{1/2}$ spaces. The modified forms of generalized closed sets and generalized continuity were studied by K. Balachandran, P. Sundaram and H. Maki [1]. M.K.R.S.Veerakumaret.al[12] introduced a new Classes of Open sets namely g* - Closed sets .This characterization paved a new pathway. Dr.S.Piousmissier and S.Jackson[8] introduced a new class of generailsed closed sets called $\hat{p}g$ closed sets. In this paper we introduce strongly and perfectly $\hat{p}g$ continuous functions. We obtain many interesting results,To substantiate these results, suitable examples are given at the respective places.

Throughout this paper, spaces (X,τ), (Y, σ), (Z,η) always mean topological space on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X. The Closure of A and the interior of A are denoted by Cl(A) and Int(A) respectively.

II. PRELIMINARIES

DEFINITION2.1: - A subset A of a topological space (X, τ) is called

(1) a **Pre-Open** set[6] if A ⊆int(Cl(A)) and a **Pre-Closed** set if Cl(int(A)) ⊆ A.

(2) a **g closed set** [3] if Cl(A) ⊆U whenever A ⊆ U , U is Open in (X, τ).The complement of g closed set is **g open**.

(3) a **Pre*-Open** set[11] if A ⊆int*(Cl(A)) and a **Pre*-Closed** set if Cl*(int(A)) ⊆ A

(4) a **$\hat{p}g$ closed set** [8] PCl*(A) ⊆ U whenever A ⊆ U , U is **Pre*-Open** in (X, τ).

DEFINITION 2.2:[8] The intersection of all P̂g Closed sets containing A is called **P̂g Closure** of A and denoted by P̂g Cl(A). That is P̂g Cl(A) = ∩{F: A ⊆ F and F∈P̂g C(X)}.

DEFINITION 2.3:[8] Let A be a subset of X. Then **P̂g Interior** of A is defined as the union of all P̂g Open sets contained in it.P̂g int(A) = ∪{V: V ⊆ A and V∈P̂g O(X)}.

DEFINITION 2.4:[9][A function f:(X,τ)→(Y,σ) is called **$\hat{p}g$ Continuous** if f^{-1}(V) is $\hat{p}g$ open in (X,τ) for every open set in (Y,σ).i.e) the pre image of every open set is $\hat{p}g$ open.

DEFINITION 2.5:[10]A function f:(X,τ)→(Y,σ) is called **$\hat{p}g$ irresolute** if f^{-1}(V) is $\hat{p}g$ open in (X,τ) for every $\hat{p}g$ open set in (Y,σ).i.e) the pre image of every $\hat{p}g$ open set is $\hat{p}g$ open.

DEFINITION 2.6:[4]

A function f:(X,τ) →(Y,σ) is called a **Strongly continuous** if f^{-1} (O) is both open and closed in (X,τ). For each subset O in (Y,σ).

DEFINITION 2.7:[7]

A function f:(X,τ) →(Y,σ) is called a **α-continuous** if f^{-1} (O) is a **α** open set of (X,τ) For every open set O of (Y,σ).

DEFINITION 2.8:[3]

A function f:(X,τ) →(Y,σ) is called a **g-continuous** if f^{-1} (O) is a g open set of (X,τ) For every open set O of (Y,σ).

DEFINITION 2.9:[4]

A function f:(X,τ) →(Y,σ) is called a **Perfectly continuous** if f^{-1} (O) is both open and closed in (X,τ).For every open set O in (Y,σ).

DEFINITION 2.10:[3]

A map f:(X,τ) →(Y,σ) is called **g-closed** if f(O) is a g closed in (Y,σ). For every closed set O in (X,τ).

DEFINITION 2.11:[8]

A topological space (X,τ) is said to be $\hat{p}$g $\hat{p}$g $T_{1/2}$*space* if every $\hat{p}g$ open set of (X,τ) is open in (X,τ).

THEOREM 2.12:

(1) Every open set is $\hat{p}$g open and every closed set is $\hat{p}$g closed.

(2) Every α open set is $\hat{p}$g open and every α closed set is $\hat{p}$g closed.

(3) Every g open set is $\hat{p}$g open and every g closed set is $\hat{p}$g closed.

III .STRONGLY $\hat{p}$g-CONTINUOUS FUNCTION

DEFINITION 3.1:

A function f:(X,τ) →(Y,σ) is called a **Strongly $\hat{p}$g-continuous** if the inverse image of every $\hat{p}$g open set in (Y,σ) is open in (X,τ).

THEOREM 3.2

If a map f:(X,τ)→(Y,σ) from a topological space (X,τ) into a topological space (Y,σ) is strongly $\hat{p}$g-continuous , Then it is continuous.

PROOF:

Let O be a open set in (Y,σ). Since, every open set is $\hat{p}$gopen, O is $\hat{p}$g open in (Y,σ). Since, f is strongly $\hat{p}$g-continuous, f^{-1} (O) is open in (X,τ). Therefore, f is continuous.

REMARK 3.3

The following example supports that the converse of the above theorem is not true in general.

EXAMPLE 3.4

Let X = Y = {a,b,c}, (X,τ) = {φ, ab,X} and (Y,σ) = {φ, a,Y}. let f:(X,τ)→(Y,σ) be defined by f(a) = a= f(b) , f(c) = b. Clearly, f is not strongly $\hat{p}$g-continuous. Since, b is $\hat{p}$g-open set in (Y,σ). But f^{-1} (b) = c which is not an open set of (X,τ). However, f is continuous.

THEOREM 3.5

If a map f :(X,τ)→(Y,σ) from a topological space (X,τ) into a topological space (Y,σ) is strongly $\hat{p}$g-continuous , iff the inverse image of every $\hat{p}$g closed set in (Y,σ) is closed in (X,τ).

PROOF:

Necessity: Assume that f is strongly $\hat{p}$g-continuous. Let O be any $\hat{p}$g closed set in (Y,σ). Then O^c is $\hat{p}$g open in (Y,σ). Since, f is strongly $\hat{p}$g-continuous, $f^{-1}(O^c)$ is open in (X,τ).
But, $f^{-1}(O^c)$ = X/f^{-1}(O) and so f^{-1} (O) is closed in (X,τ).

Sufficiency: Assume that the inverse image of every $\hat{p}$g closed set in (Y,σ) is closed in (X,τ). Then, O^c is $\hat{p}$g closed in(Y,σ). By assumption, $f^{-1}(O^c)$ is closed in (X,τ).
But $f^{-1}(O^c)$ = X/f^{-1}(O) and so f^{-1} (O) is open in (X,τ). Therefore, f is strongly $\hat{p}$g-continuous.

THEOREM 3.6

If a map f :(X,τ)→(Y,σ) is strongly continuous, Then it is strongly $\hat{p}$g-continuous.

PROOF:

Assume that f is strongly continuous. Let O be open in (Y,σ), then it is $\hat{p}$g open set in (Y,σ). Since, f is strongly continuous, f^{-1} (O) is open in (X,τ). Therefore, f is strongly $\hat{p}$g-continuous.

THEOREM 3.7

If a map f :(X,τ)→(Y,σ) is strongly $\hat{p}$g-continuous , then it is $\hat{p}$g-continuous.

PROOF:

Let O be a open set in (Y,σ). O is $\hat{p}$g open in (Y,σ). Since f is strongly $\hat{p}$g-continuous, f^{-1}(O) is open in (X,τ).f^{-1} (O) is $\hat{p}$g open in (X,τ). Therefore, f is $\hat{p}$g-continuous.

THEOREM 3.8

If a map f:(X,τ)→(Y,σ) is strongly $\hat{p}$g-continuous and g:(Y,σ)→(Z,η) is $\hat{p}$g-continuous, then gof:(X,τ)→ (Z,η) is continuous.

PROOF:

Let O be any open set in (Z,η).Since g is $\hat{p}$g-continuous, g^{-1}(O) is $\hat{p}$g open in (Y,σ). since f is strongly $\hat{p}$g-continuous $f^{-1}(g^{-1}$(O)) is open in (X,τ). But $(gof)^{-1}$(O) = $f^{-1}(g^{-1}$(O)). Therefore, gof is continuous.

THEOREM 3.9

If a map f:(X,τ)→(Y,σ) is strongly $\hat{p}$g-continuous and g:(Y,σ)→(Z,η) is $\hat{p}$g irresolute, then gof:(X,τ)→ (Z,η) is strongly $\hat{p}$g- continuous.

PROOF:

Let O be any $\hat{p}$g open set in (Z,η). Since g is $\hat{p}$g-irresolute, g^{-1}(O) is $\hat{p}$g open in (Y,σ). since f is strongly $\hat{p}$g-continuous $f^{-1}(g^{-1}$(O)) is open in (X,τ). But $(gof)^{-1}$(O) = $f^{-1}(g^{-1}$(O)). Therefore, gof is strongly $\hat{p}$g- continuous.

THEOREM 3.10

If a map f: (X,τ)→(Y,σ) is $\hat{p}$g-continuous and g:(Y,σ)→(Z,η) is strongly $\hat{p}$g-continuous, then gof:(X,τ)→(Z,η) is $\hat{p}$g irresolute.

PROOF: Let O be any $\hat{p}$g open set in (Z,η).Since g is strongly $\hat{p}$g-continuous, g^{-1}(O) is $\hat{p}$g open in (Y,σ). Since f is $\hat{p}$g-continuous, $f^{-1}(g^{-1}$(O)) is $\hat{p}$g open in (X,τ).
But $(gof)^{-1}$(O) = $f^{-1}(g^{-1}$(O)). Hence, gof:(X,τ)→(Z,η) is $\hat{p}$g irresolute.

THEOREM 3.11

If (X,τ) be any topological space and (Y,σ) be a $\hat{p}g$ $T_{1/2}$space and f:(X,τ)→(Y,σ)be a map. Then the following are equivalent.

1) f is strongly $\hat{p}g$-continuous
2) f is continuous

PROOF:

(1) => (2) Let O be any open in (Y,σ). O is $\hat{p}g$ open in (Y,σ). Then f^{-1} (O) is open in (X,τ). Hence, f is continuous.

(2) => (1) Let O be any $\hat{p}g$ open in (Y,σ).Since, (Y,σ) is a $\hat{p}g$ $T_{1/2}$space, O is open in (Y,σ). Since, f is continuous, Then f^{-1} (O) is open in (X,τ).Hence, f is strongly $\hat{p}g$-continuous.

THEOREM 3.12

Let f: (X,τ) →(Y,σ) be a map. Both (X,τ) *and* (Y,σ) are $\hat{p}g$ $T_{1/2}$space. Then the following are equivalent.

1) f is a $\hat{p}g$ irresoute
2) f is strongly $\hat{p}g$ continuous
3) f is continuous
4) f is $\hat{p}g$ continuo

PROOF:

(1) => (2) Let O be $\hat{p}g$ open set in (Y,σ). Since f is a $\hat{p}g$ irresolute, f^{-1} (O) is $\hat{p}g$ open in (X,τ). Since, (X,τ) is $\hat{p}g$ $T_{1/2}$space, f^{-1} (O) is open in (X,τ). Therefore, f is strongly $\hat{p}g$ continuous.

(2) => (3) Let O be a open set in (Y,σ).Since, every open set is $\hat{p}g$ open , O is $\hat{p}g$ open in (Y,σ). Since f is strongly continuous, f^{-1} (O) is open in (X,τ).
Hence f is continuous.

(3) => (4) Let O be open in (Y,σ). Since f is continuous,f^{-1} (O) is open in (X,τ).Since every open set is $\hat{p}g$open, f^{-1} (O)is $\hat{p}g$ open set in(X,τ). . Thus, f is $\hat{p}g$ continuous.

(4) => (1) Let O be a $\hat{p}g$ open set in (Y,σ). Since (Y,σ) is a $\hat{p}g$ $T_{1/2}$ space, O is a open set in (Y,σ). Since f is $\hat{p}g$ continuous, f^{-1} (O) is $\hat{p}g$ open in(Y,σ). Hence, f is a $\hat{p}g$ irresolute.

THEOREM 3.13

The composition of two strongly $\hat{p}g$ continuous maps is strongly $\hat{p}g$ continuous.

PROOF:

Let O be a $\hat{p}g$ open set in (Z,η).Since, g is strongly $\hat{p}g$ continuous, we get g^{-1}(O) is open in (Y,σ). g^{-1}(O) is $\hat{p}g$ open in (Y,σ).As f is also strongly $\hat{p}g$ continuous, $f^{-1}(g^{-1}$(O)) = $(gof)^{-1}$(O) is open in (X,τ).Hence, gof is strongly $\hat{p}g$ continuous.

THEOREM 3.14

If f: (X,τ) →(Y,σ) and g:(Y,σ) →(Z,η) be any two maps. Then their composition
gof: (X,τ) → (Z,η) is strongly $\hat{p}g$ continuous, if g is strongly $\hat{p}g$ continuous and f is continuous.

PROOF:

Let O be a $\hat{p}g$ open in (Z,η). Since, g is strongly $\hat{p}g$ continuous, g^{-1}(O) is open in (Y,σ). Since f is continuous, $f^{-1}(g^{-1}$(O)) = $(gof)^{-1}$(O) is open in (X,τ).Hence gof is strongly $\hat{p}g$ continuous.

IV. PERFECTLY CONTINUOUS FUNCTION

DEFINITION 4.1:

A map f: (X,τ) → (Y,σ) is said to be **Perfectly $\hat{p}$g continuous** if the inverse image of every $\hat{p}g$ open set in (Y,σ) is both open and closed in (X,τ).

THEOREM 4.2

If a map f: (X,τ) → (Y,σ) from a topological space (X,τ) into a topological space (Y,σ) is perfectly $\hat{p}g$ continuous then it is strongly $\hat{p}g$ continuous.

PROOF:

Assume that f is perfectly $\hat{p}g$ continuous. Let O be any $\hat{p}g$ open set in (Y,σ). Since, f is perfectly $\hat{p}g$ continuous, f^{-1}(O) is open in (X,τ). Therefore, f is strongly $\hat{p}g$ continuous.

REMARK 4.3: The converse of the above theorem is need not to be true.

EXAMPLE 4.4:

Let X = Y = {a,b,c}, (X,τ) = {φ, a,b,ab,bc,X} and (Y,σ) = {φ,a,b,ab,Y}. let f:(X,τ)→(Y,σ) be defined by f(a) = a, f(b)= b , f(c) = c. Clearly, f is not perfectly $\hat{p}g$-continuous. Since, b and {a,b} are $\hat{p}g$-open set in (Y,σ). But f^{-1} (b) = b and f^{-1} (ab) = {a,b} which are open in (X,τ). But, not closed in (X,τ).However, f is strongly $\hat{p}g$ continuous.

THEOREM 4.5

If a map f: (X,τ) → (Y,σ) from a topological space (X,τ) into a topological space (Y,σ) is perfectly $\hat{p}g$ continuous, then it is perfectly continuous.

PROOF:

Let O be an open set in (Y,σ).O is an $\hat{p}g$ open set in (Y,σ). Since f is perfectly $\hat{p}g$ continuous, f^{-1}(O) is both open and closed in (X,τ).Therefore, f is perfectly continuous.

THEOREM 4.6

A map f: (X,τ) → (Y,σ) from a topological space (X,τ) into a topological space (Y,σ) is perfectly $\hat{p}g$ continuous if and only if f^{-1}(O) is both open and closed in (X,τ) for every $\hat{p}g$ closed set O in (Y,σ).

PROOF:

Necessity: Let O be an $\hat{p}g$ closed set in (Y,σ). Then O^c is $\hat{p}g$ open in (Y,σ).Since, f is perfectly $\hat{p}g$ continuous, $f^{-1}(O^c)$ is both open and closed in (X,τ). But $f^{-1}(O^c)$ = X/f^{-1}(O) and so f^{-1}(O) is both open and closed in (X,τ).

Sufficiency: Assume that the inverse image of every $\hat{p}g$ closed set in (Y,σ) is both open and closed in (X,τ).Let O be any $\hat{p}g$ open in (Y,σ). Then O^c is $\hat{p}g$ closed in (Y,σ).By assumption $f^{-1}(O^c)$ is both open and closed in (X,τ). But

$f^{-1}(O^c)$ = X/f^{-1}(O) and so f^{-1}(O) is both open and closed in (X,τ). Therefore, f is perfectly $\hat{p}$g continuous.

THEOREM 4.7

Let (X,τ) be a discrete topological space and (Y,σ) be any topological space. Let
f: (X,τ) → (Y,σ) be a map, then the following statements are true.

1) f is strongly $\hat{p}$g continuous
2) f is perfectly $\hat{p}$g continuous

PROOF:

(1) => (2) Let O be any $\hat{p}$g open set in (Y,σ).By hypothesis, f^{-1}(O) is both open and closed in (X,τ).Since (X,τ) is a discrete topological space, f^{-1}(O) is closed in (X,τ). f^{-1}(O) is both open and closed in (X,τ). Hence, f is perfectly $\hat{p}$g continuous.

(2) => (1) Let O be any $\hat{p}$g open set in (Y,σ).Then ,f^{-1}(O) is both open and closed in (X,τ). Hence, f is strongly $\hat{p}$g continuous.

THEOREM4.8

If a map f: (X,τ) → (Y,σ) and g: (Y,σ) → (Z,η) are perfectly $\hat{p}$g continuous, then their composition gof: (X,τ) → (Z,η) is also perfectly $\hat{p}$g continuous.

PROOF:

Let O be a $\hat{p}$g open set in (Z,η).Since, g is perfectly $\hat{p}$g continuous. We get that g^{-1}(O) is open and closed in (Y,σ). g^{-1}(O) is $\hat{p}$g open in (Y,σ).Since f is perfectly $\hat{p}$g continuous,
$f^{-1}(g^{-1}$(O))=$(gof)^{-1}$(O) is both open and closed in (X,τ). Hence, gof is perfectly $\hat{p}$g continuous.

THEOREM 4.9

If f: (X,τ) → (Y,σ) and g: (Y,σ) → (Z,η) be any two maps. Then their composition is strongly $\hat{p}$g continuous if g is perfectly $\hat{p}$g continuous and f is continuous.

PROOF:

Let O be any $\hat{p}$g open set in (Z,η).Then, g^{-1}(O) is open and closed in (Y,σ). Since, f is continuous. $f^{-1}(g^{-1}$(O))=$(gof)^{-1}$(O) is open in (X,τ) Hence, gof is strongly $\hat{p}$g continuous.

THEOREM 4.10

If a map f: (X,τ) → (Y,σ) is perfectly $\hat{p}$g continuous and a map g: (Y,σ) → (Z,η) is strongly $\hat{p}$g continuous then the composition gof: (X,τ) → (Z,η) is perfectly $\hat{p}$g continuous.

PROOF:

Let O be any $\hat{p}$g open set in (Z,η). Then, g^{-1}(O) is open in (Y,σ). g^{-1}(O) is $\hat{p}$g open in (Y,σ).By hypothesis,.$f^{-1}(g^{-1}$(O)) =$(gof)^{-1}$(O) is both open and closed in (X,τ). Therefore, gof is perfectly $\hat{p}$g continuous.

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