More Functions Associated with α*g Closed sets in Topological Spaces

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Abstract- **The aim of this paper is to introduce two new classes of functions, namely totally α*g-continuous functions and strongly α*g-continuous functions and study its properties.**

Keywords: totally α*g-continuous functions and strongly α*g-continuous function..

I. INTRODUCTION

Continuity is an important concept in mathematics and many forms of continuous functions have been introduced over the years. RC Jain [4] introduced the concept of totally continuous functions for topological spaces.In 1960,Levine .N [6] introduced strong continuity in topological spaces. In this paper,we define totally α *g continuous functions and strongly α*g continuous functions and basic properties of these functions are investigated and obtained.

II PRELIMINARIES

Throughout this paper (X, τ), (Y,σ) and (Z,η) or X , Y , Z represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure and the interior of A respectively.

Definition 2.1:A subset A of X is said to be ***α*g-closed* set** [1] if $\alpha cl(A) \subseteq$ U Whenever A⊆ U and U is α*-open.

Definition 2.2: A function f: (X, τ) ⟶ (Y, σ) is called a ***α*–continuous** [8] if f^{-1}(O) is a α-closed set [9] of (X, τ) for every closed set O of (Y, σ).

Definition 2.3: A function f: (X, τ) ⟶ (Y, σ) is called a ***αg* –continuous** [3] if f^{-1}(O) is a αg –closed set [7] of (X, τ) for every closed set O of (Y, σ).

Definition 2.4 : A function f: (X, τ) ⟶ (Y, σ) is called a **g*α*– continuous** [5] if f^{-1}(O) a gα-closed set [5] of (X, τ) for every closed set O of (Y, σ).

Definition 2.5: A function f: (X, τ) ⟶(Y, σ) is said to be **α*g- continuous**[2] if f^{-1}(O) is α*g-closed set [1] in (X, τ) for every closed in (Y, σ).

Definition 2.6:A function f:(X,τ)→(Y,σ) is said to be **α*g-irresolute**[2] if f^{-1}(O) is a α*g-closed set [1] of (X,τ) for every α*g-closed set B of (Y,σ).

Definition 2.7: A function f: (X, τ) ⟶ (Y, σ) is called a **strongly continuous** [6]if f^{-1}(O) is both open and closed in (X, τ) for each subset O in (Y, σ).

Definition 2.8:A function f: (X, τ) ⟶ (Y, σ) is called a **totally–continuous** [4] if f^{-1}(O) is a clopen set in (X, τ) for every closed set O of (Y, σ).

Definition 2.9:A Topological space X is said to be ***α*g* T $_{1/2}$ space** [10] if every α*g-closed set of X is closed in X.

Theorem 2.10:[1] Every closed set is α*g-closed.

Theorem 2.11:[1] Every open set is α*g-open.

Theorem 2.12:[1] Every α*g-closed is gα-closed.

Theorem 2.13:[1] Every α*g-closed is αg-closed.

III TOTALLY *α*G CONTINUOUS CONTINUOUS

We introduce the following definition.

Definition 3.1:A function f:(X,τ)→(Y,σ) is said to be **totally *α*g-continuous** if the inverse image of every closed set in (Y,σ) is α*g-clopen in (X,τ).

Example3.2: LetX=Y={a,b,c,d},τ={ϕ,{a},{bcd},X},σ={ϕ,{abc},Y},α*gO(X)={ϕ,{bcd},{a},X},α*gC(X)={ϕ,{a},{bcd},X}.Let g:(X,τ)→(Y,σ)be defined by g(a)=d, g(b)=c, g(c)=b, g(d)=a. Therefore g is totally α*g continuous.

Theorem 3.3: Every totally α*g continuous functions is α*g -continuous.

Proof: Let O be any closed set of (Y, σ). Since, f is totally α*g continuous functions, f^{-1}(O) is both α*g open and α*g closed in (X, τ). Therefore, f is α*g continuous.

Remark 3.4: The converse of above theorem need not be true as the the following the example.

Example 3.5: Let X=Y={a,b,c},τ={ϕ,{ab},{a},{ac},X},σ={ϕ,{a},{ab},Y},α*gO(X)={ϕ,{ac},{ab},{a},X },α*gC(X)={ϕ,{b},{c},{bc},X}.Let g:(X,τ)→(Y,σ) be defined by g(a)=a, g(b)=c, g(c)=b.Clearly, g is α*g-continuous but g $^{-1}$({b,c})=bc is α*g-closed in X but not α*g-open in X. Therefore, g is not totally α*g continuous.

Theorem 3.6: Every totally continuous functions is α*g -continuous.

Proof: Let O be any closed set of (Y, σ). Since, f is totally continuous functions, f^{-1}(O) is both open and closed in (X, τ),Since every closed is α*g-closed,f^{-1}(O) is α*g-closed in X.Therefore, f is α*g -continuous.

Remark 3.7: The converse of above theorem need not be true as the following the example.

Example 3.8: Let X= Y={a,b,c,d},τ={ϕ,{ab},{abc},X},σ={ϕ,{abc},Y},α*gC(X)= {ϕ,{c},{d},{cd},X}.Let g:(X,τ)→(Y,σ) be defined by g(a)=b,g(b)=c,g(c)=a,g(d)=d.Clearly, g is α*g-continuous but g $^{-1}$({d})=d is closed in X but not open in X.Therefore,g is not totally continuous.

Theorem 3.9: Every totally α*g continuous functions is gα -continuous.

Proof: Let O be any closed set of (Y, σ). Since, f is totally α*g continuous functions, f^{-1}(O) is both α*g-open and α*g-closed in (X, τ),Since every α*g-closed is gα-closed,f^{-1}(O) is gα-closed in X.Therefore, f is gα-continuous.

Remark 3.10: The converse of above theorem need not be true as the following the example.

Example 3.11: Let X=Y={a,b,c,d},τ={ϕ,{ab},{abc},X},σ={ϕ,{a},,{abc},Y},gαC(X)={ϕ,{b},{c},{d},{cd },{bcd},X},α*gC(X)={ϕ,{c},{d},{cd},X}.Let g:(X,τ)→(Y,σ) be defined by g(a)=a,g(b)=c,g(c)=d,g(d)=b. Clearly, g is gα-continuous but g $^{-1}$({d})={c} is α*g-closed in X but not α*g-open in X.Therefore,g is not totally α*g continuous.

Theorem 3.12: Every totally α*g continuous functions is αg -continuous.

Proof: Let O be any closed set of (Y, σ). Since, f is totally α*g continuous functions, f^{-1}(O) is both α*g-open and α*g-closed in (X, τ),Since every α*g-closed is αg-closed,f^{-1}(O) is αg-closed in X.Therefore, f is αg-continuous.

Remark 3.13: The converse of above theorem need not be true as the following the example.

Example 3.14: Let X=Y={a,b,c,d},τ={ϕ,{a},{b},{ab},{bc},{abc},X},σ={ϕ,{a}, Y},αgC(X)={ϕ,{c},{d},{ cd},{ad},{bd},{abd},{acd},{bcd},X},α*gC(X)={ϕ,{c},{d},{cd},{ad},{acd},{bcd},X}.Let g:(X,τ)→ (Y,σ) be defined by g(a)=a,g(b)=d,g(c)=b,g(d)=c.Clearly,g is αg-continuous but g $^{-1}$({b,c,d})={b,c,d} is α*g-closed in X but not α*g-open in X.Therefore,g is not totally α*g continuous.

Theorem 3.15: Let f: X → Y and g: Y → Z be functions. Then g ∘ f : X → Z

(i) If f is α*g-irresolute and g is totally α*g continuous then g ∘ f is totally α*g continuous.

(ii) If f is totally α*g- continuous and g is continuous then g ∘ f is totally α*g continuous.

Proof:

(i) Let O be any closed set in Z. Since g is totally α*g continuous, g $^{-1}$(O) is α*g clopen in Y. Since f is α*g -irresolute, f^{-1}(g $^{-1}$(O)) is α*g-open and α*g-closed in X. Since,(g ∘ f) $^{-1}$ (O) = f^{-1}(g $^{-1}$(O)).Therefore, g ∘ f is totally α*g continuous.

(ii) Let O be any closed set in Z. Since g is continuous, g $^{-1}$(O) is closed in Y. Since, f is totally α*g continuous, f^{-1}(g $^{-1}$(O)) is α*g clopen in X. Hence, g ∘f is totally α*g continuous.

IV STRONGLY α *g CONTINUOUS FUNCTION

Definition 4.1: A mapping f:X → Y is said to be **strongly α*g continuous** if the inverse image of every α*g-closed set in Y is closed in X.

Example 4.2: Let X = Y={a,b,c,d}, τ={ϕ,{a},{b},{ab},{bc},{abc},X},σ={ϕ,{abc},Y},α*gC(X) ={ϕ,{c},{d},{cd},{ad},{acd},{bcd},X},α*gC(Y)={ϕ,{d},Y} .Let g:(X,τ)→(Y,σ)be defined by g(a)=b,g(b)=c,g(c)=a, g(d)=d.Therefore g is strongly α*g continuous function.

Theorem 4.3: If a map f: X ⟶ Y from a topological spaces X into a topological spaces Y is strongly α*g continuous then it is continuous.

Proof: Let O be a closed set in Y. Since every closed set is α*g-closed, O is α*g-closed in Y. Since f is strongly α*g continuous , f^{-1}(O) is closed in X. Therefore f is continuous.

Remark 4.4: The following example that the converse of the above theorem is not true in general.

Example 4.5: Let X = Y={a,b,c}, τ={ϕ,{a},{ab},X},σ={ϕ,{a},Y},α*gC(X)={ϕ,{b},{c},{ac},{bc},X},α*g C(Y)={ϕ,{b},{c},{bc},Y}.Let g:(X,τ)→(Y,σ) be defined by g(a)=d, g(b)=c, g(c)=b,g(d)=c.Clearly g is continuous.but g $^{-1}$ ({c})={b} is closed in X. Therefore, g is not strongly α*g continuous.

Theorem 4.6: A map f: X ⟶ Y from a topological spaces X into a topological spaces Y is strongly α*g- continuous if and only if the inverse image of every α*g- open set in Y is open in X.

Proof: Assume that f is strongly α*g continuous. Let O be any α*g-open set in Y.Then O ^{c} is α*g-closed in Y.Since f is strongly α*g continuous, f^{-1}(O ^{c}) is closed in X .But f^{-1}(O ^{c})= X/ f^{-1}(O) and so f^{-1}(O) is open in X.

Conversely, assume that the inverse image of every α*g-open set in Y is open in X.Then O ^{c} is α*g-closed in Y. By assumption, f^{-1}(O ^{c}) is closed in X, but f^{-1}(O ^{c})= X/ f^{-1}(O) and so f^{-1}(O) is open in X. Therefore, f is strongly α*g continuous.

Theorem 4.7: If a map f: X ⟶ Y is strongly continuous then it is strongly α*g continuous.

Proof: Assume that f is strongly continuous.Let O be an closed set in Y.Since every closed set is α*g-closed,implies O is α*g-closed set in Y.Since f is strongly continuous, f^{-1}(O) is closed in X. Therefore, f is strongly α*g continuous.

Remark 4.8:The converse of above theorem need not be true as the following the example.

Example 4.9: Let X =Y={a,b,c,d},τ={ϕ,{a},{b},{ab},{abc},X},σ={ϕ,{abc},Y}, α*gC(Y)={ϕ,{d},Y}.Let g:(X,τ)→(Y,σ) be defined by g(a)=b, g(b)=c,g(c)=a,g(d)=d.Clearly g is strongly α*g continuous. but g $^{-1}$({d})={d} is closed in X but not open in X.Therefore g is not strongly continuous function.

Theorem 4.10: If a map f: X ⟶ Y is strongly α*g continuous then it is α*g- continuous.

Proof: Let O be any closed set in Y.Since every closed set is α*g-closed,O is α*g -closed in Y. Since f is strongly α*g continuous implies f^{-1}(O) is closed in X. By [1] f^{-1}(O) is α*g closed in X. Therefore, f is α*g continuous.

Remark 4.11: The converse of above theorem need not be true as the following the example.

Example 4.12: Let X = Y ={a,b,c,d},τ={ϕ,{a},{ab},{abc},X},σ={ϕ,{abc},Y},α*gC(X)={ϕ,{b},{c},{d},{ bc},{bd},{cd},{bcd},X}, α*gC(Y)={ϕ,{d},Y}. Let g:(X,τ)→(Y,σ) be defined by g(a)=c, g(b)=d, g(c)=b,g(d)=a.Clearly g is α*g-continuous. but g $^{-1}$({d})={b} is not closed in X.Therefore g is not strongly α*g continuous function.

Theorem 4.13: If a map f: X ⟶ Y is strongly α*g continuous and a map g: Y ⟶ Z is α*g continuous then g ∘ f: X ⟶ Z is continuous.

Proof: Let O be any closed set in Z. Since g is α*g continuous, g $^{-1}$(O) is α*g-closed in Y . Since f is strongly α*gcontinuous f^{-1}(g $^{-1}$(O)) is closed in X. But (g∘f) $^{-1}$(O)=f^{-1}(g $^{-1}$(O)). Therefore, g ∘f is continuous.

Theorem 4.14: If a map f: X ⟶ Y is strongly α*g continuous and a map g: Y ⟶ Z is α*g- irresolute, then g ∘ f: X ⟶ Z is strongly α*g continuous.

Proof: Let O be any α*g-closed set in Z. Since g is α*g-irresolute, g $^{-1}$(O) is α*g closed in Y. Also, f is strongly α*g continuous f^{-1}(g $^{-1}$(O)) is closed in X. But (g ∘f) $^{-1}$(O) = f^{-1}(g $^{-1}$(O)) is closed in X. Hence, g ∘ f: X ⟶ Z is strongly α*g continuous.

Theorem 4.15: If a map f: X ⟶ Y is α*g continuous and a map g: Y ⟶ Z is strongly α*g continuous,then g ∘ f: X ⟶ Z is α*g irresolute.

Proof: Let O be any α*g-closed set in Z. Since g is strongly α*g continuous, g $^{-1}$(O) is closed in Y. Also, f is α*g continuous, f^{-1}(g $^{-1}$(O)) is α*g-closed in X. But (g ∘f) $^{-1}$(O) = f^{-1}(g $^{-1}$(O)). Hence, g ∘ f: X ⟶ Z is α*g-irresolute.

Theorem 4.16: Let X be any topological spaces and Y be a α*g T $_{1/2}$ space and f: X ⟶ Y be a map. Then the following are equivalent.

1) f is strongly α*g continuous

2) f is continuous

Proof: (1) ⟹(2) Let O be any closed set in Y.Since every closed set is α*g-closed,O is α*g -closed in Y. Then f^{-1}(O) is closed in X. Hence , f is continuous.

(2) ⟹(1) Let O be any α*g-closed in (Y,σ).Since, (Y,σ) is a α*g T $_{1/2}$ space,O is closed in (Y,σ).Since, f is continuous.Then f^{-1}(O) is closed in (X,τ).Hence, f is strongly α*g continuous.

Theorem 4.17: Let f: (X, τ) ⟶ (Y, σ) be a map.Both (X, τ) and (Y,σ) are α*g $T_{1/2}$ space .Then the following are equivalent.

1) f is α*g-irresolute

2) f is strongly α*g continuous

3) f is continuous

4) f is α*g-continuous

Proof: The proof is obvious.

Theorem 4.18: The composition of two strongly α*g continuous maps is strongly α*g continuous.

Proof: Let O be a α*g closed set in (Z, η). Since, g is strongly α*g continuous, we get g^{-1}(O) is closed in (Y, σ).Since every closed set is α*g-closed, g^{-1}(O) is α*g-closed in (Y, σ). As f is also strongly α*g continuous, $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}$ (O) is closed in (X, τ). Hence, (g ∘ f) is strongly α*g continuous.

Theorem 4.19: If f:(X,τ) ⟶(Y,σ)and g:(Y, σ) ⟶ (Z, η) be any two maps.Then their composition g ∘ f:(X,τ)⟶(Z, η) is strongly α*g continuous if g is strongly α*g continuous and f is continuous.

Proof: Let O be a α*g closed in (Z, η) .Since, g is strongly α*g continuous, g^{-1}(O) is closed in (Y,σ).Since f is continuous,$f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is closed in (X,τ).Hence, (g ∘ f) is strongly α*g continuous.

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