M. Parvathi Banu et al. International Journal of Recent Research Aspects ISSN: 2349-7688, Vol. 5, Issue 1, April 2018, pp.1040-1044

Left Singularity and Left Regularity in near Idempotent Γ - Semi group

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Abstract: - In this paper, left singularity and left regularity in a near-idempotent Γ – semigroup are defined. In a near-idempotent Γ– semigroup λ_a is left singular and it is also proved that every δ class in a near-idempotent Γ – semigroup is left (right) singular if and only if S is left (right) regular. ξ- class is defined and proved that it is a near null semigroup. Also $\xi_a\ \xi_b \subset \xi$ for all a,b in S and $\xi_{ab}= \xi_a$ in a left singular near-idempotent Γ – semigroup. Any near-idempotent Γ – semigroup is left regular if and only if $\rho = \xi$ and right regular if $\lambda = \xi$. Also any near idempotent Γ – semigroup is near-commutative if $\delta = \xi$. Any near-commutative Γ – semigroup is near commutative if only and only if it is both left and right regular.

Keywords: Near- idempotent, Γ – semigroup, regular, singular semigroup, δ class, λ-class in near-idempotent Γ – semigroup, near-commutative Γ – semigroup.

I. INTRODUCTION

David Mclean[10] has obtained a decomposition of a band into more special bands. He has obtained a band as a semilattice union of rectangle bands. Motivated by this result, we have attempted to obtain a near idempotent Γ-semigroup as a union of more special near idempotent Γ-semigroups. We obtain each δ-class as a rectangular near-idempoent Γ- semigroup and each $\lambda(\rho)$ class as a left (right) singular near idempotent Γ-semigroups. We also show that a left(right) singular near idempotent Γ-semigroup is a semilattice union of left(right) singular near idempotent Γ-semigroups. We characterize left(right) regular Γ-semigroup in terms of the relations defined on it.

II. PRELIMINARY

DEFINITION II.1: Let S be a Γ- semigroup. Then S is said to be a near – idempotent Γ- semigroup if $x\gamma_1 y^2\gamma_2 z = x\gamma_1 y\gamma_2 z$ for all x,y,z ∈ S and $\gamma_1, \gamma_2 \in \Gamma$

DEFINITION II.2: Let S be a Γ- semigroup. Then S is said to be left-regular near-idempotent Γ- semigroup if $x\gamma_1 y\gamma_2 z\gamma_3 y\gamma_4 w = x\gamma_1 y\gamma_2 z\gamma_3 w$ for all x,y,z,w ∈ S and $\gamma_1, \gamma_2, \gamma_3, \gamma_4 \in \Gamma$

DEFINITION II.3: Let S be a Γ- semigroup. Then S is said to be left-singular near-idempotent Γ- semigroup if $x\gamma_1 y\gamma_2 z\gamma_3 w= x\gamma_1 y\gamma_2 w$ for all x,y,z,w∈ S and $\gamma_1, \gamma_2, \gamma_3, \in \Gamma$

DEFINITION II.4: A semigroup R is called a rectangular near idempotent Γ –semigroup if R is a near idempotent semigroup and it satisfy the identity

$x\gamma_1 y\gamma_2 z\gamma_3 y\gamma_4 w= x\gamma_1 y\gamma_2 w$ for all x,y,z,w∈ S and $\gamma_1, \gamma_2, \gamma_3, \gamma_4 \in \Gamma$

DEFINITION II.5: Let S be a near - idempotent Γ-semigroup and a and b, elements of S. We define the relation λ and ρ on S as follows:

a λ b if and only if $x\gamma_1 a\gamma_2 b\gamma_3 y = x\gamma_1' a\gamma_2' y$ and $x\gamma_1 b\gamma_2 a\gamma_3 y = x\gamma_1' b\gamma_2' y$ for all x, y ∈ S and $\gamma_1, \gamma_2, \gamma_3, \gamma_1', \gamma_2' \in \Gamma$

a ρ b if and only if $x\gamma_1 a\gamma_2 b\gamma_3 y = x\gamma_1' b\gamma_2' y$ and $x\gamma_1 b\gamma_2 a\gamma_3 y = x\gamma_1' a\gamma_2' y$ for all x ,y∈ S and $\gamma_1, \gamma_2, \gamma_3, \gamma_1', \gamma_2' \in \Gamma$

Both λ and ρ turn out to be an equivalence relation on S.

LEMMA II.6: Let S be a near-idempotent Γ- semigroup. Then the relation λ is an equivalence relation on S.

Proof: $x\gamma_1 a^2\gamma_2 z = x\gamma_1 a\gamma_2 z$ for all x,y,a ∈ S and $\gamma_1, \gamma_2 \in \Gamma$, by the definition of near-idempotent semigroup, so that a λ a for all a in S. Hence, λ is reflexive.

Let a λ b. Then, $x\gamma_1 a\gamma_2 b\gamma_3 y = x\gamma_1 a\gamma_2 y$ and $x\gamma_1 b\gamma_2 a\gamma_3 y = x\gamma_1 b\gamma_2 y$ for all x,y ∈ S and $\gamma_1, \gamma_2, \gamma_3 \in \Gamma$ which also implies b λ a. Hence, λ is symmetric.

Let a λ b and b λ c. Then, for all x, y ∈ S. We have, $x\gamma_1 a\gamma_2 b\gamma_3 y = x\gamma_1 a\gamma_2 y$ and $x\gamma_1 b\gamma_2 a\gamma_3 y = x\gamma_1 b\gamma_2 y$ and $x\gamma_1 b\gamma_2 c\gamma_3 y = x\gamma_1 b\gamma_2 y$ and $x\gamma_1 c\gamma_2 b\gamma_3 y = x\gamma_1 c\gamma_2 y$. Hence $x\gamma_1 a\gamma_2 c\gamma_3 y = x\gamma_1 a\gamma_2 c\gamma_3 y = x\gamma_1 a\gamma_2 b\gamma_3 c\gamma_4 y =$

$x\gamma_1\ a\gamma_2\ b\gamma_3c\gamma_4\ y = x\gamma_1\ a\gamma_2 b\gamma_3\ y = x\gamma_1 a\gamma_2 y$ for all $x,y \in S$.

Similarly, $x\gamma_1 c\gamma_2 a\gamma_3 y = x\gamma_1\ c\gamma_2 b\gamma_3\ a\gamma_4\ y = x\gamma_1\ c\gamma_2\ b\gamma_3 a\gamma_4\ y = x\gamma_1\ c\gamma_2\ b\gamma_3\ y = x\gamma_1 c\gamma_2 y$ for all $x, y \in S$. which implies $a\,\lambda\,c$. Hence λ is transitive. Thus λ is an equivalence relation on S.

Dually, we can prove that ρ is an equivalence relation on the near – idempotent Γ- semigroup on S.

LEMMA II.7: Let S be a near-idempotent Γ- semigroup. Let $a\,\lambda\,b$. Then, $a\gamma_1 c = b\gamma_2 c$ for all $c \in S$.

Proof: Let $a\ \lambda\ b$ where $a, b \in S$. we claim that for any $c \in S$, $a\gamma_1 c = b\gamma_2 c$

$a\ \lambda\ b \Rightarrow x\gamma_1 a\gamma_2 b\gamma_3 y = x\gamma_1 a\gamma_2 y$ and $x\gamma_1 b\gamma_2 a\gamma_3 y = x\gamma_1 b\gamma_2 y$ for all $x, y \in S$. Then for all $x, y \in S$ we have $x\gamma_1 a\gamma_2 c\gamma_3\ b\gamma_4 c\gamma_5 y = x\gamma_1 a\gamma_2\ c\gamma_3\ b\gamma_4 c\gamma_5\ y = x\gamma_1 a\gamma_2 b\gamma_3 c\gamma_4 b\gamma_5 c\gamma_6\ y = x\gamma_1\ a\gamma_2 (b\gamma_3 c\gamma_4)^2 y = x\gamma_1 a\gamma_2 b\gamma_3 c\gamma_4 y$ (by the definition of S) $= x\gamma_1 a\gamma_2 b\gamma_3\ c\gamma_4\ y = x\gamma_1 a\gamma_2 c\gamma_3 y$ and $x\gamma_1\ b\gamma_2 c\gamma_3\ a\gamma_4 c\gamma_5\ y = x\gamma_1\ b\gamma_2\ c\gamma_3 a\gamma_4 c\gamma_5\ y = x\gamma_1\ b\gamma_2 a\gamma_3 c\gamma_4 a\gamma_5 c\gamma_6\ y = x\gamma_1 b\gamma_2 (a\gamma_3 c\gamma_4)^2 y = x\gamma_1\ b\gamma_2 a\gamma_3 c\gamma_4 y$(by the definition of S)$= x\gamma_1\ b\gamma_2 a\gamma_3 c\gamma_4 y = x\gamma_1 b\gamma_2 c\gamma_3 y$ leading to $a\gamma_1 c = b\gamma_2 c$ for all $c \in S$ .Hence λ is a right congruence on S.

Dually, ρ is a left congruence on S.

RESULT II.8: We now consider the composition of two relations λ and ρ as follows

Let S be a near-idempotent Γ- semigroup. Then for any $a, b \in S$, we say that

$a\,\lambda \circ \rho\,b$ if there exists $c \in S$, such that $a\ \lambda\ c$ and $c\ \rho\ b$

LEMMA II.9: If S is a near-idempotent Γ- semigroup, then $\lambda \circ \rho = \rho \circ \lambda$ in S.

Proof: we first prove that $\lambda \circ \rho \subset \rho \circ \lambda$. Let $a\,\lambda \circ \rho b$.Then there exists $c \in S$ such that $a\,\lambda\,c$ and $c\,\rho\,b$.

$a\,\lambda\,c \Rightarrow x\gamma_1 a\gamma_2 c\gamma_3 y = x\gamma_1 a\gamma_2 y$ and $x\gamma_1 c\gamma_2 a\gamma_3 y = x\gamma_1 c\gamma_2 y$ for all $x,y \in S$,$\gamma_1, \gamma_2, \gamma_3, \in \Gamma$. Choose $d = a\gamma_1 c\gamma_2 b$. Then for all $x,y \in S$, $x\gamma_1 a\gamma_2 d\gamma_3 y = x\gamma_1\ a\gamma_2\ a\gamma_3 c\gamma_4 b\gamma_5\ y = x\gamma_1\ a^2\gamma_2\ c\gamma_3 b\gamma_4\ y = x\gamma_1\ a\gamma_2\ c\gamma_3 b\gamma_4\ y = x\gamma_1\ a\gamma_2 c\gamma_3 b\gamma_4\ y = x\gamma_1 d\gamma_2 y$ and

$x\gamma_1 d\gamma_2 a\gamma_3 y = x\gamma_1\ a\gamma_2\ c\gamma_3 b\gamma_4\ a\gamma_5 y = x\gamma_1\ a\gamma_2 b\gamma_3\ a\gamma_4 y = x\gamma_1 a\gamma_2 b\gamma_3\ a\gamma_4 c\gamma_5\ y = x\gamma_1 a\gamma_2 c\gamma_3 y$ (since $b\ \rho\ c$, ρ is a left congruence) $= x\gamma_1 a\gamma_2 y$.

But $x\gamma_1 a\gamma_2 c\gamma_3 y = x\gamma_1 a\gamma_2 y$. So that finally we get $x\gamma_1 d\gamma_2 a\gamma_3 y = x\gamma_1 a\gamma_2 y$. Therefore $a\ \rho\ d$ .Similarly, $x\gamma_1 d\gamma_2 b\gamma_3 y = x\gamma_1 a\gamma_2 c\gamma_3 b\gamma_4 b\gamma_5 y = x\gamma_1 a\gamma_2 c\ \gamma_3 b^2\gamma_4 y = x\gamma_1 a\gamma_2 c\gamma_3 b\gamma_4 y = x\gamma_1 d\gamma_2 y$ for all x, y in S. $x\gamma_1 b\gamma_2 d\gamma_3 y = x\gamma_1\ b\gamma_2\ a\gamma_3 c\gamma_4 b\gamma_5\ y = x\gamma_1 b\gamma_2 a\gamma_3\ c\gamma_4 b\gamma_5\ y = x\gamma_1 b\gamma_2 a\gamma_3 b\gamma_4\ y = x\gamma_1\ b\gamma_2\ a\gamma_3 b\ \gamma_4 y = x\gamma_1\ c\gamma_2 b\gamma_3\ a\gamma_4 b\gamma_5 y = x\gamma_1 c\gamma_2 b\gamma_3 y$ (since λ is a right congruence $a\gamma_1 b\ \lambda\ c\gamma_2 b$). But $x\gamma_1 c\gamma_2 b\gamma_3 y = x\gamma_1 b\gamma_2 y$, so that we get $x\gamma_1 b\gamma_2 d\gamma_3 y = x\gamma_1 b\gamma_2 y$. Hence $d\ \lambda\ c$. Thus $a\,\rho\,d$, $d\ \lambda\ b$ so that $a\ \rho \circ \lambda$ b. This gives $\lambda \circ \rho \subset \rho \circ \lambda$. By similar argument, we can prove that

$\rho \circ \lambda \subset \lambda \circ \rho$. Thus we get $\lambda \circ \rho = \rho \circ \lambda$.

We now define the relation δ on S as follows

DEFINITION II.10: Let S be a near –idempotent Γ-semigroup. Let $a, b \in S$. We define $\delta = \lambda \circ \rho$. In other words, $a\ \delta\ b$ if and only if there exists $c \in S$ such that $a\,\lambda\,c$ and $c\ \rho\ b$

We have already prove that $\lambda \circ \rho = \rho \circ \lambda$. Hence we can write $a\,\lambda \circ \rho\,b$ or $a\,\rho \circ \lambda\,b$ instead for $a\,\delta\,b$.

LEMMA II.11: Let S be a near-idempotent Γ- semigroup. δ is an equivalence relation on S.

Proof: For all a in S, $a\,\lambda\,a$ and $a\,\rho\,a$. Since λ and ρ are reflexive so that $a\,\lambda \circ \rho\,b$ which means $a\,\delta\,a$. Hence δ is reflexive.

$a\ \delta\ b \Rightarrow a\,\lambda \circ \rho\,b \Rightarrow$ there exists $u \in S$ such that $a\,\lambda\,u$ and $u\,\rho\,b \Rightarrow$ there exists $u \in S$ such that $b\,\rho\,u$ and $u\,\lambda\,a$ since λ and ρ are symmetric $\Rightarrow b\,\rho \circ \lambda\,a \Rightarrow b\,\delta\,a$ [since $\lambda \circ \rho = \rho \circ \lambda = \delta$]. Hence δ is symmetric.

$a\ \delta\ b$, $b\ \delta\ c \Rightarrow$ there exists $u, v \in S$ such that $a\,\lambda\,u$ and $u\,\rho\,b$, $b\,\lambda\,v$ and $v\,\rho\,c$ since $u\,\rho\,b$ and $b\,\lambda\,v$ we have $u\,\rho \circ \lambda\,v$ we have $u\,\lambda \circ \rho\,v$. Since $\lambda \circ \rho = \rho \circ \lambda$. Thus there exists $w \in S$ such that $u\,\lambda\,w$ and $w\,\rho\,v$.

$a\ \lambda\ u$ and $u\ \lambda\ w$ so that $a\ \lambda\ w$; $w\ \rho\ v$ and $v\ \rho\ c$ so that $w\,\rho\,c$. Therefore $a\,\lambda \circ \rho\,c$

i.e., $a\,\delta\,c$. Thus δ is transitive. Hence δ is an equivalence relation on S.

III. DECOMPOSITION OF NEAR IDEMPOTENT Γ-SEMIGROUP

Theorem III.1 : { δ_a / $a \in S$ } is a semigroup under the operation $\delta_a * \delta_b = \delta_{ab}$

We now prove that every δ- class is a Γ- subsemigroup of S.

Theorem III.2: Let S be a near-idempotent Γ-semigroup and $a \in S$.Then δ_a is rectangular near-idempotent Γ-semigroup.

Proof: Let $x , y , z, w \in \delta_a$. $x\,\delta\,a$, $y\,\delta\,a$, $z\,\delta\,a$, $w\,\delta\,a$. By transitivity $y\,\delta\,z$. Hence for all $x, w \in S$. $x\gamma_1 y\,\gamma_2 z\gamma_3 y\gamma_4 w = x\gamma_1 y\,\gamma_2 w$ and $x\gamma_1 z\,\gamma_2 y\gamma_3 z\gamma_4 w = x\gamma_1 z\,\gamma_2 w$. This result is true when $x, w \in \delta_a$ also. Thus we have $x\gamma_1 y\,\gamma_2 z\gamma_3 y\gamma_4 w = x\gamma_1 y\,\gamma_2 w$ for all $x,y,z,w \in \delta_a$ .Hence δ_a is rectangular near idempotent Γ-semigroup.

Theorem III.3: Let S be a near-idempotent Γ- semigroup. Then for a∈ S, λ_a is left-singular near idempotent Γ-semigroup.

Proof: Let S be a near-idempotent Γ- semigroup. Consider the relation λ on S. For a, b ∈ S

a λ b if and only if $x\gamma_1 a\gamma_2 b\gamma_3 y = x\gamma_1 a\gamma_2 y$ and $x\gamma_1 b\gamma_2 a\gamma_3 y = x\gamma_1 b\gamma_2 y$ for all x ,y ∈ S and $\gamma_1, \gamma_2, \gamma_3 \in \Gamma$ .Consider an equivalence relation λ_a where a ∈ S.We claim that λ_a is a near left – singular near-idempotent Γ-semigroup. Let u , v ∈ λ_a . a λ u and a λ v . For all x, y ∈ S $x\gamma_1 a\gamma_2 u\gamma_3 y = x\gamma_1 a\gamma_2 y$, $x\gamma_1 u\gamma_2 a\gamma_3 y = x\gamma_1 u\gamma_2 y$ and $x\gamma_1 a\gamma_2 v\gamma_3 y = x\gamma_1 a\gamma_2 y$, $x\gamma_1 v\gamma_2 a\gamma_3 y = x\gamma_1 a\gamma_2 y$. For all x , y ∈ S. $x\gamma_1 u\gamma_2 v\gamma_3 a\gamma_4 y = x\gamma_1 u\gamma_2 . v\gamma_3 a\gamma_4 y = x\gamma_1 u\gamma_2 v\gamma_3 y$ and $x\gamma_1 a\gamma_2 u\gamma_3 v\gamma_4 y = x\gamma_1 a\gamma_2 u\gamma_3 \cdot v\gamma_4 y = x\gamma_1 a\gamma_2 v\gamma_3 y = x\gamma_1 a\gamma_2 y$. Here, $u\gamma v$ λ a. Hence $u\gamma v \in \lambda_a$.

λ_a is a subsemigroup of S. Also $x\gamma_1 u\gamma_2 v\gamma_3 = x\gamma_1 u\gamma_2 a\gamma_3 v\gamma_4 y = x\gamma_1 u\gamma_2 a\gamma_3 v\gamma_4 y = x\gamma_1 u\gamma_2 a\gamma_3 y = x\gamma_1 u\gamma_2 y$ and $x\gamma_1 v\gamma_2 u\gamma_3 y = x\gamma_1 v\gamma_2 a\gamma_3 u\gamma_4 y = x\gamma_1 v\gamma_2 a\gamma_3 u\gamma_4 y = x\gamma_1 v\gamma_2 a\gamma_3 y = x\gamma_1 v\gamma_2 y$ for all x, y in S. Hence it is also true for all x , y ∈ λ_a . Thus for x , u, v, y ∈ λ_a , $x\gamma_1 u\gamma_2 v\gamma_3 y = x\gamma_1 u\gamma_2 y$. Hence λ_a is a left – singular near-idempotent Γ- semigroup.

Theorem III.4: Let R be a rectangular near-idempotent Γ-semigroup. Then for a, b ∈ R, $\lambda_a \lambda_b \subset \lambda_b$

Proof: Let u ∈ λ_a and v ∈ λ_b .Then $x\gamma_1 u\gamma_2 a\gamma_3 y = x\gamma_1 u\gamma_2 y$ and $x\gamma_1 a\gamma_2 u\gamma_3 y = x\gamma_1 a\gamma_2 y$, $x\gamma_1 v\gamma_2 b\gamma_3 y = x\gamma_1 v\gamma_2 y$ and $x\gamma_1 b\gamma_2 v\gamma_3 y = x\gamma_1 b\gamma_2 y$.

$x\gamma_1 u\gamma_2 v\gamma_3 b\gamma_4 y = x\gamma_1 u\gamma_2 v\gamma_3 b\gamma_4 y = x\gamma_1 u\gamma_2 v\gamma_3 y$ and $x\gamma_1 b\gamma_2 u\gamma_3 v\gamma_4 y = x\gamma_1 b\gamma_2 v\gamma_3 u\gamma_4 v\gamma_5 y = x\gamma_1 b\gamma_2 v\gamma_3 u\gamma_4 v\gamma_5 y = x\gamma_1 b\gamma_2 v\gamma_3 y = x\gamma_1 b\gamma_2 y$ [since u, v ∈ R and hence $x\gamma_1 v\gamma_2 u\gamma_3 v\gamma_4 y = x\gamma_1 v\gamma_2 y$]. Thus, $u\gamma v \in \lambda_b$ i.e, $\lambda_a \lambda_b \subset \lambda_b$ for all a ,b in R

Note III.5: If we define an operation ⋄ on { λ_a / a ∈ R } such that $\lambda_a \diamond \lambda_b = \lambda_c$ if and only if $\lambda_a \lambda_b \subset \lambda_c$ then from the above discussions of this theorem it is clear that $\lambda_a \diamond \lambda_b = \lambda_b$. Thus R is right-singular band of left – singular near-idempotent Γ- semigroup.

Now we move on to verify that left (right) regular near-idempotent Γ- semigroup is a semilattice of left (right) singular near-idempotent Γ- semigroup.

Theorem III.6: S is a left (right) regular near-idempotent Γ-semigroup if and only if every δ-class in S is a near left (right) singular near-idempotent Γ- semigroup.

Proof: Let S be a left (right)regular near-idempotent Γ-semigroup. Then $x\gamma_1 y\gamma_2 z\gamma_3 y\gamma_4 w = x\gamma_1 y\gamma_2 z\gamma_3 w$ for all x, y, z, w ∈ S and $\gamma_1, \gamma_2, \gamma_3, \gamma_4 \in \Gamma$ ---------(1). Let a ∈ S. δ_a is a rectangular near-idempotent Γ-semigroup of S. Hence, $x\gamma_1 y\gamma_2 z\gamma_3 y\gamma_4 w = x\gamma_1 y\gamma_2 w$ for all x, y, w, z in δ_a ----------- (2)

(1) and (2) gives $x\gamma_1 y\gamma_2 z\gamma_3 w = x\gamma_1 y\gamma_2 w$ for all x,y, z, w in δ_a . Hence δ_a degenerates into a near left singular near-idempotent Γ-semigroup .

Conversely, let a, b∈S. $a\gamma_1 b$ δ $b\gamma_2 a$, ab, ba are in the same δ-class. They are in a near- idempotent Γ-semigroup. For all x, y ∈S. $x\gamma_1 a\gamma_2 b\gamma_3 b\gamma_4 a\gamma_5 y = x\gamma_1 a\gamma_2 b\gamma_3 y \Rightarrow x\gamma_1 a\gamma_2 b^2\gamma_3 a\gamma_4 y = x\gamma_1 a\gamma_2 b\gamma_3 y \Rightarrow x\gamma_1 a\gamma_2 b\gamma_3 a\gamma_4 y = x\gamma_1 a\gamma_2 b\gamma_3 y$.

Therefore S is a left –regular near idempotent Γ-semigroup.

IV. LEFT SINGULARITY AND LEFT REGULARITY IN NEAR IDEMPOTENT Γ- SEMIGROUP

DEFINITION IV.1: Let S be a near-idempotent Γ-semigroup. Let a, b ∈ S.We say that a ξ b if and only if a λ b and a ρ b. In other words, $\xi = \lambda \cap \rho$.

LEMMA IV.2: Let S be a near-idempotent Γ-semigroup. Let a, b ∈ S. Then a ξ b if and only if $x\gamma_1 a\gamma_2 y = x\gamma_1 b\gamma_2 y$ for all x, y ∈ S.

Proof: let a ξ b. Then a λ b and a ρ b. Hence for all x, y ∈ S. $x\gamma_1 a\gamma_2 b\gamma_3 y = x\gamma_1 a\gamma_2 y$; $x\gamma_1 b\gamma_2 a\gamma_3 y = x\gamma_1 b\gamma_2 y$ and $x\gamma_1 a\gamma_2 b\gamma_3 y = x\gamma_1 b\gamma_2 y$; $x\gamma_1 b\gamma_2 a\gamma_3 y = x\gamma_1 a\gamma_2 y$. From the above equation it is clear that, $x\gamma_1 a\gamma_2 y = x\gamma_1 b\gamma_2 y$ for all x,y in S. Conversely, suppose that $x\gamma_1 a\gamma_2 y = x\gamma_1 b\gamma_2 y$ for all x,y ∈ S. For all x,y ∈ S. $x\gamma_1 a\gamma_2 b\gamma_3 y = x\gamma_1 b\gamma_2 b\gamma_3 y = x\gamma_1 b^2\gamma_2 y = x\gamma_1 b\gamma_2 y$ and $x\gamma_1 b\gamma_2 a\gamma_3 y = x\gamma_1 a\gamma_2 a\gamma_3 y = x\gamma_1 a^2\gamma_2 y = x\gamma_1 a\gamma_2 y$ so that a ρ b. Also for all x, y ∈ S. $x\gamma_1 a\gamma_2 b\gamma_3 y = x\gamma_1 a\gamma_2 a\gamma_3 y = x\gamma_1 a^2\gamma_2 y = x\gamma_1 a\gamma_2 y$ and $x\gamma_1 b\gamma_2 a\gamma_3 y = x\gamma_1 b\gamma_2 b\gamma_3 y = x\gamma_1 b^2\gamma_2 y = x\gamma_1 b\gamma_2 y$. So that, a λ b.Thus a ($\lambda \cap \rho$) i.e a ξ b.

LEMMA IV.3: Let S be a near-idempotent Γ-semigroup. Let a ∈ S, then every ξ- class is a near null semigroup.

Proof: Define ξ on S. Let a ∈ S. Let u, v ∈ ξ_a . $x\gamma_1 u\gamma_2 y = x\gamma_1 a\gamma_2 y = x\gamma_1 v\gamma_2 y$ for all x , y∈ S. For all x, y ∈ S. $x\gamma_1 u\gamma_2 v\gamma_3 y = x\gamma_1 u\gamma_2 . v\gamma_3 y = x\gamma_1 a\gamma_2 v\gamma_3 y = x\gamma_1 a\gamma_2 . v\gamma_3 y = x\gamma_1 a\gamma_2 . a\gamma_3 y = x\gamma_1 a^2\gamma_2 y = x\gamma_1 a\gamma_2 y$. Then $u\gamma v \in \xi_a$ so that ξ_a is subsemigroup of S. Also $x\gamma_1 u\gamma_2 y = x\gamma_1 v\gamma_2 y$ for all x, y ∈ S. Hence $x\gamma_1 u\gamma_2 y = x\gamma_1 v\gamma_2 y$ for all x, y ∈ ξ_a also. In other words if x, y, z, w ∈ ξ_a . $x\gamma_1 y\gamma_2 w = x\gamma_1 z\gamma_2 w$. Hence ξ_a is a near null semigroup. Also, if u ∈ ξ_a and v ∈ ξ_a. For all x, y in S, $x\gamma_1 u\gamma_2 y = x\gamma_1 a\gamma_2 y$ and $x\gamma_1 v\gamma_2 y = x\gamma_1 b\gamma_2 y$. $x\gamma_1 u\gamma_2 v\gamma_3 y = x\gamma_1 a\gamma_2 v\gamma_3 y = x\gamma_1 a\gamma_2 b\gamma_3 y$. So that $u\gamma v \in \xi_{ab}$. Hence $\xi_a \xi_b \subset \xi_{ab}$.

LEMMA IV.4: Let S be a near-idempotent Γ-semigroup and a, b ∈ S. Then $\xi_a \xi_b \subset \xi_{ab}$.

LEMMA IV.5: Let Ξ = { ξ_a / a ∈ S }. Define ▫ on Ξ such that ξ_a ▫ ξ_b = ξ_c if and only if $\xi_a \xi_b \subset \xi_c$. Then Ξ is a semigroup under ▫ .

Proof: By the last lemma $\xi_a \xi_b \subset \xi_{ab}$. Hence $\xi_a \circ \xi_b = \xi_{ab}$. Hence Ξ is a semigroup under $\circ$.

LEMMA IV.5: Let L be a left singular near-idempotent Γ-semigroup a, b in L. Then $\xi_{ab} = \xi_a$.

Proof: Let x, y, a, b ∈ L. Then $x\gamma_1 a\gamma_2 b\gamma_3 y = x\gamma_1 a\gamma_2 y$ for all x, y ∈ L. Hence $\xi_{ab} = \xi_a$. Thus $\xi_a \circ \xi_b = \xi_{ab} = \xi_a$ for all a, b ∈ L. Hence a left singular near-idempotent Γ-semigroup is a left singular union of near null semigroups.

LEMMA IV.6: A right singular near-idempotent Γ-semigroup is a right singular union of near null semigroups.

LEMMA IV.7: A near-idempotent Γ-semigroup S is a left regular near-idempotent Γ-semigroup if and only if $\lambda = \delta$ on S.

LEMMA IV.8: A near-idempotent Γ-semigroup S is a left regular near-idempotent Γ-semigroup if and only if $\rho = \xi$ on S.

Proof: In a left regular near-idempotent Γ-semigroup $\xi = \lambda \cap \rho = \delta \cap \rho$ [by last lemma] $= \rho$ since $\rho \subset \delta$. Let S be a near-idempotent Γ-semigroup in which $\rho = \xi$. $x\gamma_1 u\gamma_2 v\gamma_3 u\gamma_4 . u\gamma_5 v\gamma_6 \; y = x\gamma_1 \; u\gamma_2 v\gamma_3 \; u^2\gamma_4 v\gamma_5 y = x\gamma_1 u\gamma_2 v\gamma_3 u\gamma_4 v\gamma_5 y = x\gamma_1(u\gamma_2 v\gamma_3)^2 y$ for all x, y, u, v ∈ S $= x\gamma_1 u\gamma_2 v\gamma_3 y$.

$x\gamma_1 \; u\gamma_2 v\gamma_3 . u\gamma_4 u\gamma_5 u\gamma_6 \; y = x\gamma_1(u\gamma_2 v\gamma_3)^2 u\gamma_4 y = x\gamma_1 \, u\gamma_2 v\gamma_3 u\gamma_4 y$. Thus $u\gamma_1 v \; \rho \; u\gamma_2 v\gamma_3 u$. Since $\rho = \xi$, $x\gamma_1 u\gamma_2 v\gamma_3 u\gamma_4 \; y = x\gamma_1 u\gamma_2 v\gamma_3 y$ for all u, v ∈ S. Hence S is left regular near-idempotent Γ-semigroup.

Lemma IV.9: λ is a congruence relation in a left regular near-idempotent Γ-semigroup S.

Proof: Let S be a left regular near-idempotent Γ-semigroup. Let a λ b. Then $x\gamma_1 a\gamma_2 b\gamma_3 y = x\gamma_1 a\gamma_2 y$; $x\gamma_1 b\gamma_2 a\gamma_3 y = x\gamma_1 b\gamma_2 y$; Let c ∈ S. $x\gamma_1 c\gamma_2 a\gamma_3 c\gamma_4 b\gamma_5 y = x\gamma_1 c\gamma_2 a\gamma_3 c\gamma_4 . b\gamma_5 y = x\gamma_1 c\gamma_2 a\gamma_3 b\gamma_4 y = x\gamma_1 c\gamma_2 a\gamma_3 y$.

$x\gamma_1 c\gamma_2 b\gamma_3 c\gamma_4 a\gamma_5 y = x\gamma_1 c\gamma_2 b\gamma_3 a\gamma_4 y = x\gamma_1 c\gamma_2 b\gamma_3 y$. Thus we get that a λ b $\Rightarrow$ $c\gamma_1 a \; \lambda \; c\gamma_2 b$. Therefore λ is a left congruence. We know that λ is a right congruence in a near-idempotent Γ-semigroup. Thus λ is a congruence relation on S.

Lemma IV.10: In a near-idempotent Γ-semigroup S, $\delta = \xi$ implies that S is a near-commutative near idempotent Γ-semigroup.

Proof: Let a, b ∈ S. In any near– idempotent Γ-semigroup $a\gamma_1 b \; \lambda \; b\gamma_2 a$. But $\delta = \xi$. Hence $a\gamma_1 b \; \xi \; b\gamma_2 a$. Thus $x\gamma_1 a\gamma_2 b\gamma_3 y = x\gamma_1 b\gamma_2 a\gamma_3 y$ for all x, y in S. Hence S is near-commutative.

Theorem IV.11: A near-idempotent Γ-semigroup S is a near-commutative if and only if $\delta = \xi$ on S.

Theorem IV.12: A near-idempotent Γ-semigroup S is near-commutative if and only if it is both a left regular and a right regular near-idempotent Γ-semigroup.

Proof: Suppose that near-idempotent Γ-semigroup S is a near-commutative near-idempotent Γ-semigroup. Then $x\gamma_1 y\gamma_2 z\gamma_3 w = x\gamma_1 z\gamma_2 y\gamma_3 w$ for all x, y, z, w in S.

$x\gamma_1 y\gamma_2 z\gamma_3 y\gamma_4 w = x\gamma_1 \; y\gamma_2 z\gamma_3 . \; y\gamma_4 \; w = x\gamma_1 \; y^2\gamma_2 \; z\gamma_3 w = x\gamma_1 y\gamma_2 z\gamma_3 w$. Therefore S is a left regular near – idempotent Γ-semigroup. $x\gamma_1 y\gamma_2 z\gamma_3 y\gamma_4 w = x\gamma_1 y\gamma_2 . z\gamma_3 y\gamma_4 \; w = x\gamma_1 z\gamma_2 y^2 \gamma_3 w = x\gamma_1 z\gamma_2 y \gamma_3 w$

Therefore S is a right regular near-idempotent Γ-semigroup. Therefore S is both a left regular and a right regular near-idempotent Γ-semigroup.

Conversely, Let S be both a left regular and a right regular near-idempotent Γ-semigroup. $x\gamma_1 y\gamma_2 z\gamma_3 \; y\gamma_4 w = x\gamma_1 y\gamma_2 z\gamma_3 w$ by near left regularity $x\gamma_1 y\gamma_2 z\gamma_3 y\gamma_4 w = x\gamma_1 z\gamma_2 y\gamma_3 w$ by near right regularity. Therefore $x\gamma_1 y\gamma_2 z\gamma_3 w = x\gamma_1 z\gamma_2 y\gamma_3 w$. So that S is near-commutative.

Conclusion: In this paper, the class δ_a is proved as a rectangular near-idempotent Γ-semigroup and the class λ_a is proved as a left singular near-idempotent Γ-semigroup and for any a, b in a rectangular near-idempotent Γ-semigroup, $\lambda_a \lambda_b$ is contained in λ_b. Also, R is a right singular band of left singular near-idempotent Γ-semigroup. Also a relation ξ is defined and is proved that $\xi = \lambda \cap \rho$ along with the property that $\xi_a \xi_b \subseteq \xi_{ab}$ for any a, b in S. Also, if S is left-singular then $\xi_a \xi_b = \xi_a$.

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