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Some Graphs On Near Divisor Cordial-II

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Abstract: - A Near divisor cordial labeling of a graph G with vertex set V is a bijection f from V to {1,2,..., │V│-1,│V│+1},such that if each edge uv is assigned the label 1 if f (u) divides f (v) (or) if f (v) divides f (u) and 0 otherwise, then the number of edges labelled with 0 and the number of edges labelled with 1 differ by almost 1. If a graph admits Near divisor cordial labeling then it is called Near divisor cordial graph. In this paper , We proved graphs such as J(n+1,n),S_n,$B_{n,m}^2$,$K_{1,n}$,S($K_{1,n}$),$K_{2,n}$,$K_{3,n}$, <$K_{1,n}^{(1)}$,$K_{1,n}^{(2)}$>and<$K_{1,n}^{(1)}$,$K_{1,n}^{(2)}$,$K_{1,n}^{(3)}$> are Near divisor cordial (NDC).

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Keywords: Cordial labelling , Divisor cordial labelling and Near divisor cordial labelling.

I. INTRODUCTION

By a graph , we mean a finite undirected graphs without loops and multiple edges for terms not defined here.We refer to Harary [3]

Definition 1.1 [1]:
Let G = (V,E) be a graph. A mapping f : V(G) → { 0,1 } is called binary vertex labelling of G and f (v) is called the label of the vertex v of G under f.
Cahit [1] defined cordial labelling as follows

Definition 1.2 :
A binary vertex labelling of a graph G is called a cordial labelling if │v_f (0) – v_f (1) │≤ 1 and │e_f (0) – e_f (1) │≤ 1 . A graph G is cordial if it admits cordial labeling.

Here f^* : E(G) → { 0 , 1 } is given by f^*(e) =│f (u) – f (v) │. v_f (0) , v_f (1) be the number of vertices of G having labels 0 and 1 respectively under f and e_f (0) , e_f (1) be the number of edges of G having 0 and 1 respectively under f^*. The concept of divisor cordial labelling is introduced by R.Varatharajan, S.Navaneetha Krishnan and K.Nagarajan [5] and defined as follows:

Definition 1.3 [5] :
Let G = (V,E) be a simple graph and f : v → {1,2,.....,|V|} be a bijection. For each edge uv , assign the label 1 if either f (u) | f (v) or if f (v) | f(u) and the label 0 otherwise. f is called divisor cordial labelling if │e_f (0) – e_f (1) │≤ 1.
The concept of Near graceful labelling is introduced by Frucht [4] with edge labelling {1,2,...,q-1,q+1}.Motivated by the above definitions, we introduce the concept called Near divisor cordial.

2. MAIN RESULTS:

Definition 2.1:
Let G = (V,E) be a simple graph and f : V (G) → {1,2,....,|V|-1,|V|+1} be a bijection. For each edge uv, assign the label 1 if either f (u) | f (v) or f (v) | f(u) and the label 0 otherwise. f is called Near divisor cordial labelling if │e_f (0) – e_f (1) │≤ 1.
Note that K_7 is not divisor cordial but it is Near divisor cordial and $K_{1,2m}$ is divisor cordial but it is not Near divisor cordial. Hence the above definition is meaningful.
The following definitions are useful for proving theorems.

Definition 2.2 : For integers m,n ≥ 0 , we consider the graph ***jellyfish J(m,n)*** with vertex set V(J(m,n))={u,v,x,y} U { x_1,x_2,. . . . . ,x_m } U {y_1,y_2, ... ,y_n } and the edge set E(J(m,n)) = {(u,x) , (u,y), (u,v), (v,x), (v,y)} U {(x_i,x) / 1≤ i ≤ m} U {(y_i,y) / 1≤ i ≤ n}.

Definition 2.3: The graph P_n+ K_1 is called a shell

Definition 2.4 : The Bistar $B_{m,n}$ is the graph obtained from K_2 by identifying the center vertices of $K_{1,m}$ and $k_{1,n}$ at the vertices of K_2 respectively. $B_{m,m}$ is often denoted by B(m).

The Complete bipartite graph $K_{1,n}$ is called a Star Graph and it is demoted by S_m.

S($K_{1,n}$) the sub division of the star $k_{1,n}$ is a tree obtained from the star $k_{1,n}$ by adding a new pendent edge to each of the existing n pendent vertices.

Definition 2.5: Consider two stars $K_{1,n}^{(1)}$ and $K_{1,n}^{(2)}$ .Then G = < $K_{1,n}^{(1)}$,$K_{1,n}^{(2)}$ > is the graph obtained by joining apex vertices of star to a new vertex x. Note that G has 2n+3 vertices and 2n+2 edges.

Definition 2.6: Consider t copies of stars namely $K_{1,n}^{(1)}$,$K_{1,n}^{(2)}$,. . . . . , $K_{1,n}^{(t)}$. Then G = < $K_{1,n}^{(1)}$,$K_{1,n}^{(2)}$, ... , $K_{1,n}^{(t)}$ > is the graph obtained by joining apex vertices

of each $K_{1,n}^{(m-1)}$ and $K_{1,n}^{(m)}$ to a new vertex x_{m-1} where $2 \leq m \leq t$.
Note that G has t(n+2)-1 vertices and t(n+2)-2 edges.

THEOREM 2.7:
The graph J(n+1,n) is Near divisor cordial
Proof:
Let $V(J(n+1,n))=\{u_1,u_2,\ldots,u_n,w_1,w_2,w_3,w_4,v_1,v_2,v_3,\ldots,v_{n+1}\}$
$E(J(n+1,n)) = \{ w_1w_2, w_2w_3, w_3w_4, w_4w_1, w_2w_4\} \cup \{ w_3u_i / 1\leq i \leq n \} \cup \{w_1v_i / 1 \leq i \leq n+1\}$
Define $f(w_1) = 1$ and $f(w_3) = S$ such that s is the largest prime number such that $S \leq 2n+6$ and $S \neq 2n+5$
Label the remaining vertices from {2,3,............,s-1,s+1,........,2n+4,2n+6} in that order.
Then ,$e_f(0) = e_f(1) = k$ where $k = \frac{2n+6}{2}$
Hence $|e_f(0) - e_f(1)| = 0$
Hence, the graph J(n+1,n) is a Near divisor cordial

Example 2.8:

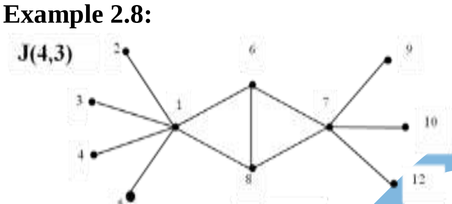


Fig 2.1

THEOREM 2.9:
The shell S_n is a Near divisor cordial
Proof:
Let $V(S_n) = \{v_0, v_1, v_2, \ldots, v_{n-1}\}$
$E(S_n)=\{v_0v_i ,\ 1 \leq i \leq n-1 \}$ and $i\neq n\} \cup \{ v_iv_{i+1} ,\ 1 \leq i \leq n-2 \}$
Fix $f(v_0) = 1$
Label the remaining vertices from {2,3,4,........,n-1,n+1} in that order.
Then $e_f(0)=n-2$, $e_f(1)=n-1$
Hence $|e_f(0) - e_f(1)| = 1$
Hence, S_n is a Near divisor cordial

Example 2.10:

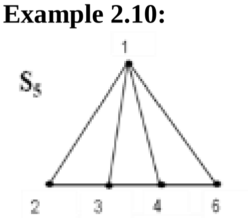


Fig 2.2

THEOREM 2.11:
The Graph $B_{n,m}^2$ is Near divisor cordial

Proof: $|V(B_{n,m}^2)| = n+m+2$ and $|E(B_{n,m}^2)| = 4n+1$
Always fix $f(v_0)=1$ and $f(u_0)=S$, where s is the largest prime such that $S\leq n+m+3$ and $S\neq n+m+2$.
Then label the remaining vertices from {2,3,.....,n+m+1,n+m+3}
Then $e_f(0) = n + m$, $e_f(1) = n+m+1$
Hence $|e_f(0) - e_f(1)| = 1$
Hence, The Graph $B_{n,m}^2$ is Near divisor cordial

Example 2.12:

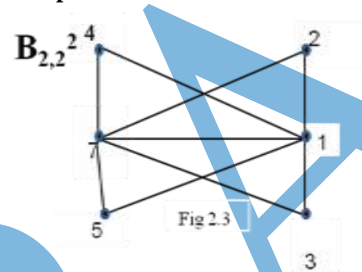


Fig 2.3

THEOREM 2.13:
The star graph $K_{1,n}$ is Near divisor cordial iff n is odd, $n \geq 5$
Proof:
Let $V(K_{1,n}) = \{v_1,v_2,v_3,\ldots,v_n\}$ and $E(K_{1,n}) = \{ vv_i : 1 \leq i \leq n\}$ and n=4.
Now assign label 2 to the vertex v and label the remaining vertices $v_1, v_2, v_3, \ldots, v_n$ by 1,3,4,. . . . . , n-1 and n+1 respectively.
We have, $e_f(0) = k+1$, $e_f(1) = k$, where n= 2k+1
Hence , Hence $|e_f(0) - e_f(1)| = 1$.
Therefore, $K_{1,n}$ is Near divisor cordial for n is odd and $n \geq 5$, n = 4,6
Conversely,
Suppose $K_{1,n}$ is Near divisor cordial
Suppose n is even and $n \geq 8$
Let n = 2k , there are k+1 even numbers and k odd numbers as labels.
Assigning any odd number to the central vertex as label, then it does not satisfy the condition
$|e_f(0) - e_f(1)| \leq 1$.
If f (v) = 2 for a labelling f in that case also $e_f(1) = k+1$ and $e_f(0) = k-1$. It can be easily verifies that by assigning any even number > 2 to the central vertex as label, then it does not satisfy the condition $|e_f(0) - e_f(1)| \leq 1$.
$\therefore$ $K_{1,n}$ is not Near divisor cordial When n is even & $n \geq 8$.
Clearly , $K_{1,n} \cong P_3$ is not near divisor cordial.
Therefore if $K_{1,n}$ is Near divisor cordial then n should be odd and $n \geq 3$ and n=4,6.
Example 2.14:

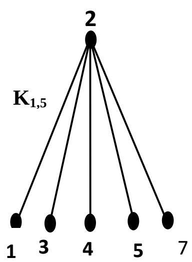


fig 2.4

THEOREM 2.15:
$S(K_{1,n})$ the sub division of the star $k_{1,n}$ is near divisor cordial
Proof:
Let $V(S(K_{1,n})) = \{ v, v_i, u_i : 1 \leq i \leq n\}$ and
$E(S(K_{1,n})) = \{vv_i, v_iu_i : 1 \leq i \leq n \}$
Define f by $f(v) = 1$
$f(v_i) = 2i \ (1 \leq i \leq n)$
$f(u_i) = 2i+1 \ (1 \leq i \leq n-1)$
and $f(u_n) = 2i+2$, where i = n
here $e_f(0) = e_f(1) = n$
Hence $|e_f(0) - e_f(1)| = 0$
Therefore, $S(K_{1,n})$ is near divisor cordial.
Example 2.16:

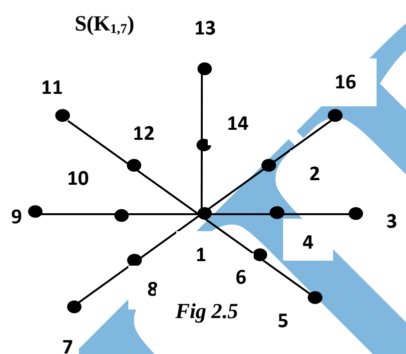


Fig 2.5

THEOREM 2.17:
The complete bipartite graph $K_{2,n}$ is near divisor cordial.
proof:
Let $V(K_{2,n}) = V_1 U V_2$
Such that $|V(K_{2,n})| = n+2$ and $|E(K_{2,n})| = 2n$.
$V_1 = \{x_1, x_2\}$ and
$V_2 = \{v_1, v_2, v_3, \ldots, v_n\}$.Now assign the label 1 to x_1 and the largest prime number S to x_2 such that $S \leq n+2$, $S \neq n+1$.Then the remaining vertices $y_1, y_2, y_3, \ldots, y_n$ is labelled from $\{2,3,4,\ldots,n+1,n+3\}-\{s\}$.
Clearly, $e_f(0) = e_f(1) = n$
Hence $|e_f(0) - e_f(1)| = 0$
Hence, $K_{2,n}$ is a Near divisor cordial
Example 2.18:

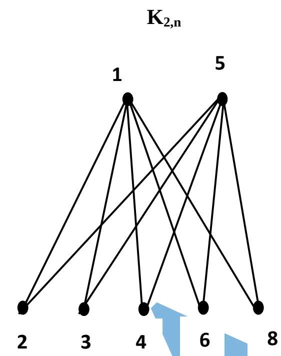
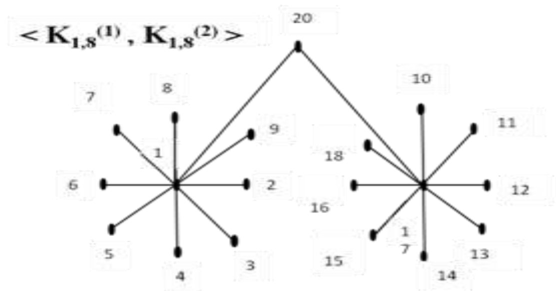


fig 2.6

THEOREM 2.19:
The complete bipartite graph $K_{3,n}$ is Near divisor cordial, n is odd.
proof:
Let $V(K_{3,n}) = V_1 U V_2$
Such that $|V(K_{3,n})| = n+3$ and $|E(K_{3,n})=3n$. $V_1 = \{x_1, x_2, x_3\}$ and $V_2 = \{v_1, v_2, v_3, \ldots, v_n\}$.Now define $f(x_1) = 1$, $f(x_2) = 2$ and $f(x_3) = s$,where s is the largest prime number such that $s \leq n+4$ and $s \neq n+3$.
And assign the remaining labels to the vertices $y_1, y_2, y_3, \ldots, y_n$
Then, $e_f(0) = n$,$e_f(1) = n-1$
Hence $|e_f(0) - e_f(1)| = 1$
Thus, $K_{3,n}$ is a Near divisor cordial.
Remark 2.21:
For $K_{m,n}$, $m \geq 4$, $e_f(0)$ value increases drastically than $e_f(1)$ and it is true for any Near divisor cordial labelling f except for some particular values of m & n.

THEOREM 2.22:
The graph $G = \langle K_{1,n}^{(1)}, K_{1,n}^{(2)} \rangle$ is Near divisor cordial
Proof :
Let $v_1^{(1)}, v_2^{(1)}, \ldots, v_n^{(1)}$ be the pendent vertices of $K_{1,n}^{(1)}$ and $v_1^{(2)}, v_2^{(2)}, \ldots, v_n^{(2)}$ be the pendent vertices of $K_{1,n}^{(2)}$
Let c_1 and c_2 be the apex vertices of $K_{1,n}^{(1)}$ and $K_{1,n}^{(2)}$ respectively and they are adjacent to a common vertex w. $|V(G)| = 2n+3$ $|E(G)| = 2n+2$. Let $f : V(G) \rightarrow \{1,2,3,\ldots,2n+2, 2n+4\}$
Now, assign the label 1 to c_1 and the largest prime number S such that $S \leq 2n+4$ (and $S \neq 2n+3$) to c_2 and the remaining numbers to be labelled to the remaining vertices of G. Since 1 divides any integer, and S does not divide any integer, then $e_f(0) = n+1$ and $e_f(1) = n+1$
Hence, $|e_f(0) - e_f(1)| = 0$.
Hence, the graph $G = \langle K_{1,n}^{(1)}, K_{1,n}^{(2)} \rangle$ is Near divisor cSordial

Example 2.23:



Theorem 2.24:
The graph G = < $K_{1,n}^{(1)}$,$K_{1,n}^{(2)}$, $K_{1,n}^{(3)}$ > is Near divisor cordial

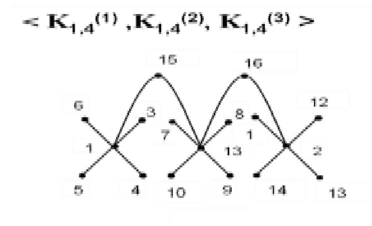
Proof :
Let $v_1^{(1)}$,$v_2^{(1)}$, ... , $v_n^{(1)}$ be the pendent vertices of $K_{1,n}^{(1)}$ and $v_1^{(2)}$,$v_2^{(2)}$, ... ,$v_n^{(2)}$ be the pendent vertices of $K_{1,n}^{(2)}$ and $v_1^{(3)}$,$v_2^{(3)}$, ... ,$v_n^{(3)}$ be the pendent vertices of $K_{1,n}^{(3)}$.
Let c_1 and c_2 and c_3 be the apex vertices of $K_{1,n}^{(1)}$ and $K_{1,n}^{(2)}$ and $K_{1,n}^{(3)}$ respectively and they are adjacent to a common vertex w_1 and w_2.such that w_1 is adjacent to c_1 and c_2 and w_2 is adjacent to c_2 and c_3.
Note that G has 3n+5 vertices and 3n+4 edges.

Case 1: n is odd
Now assign the label 1 to c_1 , 2 to c_2 and S to c_3 where S is the largest prime number such that $S \leq 3n+5$ (and $S \neq 3n+4$). Then assign the remaining numbers to the pendent vertices, we get ,
$e_f(1) = \frac{3n+5}{2}$ and $e_f(0) = \frac{3n+3}{2}$ (See fig)
Then, $|e_f(0) - e_f(1)| = 1$.

Case 2: n is even
Now assign the label 1 to c_1 , S to c_2 , where S is the largest prime number such that $S \leq 3n+5$ and 2 to c_3 . Then assign the remaining numbers to the pendent vertices in such a way that $\frac{n+1}{2}$ vertices adjacent to C_3 is assigned even numbers and remaining $\frac{n+1}{2}$ vertices adjacent to C_3 is assigned odd numbers and the remaining labels are assigned to the left over pendent vertices.
We get,$e_f(1) = \left\lfloor \frac{3n+4}{2} \right\rfloor = e_f(0)$ (See fig)
Then, $|e_f(0) - e_f(1)| = 1$.

Hence, The graph G = < $K_{1,n}^{(1)}$, $K_{1,n}^{(2)}$, $K_{1,n}^{(3)}$ > is Near divisor cordial.

Example 2.25:



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