Analysis Of An N-Policy Interdependent Finite Capacity Queueing Model With Controllable Arrival Rates

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***Abstract*-A finite capacity queue with interdependent arrival and service processes with controllable arrival rates is considered under a control policy. The steady state solutions and the system characteristics are derived and analyzed for this model.**

Keywords- Queueing system, control policy, inter-dependent arrival and service process.

1. INTRODUCTION

Several researchers have studied the N-policy for queues. By N-policy we mean that the server remains idle until there are N-customers waiting in the queue. Service starts with the arrival of the Nth customer and the busy period continues till the system is empty.

In general, the arrival and service processes are considered to be independent. But there are many practical situations in which they are dependent. This dependency can have a marked effect on system performance and must be accounted for any realistic analysis. Borthakur et.al. [1] studied poisson input queueing system with startup time and under control-operating policy. Heyman [2] studied the T-policy for the M/G/1 Queue. Chang and Ke [3] investigated the Cost Analysis of a two-phase Queue System with Randomized Control Policy. Ayyappan et.al. [4] investigated M/M/1 Retrial Queueing System with N-PolicyMultiple Vacation under Non-Pre-Emtive PriorityService by Matrix Geometric Method. Chaudhary [5] discussed a poisson queue under N-policy with a general setup time. Chaudhary and Baruah [6] studied Analysis of a poisson queue with a threshold policy and a grand vacation process: an analytic approach.. Ksirhnareaddy et.al. [7] investigated Analysis of bulk queue N-policy multiple vacations and setup times. Balchandran[8] studied Control policies for single server server system. Yadin, and Naor [9] discussed Queueing systems with Removal Service Station. Zhang and. Tian [10] studied the N threshold policy for the GI/M/1 queue.

In this paper we consider a single server finite capacity queueing system under N-policy with the assumption that the arrival and service processes of the system are correlated and follow a bivariate Poisson process.

II. DESCRIPTION AND POSTULATES OF THE MODEL

We consider a single server finite capacity queueing system with the following assumptions.

(i) The arrival process X_1 (t) and the service process X_2 (t) of the system are correlated and follow a bivariate Poisson process having the joint probability mass function of the form

$$P\left[X_1(t)=x_1,\, X_2(t)=x_2\right]$$
$$= e^{-(\lambda_i+\mu-\varepsilon)t}\sum_{j=0}^{\min(x_1,x_2)} (\varepsilon t)^j\left[(\lambda_i-\varepsilon)t\right]^{x_1-j}$$
$$\times\left[(\mu_n-\varepsilon)1\right]^{x_2-j}\frac{1}{!\,j\,!\,x_1-j\,!\,x_2-j}$$

$$x_1, x_2 = 0,1,2..............;\ \lambda_i > 0, \quad i = 0,1$$
$$\mu_n > 0,\ \ 0 \le \varepsilon < \min\ (\lambda_i, \mu_n),\ \ i = 0,1$$

with parameter λ_0, λ_1, μ_n and ε

where λ_0 : mean faster rate of arrivals, mean slower rate of arrivals, mean service rate and mean dependent rate (covariance between arrival and service process) respectively.

Server remains idle until the arrival of N customers.

Initially the customers arrival rate is λ_0 which reduces to λ_1 whenever the system size reaches a prescribed number R (R>N). The system continues with the reduced rate λ_1 as long as the number of customers in the queue is greater than some other prescribed integer r (r≥0 and r<R)

When the content in the system reaches r the arrival rate switches back to λ_0 and the same process is repeated.

The postulates of the model are as under:

The rate of no arrival and no service during a small interval of time 'h' is $1-(\lambda_0+\mu_n-2\varepsilon)h+O(h)$

The rate of one arrival and no service completion during a small interval of time 'h' when the system is in faster rate λ_0 of arrivals is $(\lambda_0-\varepsilon)h+0(h)$

The rate that there is no arrival and no service completion during a small interval of time 'h' when the system is in slower rate λ_1 of arrivals is

$1-(\lambda_1+\mu_n-2\,\varepsilon)h+O(h)$

The rate of one arrival and no service completion during a small interval of time h, when the system is in slower rate of arrivals is $(\lambda_1-\varepsilon)h+O(h)$

The rate that there is no arrival and one service completion during a small interval of time h when the system is either in faster or slower rate of arrivals in $(\mu_n-\varepsilon)h+O(h)$

The rate that there is one arrival and one service completion during a small interval of time h when the system is either in faster or slower rate of arrivals is $\varepsilon h+O(h)$

The rate that the occurrence of other than the above events during a small interval of time 'h' is 0 (h) and the events in non-overlapping intervals of time are statistically independent.

III. STEADY STATE RESULTS

In steady state, the following notations are used

$P_{0n}(0)$= The probability that there are n customers in the queue when the system is in faster rate of arrivals and the server is idle.

$P_{1n}(0)$= The probability that there are n customers in the queue when the system is in faster rate of arrivals and the server is busy.

$P_{1n}(1)$= The probability that there are n customers in the queue when the system is in slower rate of arrivals and the server is busy.

Here $P_{0n}(0)$ exists for $0\le n\le N-1$, $P_{1n}(0)$ exists for $0\le n\le R-1$ and $P_{1n}(1)$ exists for $n\ge r+1$

The steady state equations which are written through the matrix of densities are given by

$$-(\lambda_0-\varepsilon)\ P_{0n}(0)+(\lambda_0-\varepsilon)P_{0n-1}(0)=0 \qquad 1\le n\le N-1 \qquad \text{Eq. 1}$$

$$-(\lambda_0-\varepsilon)\ P_{00}(0)+(\mu-\varepsilon)P_{11}(0)=0 \qquad \text{Eq. 2}$$

$$-(\lambda_0+\mu-2\varepsilon)\ P_{11}(0)+(\mu-\varepsilon)P_{12}(0)=0 \qquad \text{Eq. 3}$$

$$-\left[\begin{matrix}(\lambda_0+\mu-2\varepsilon)\ P_{1n}(0)+(\mu-\varepsilon)P_{1n+1}(0)\\ +(\lambda_0-\varepsilon)P_{1n-1}(0)=0\end{matrix}\right] \qquad 2\le n\le N-1 \qquad \text{Eq. 4}$$

$$-(\lambda_0+\mu-2\varepsilon)\ P_{1N}(0)+(\lambda_0-\varepsilon)P_{0N-1}(0)+(\mu-\varepsilon)P_{1N+1} +(\lambda_0-\varepsilon)P_{1N-1}(0)=0 \qquad \text{Eq. 5}$$

$$\left.\begin{matrix}-(\lambda_0+\mu-2\varepsilon)\ P_{1n}(0)+(\lambda_0-\varepsilon)P_{1n-1}(0)\\ +(\mu-\varepsilon)P_{1n+1}(0)=0\end{matrix}\right] \qquad N+1\le n\le r-1 \qquad \text{Eq. 6}$$

$$\left.\begin{matrix}-(\lambda_0+\mu-2\varepsilon)\ P_{1r}(0)+(\lambda_0-\varepsilon)P_{1r-1}(0)\\ +(\mu-\varepsilon)P_{1r+1}(0)+(\mu-\varepsilon)P_{1r+1}(1)=0\end{matrix}\right] \qquad \text{Eq. 7}$$

$$\left.\begin{matrix}-(\lambda_0+\mu-2\varepsilon)\ P_{1n}(0)+(\lambda_0-\varepsilon)P_{1n-1}(0)\\ +(\mu-\varepsilon)P_{1n+1}(0)=0\end{matrix}\right] \qquad \text{Eq. 8} \qquad r+1\le n\le R-2$$

$$-(\lambda_0+\mu-2\varepsilon)\ P_{1R-1}(0)+(\lambda_0-\varepsilon)P_{1R-2}(0)=0 \qquad \text{Eq. 9}$$

$$-(\lambda_1+\mu-2\varepsilon)\ P_{1r+1}(1)+(\mu-\varepsilon)P_{1r+2}(1)=0 \qquad \text{Eq. 10}$$

$$-(\lambda_1+\mu-2\varepsilon)\ P_{1n}(1)+(\lambda_1-\varepsilon)P_{1n-1}(1)+(\mu-\varepsilon)P_{1n+1}(1)=0$$

Eq. 11 $\qquad r+2\le n\le R-1$

$$\left.\begin{matrix}-(\lambda_1+\mu-2\varepsilon)\ P_{1R}(1)+(\lambda_0-\varepsilon)P_{1R-1}(0)\\ +(\mu-\varepsilon)P_{1R+1}(1)+(\lambda_1-\varepsilon)P_{1R-1}(1)=0\end{matrix}\right] \qquad \text{Eq. 12}$$

$$-(\lambda_1+\mu-2\varepsilon)\ P_{1n}(1)+(\lambda_1-\varepsilon)P_{1n-1}(1)+(\mu-\varepsilon)P_{1n+1}(1)=0$$

R+1≤n≤K-1 Eq. 13

$$-(\lambda_1-\varepsilon)P_{1K-1}(1)+(\mu-\varepsilon)P_K(1)=0 \qquad \text{Eq. 14}$$

$$\text{Let } s=\left(\frac{\lambda_0-\varepsilon}{\mu-\varepsilon}\right) \text{ and } t=\left(\frac{\lambda_1-\varepsilon}{\mu-\varepsilon}\right)$$

From equation Eq. 1

$$P_{0n}(0)=P_{00}(0), \qquad n=0,1.........N-1 \qquad \text{Eq. 15}$$

From equations Eq. 2, Eq. 3 and Eq. 4

$$P_{1n}(0)=\frac{s(1-s^n)}{1-s}P_{00}(0),\ n=1,2,..........N\ if\ \ s\neq1$$

Eq. 16

From equation Eq. 5 and Eq. 6 we recursively derive

$$P_{1n}(0)=s^{n-N+1}\left(\frac{1-s^N}{1-s}\right)P_{00}(0), \qquad \text{Eq. 17}$$

$n=N+1,\ N+2.........r\ if\ \ s\neq1$

Using equation Eq. 7

$$P_{1r+1}(0)=s^{r-N+2}\left(\frac{1-s^N}{1-s}\right)P_{00}(0)-P_{1r+1}(1) \qquad \text{Eq. 18}$$

From equations Eq. 7 and Eq. 8

$$P_{1n}(0)=s^{n-N+1}\left(\frac{1-s^N}{1-s}\right)P_{00}(0)-\left(\frac{1-s^{n-r}}{1-s}\right)P_{1r+1}(1) \qquad \text{Eq. 19}$$

$$n=r+1,\ r+2,\ ....\ R-1$$

From equations Eq. 7 , Eq. 8 and Eq. 9

$$P_{1r+1}(1)=\frac{s^{R-N+1}(1-s^N)}{1-s^{R-r}}P_{00}(0) \qquad \text{Eq. 20}$$

$$=AB\,P_{00}(0)\ \ where\ \ A=s^{-N+1}(1-s^N)\ \ and\ \ B=\frac{s^{R+r}}{s^r-s^R}$$

From equations Eq. 10 and Eq. 11 we recursively derive

$$P_{1n}(1)=\frac{1-t^{n-r}}{1-t}P_{1r+1}(1),\quad n=r+1,\ r+2........R$$

$If\ \ s\neq1,\ t\neq1$ Eq. 21

Where $P_{r+1}(1)$ is given by Eq. 20

From equations Eq. 13 and Eq. 14we recursively derive

$$P_{1n}(1)=t^n\left(\frac{t^{-R}-t^{-r}}{1-t}\right)P_{1r+1}(1),\quad n\ge R+1,\ t\neq1,\ s\neq1$$

Eq. 22

Where P_{1r+1} is given by Eq. 20

Thus from Eq. 15 to Eq. 15,we find that all the steady state probabilities are expressed in term of $P_{00}(0)$

Under the steady state conditions let $P_0(0)$ be the probability that the server is idle and $P_1(0)$ be the probability that the server is busy with faster arrival rate respectively. Then

$$P_0(0)=\sum_{n=0}^{N-1}P_{0n}(0)=N\ P_{00}(0) \qquad \text{Eq. 23}$$

$$P_1(0)=\sum_{n=1}^{N}P_{1n}(0)+\sum_{n=N+1}^{r}P_{1n}(0)+\sum_{n=r+1}^{R-1}P_{1n}(0)$$

From equations Eq. 16, Eq. 17 and Eq. 19

$$P_1(0)=\sum_{n=1}^{N}\frac{s(1-s^n)}{1-s}P_{00}(0)+\sum_{n=N+1}^{r}\left(\frac{1-s^N}{1-s}\right)s^{n-N+1}P_{00}(0)+\sum_{n=r+1}^{R-1}\left[s^{n-N+1}\frac{(1-s^N)}{1-s}P_{00}(0)-\frac{(1-s^{n-r})}{1-s}P_{1r+1}(1)\right]$$

Using Eq. 20 and simplifying, we get

$$P_1(0)=\left[\frac{N}{1-s}-\frac{(R-r)\left(1-s^N\right)s^{R-N+1}}{1-s^{R-r}}\right]P_{00}(0)$$

$$\text{where } s\neq 1$$

$$=\left[\frac{N}{1-s}-AB(R-r)\right]P_{00}(0)\ \ \text{where } A=\left(1-s^N\right)s^{-N+1}$$

$$\text{and } B=\frac{s^{r+R}}{s^r-s^R}$$

Eq. 24

Hence
P (0) = the probability that the system is in faster rate of arrival mode

$$=P_0(0)+P_1(0)$$

$$=\left[\frac{N}{1-s}-\frac{(R-r)\left(1-s^N\right)s^{R-N+1}}{1-s^{R-r}}\right]P_{00}(0)\ \ \text{where } s\neq 1$$

$$=\left[\frac{N}{1-s}-AB\ (R-r)\right]P_{00}(0) \qquad \text{Eq. 25}$$

Again P(1)= The probability that the system is in slower rate of arrival mode

$$=\sum_{n=r+1}^{R}P_{1n}(1)+\sum_{n=R+1}^{K}P_{1n}(1)$$

From equations Eq. 20 and Eq. 21

$$P(1)=\sum_{n=r+1}^{R}\left(\frac{1-t^{n-r}}{1-t}\right)P_{1r+1}+\sum_{n=R+1}^{K}\frac{t^n\left(t^{-R}-t^{-r}\right)}{(r-t)}P_{1r+1}(1)$$

$$=\left[(R-r)(1-t)-\left(t^{K-R+1}-t^{K-r+1}\right)\right]\frac{AB}{(1-t)^2}P_{00}$$

Eq. 26

Using the normalizing condition
P(0)+P(1)=1 and substituting for P(0) and P (1) and simplifying we get

$$\left[P_{00}(0)\right]^{-1}=\left\{\frac{N}{(1-S)}+\left[\frac{(R-r)t}{(1-t)}-\frac{\left(t^{K-R+1}-t^{K-r+1}\right)}{(1-t)^2}\right]\frac{AB}{(1-t)^2}\right\},$$

$$s\neq 1, t\neq 1 \qquad \text{Eq. 27}$$

IV. EXPECTED NUMBER OF UNITS IN THE SYSTEM

The expected number of units in the system when if is in different states can be determined as follows:
L_1= the expected number of customers in the system when the server is idle

$$L_1=\sum_{n=1}^{N-1}n\,P_{0n}(0)=\frac{N(N-1)}{2}P_{00}(0) \qquad \text{Eq. 28}$$

Where $P_{00}(0)$ is given by Eq. 27
L_2 = the expected number of customers in the system when the server is busy with faster arrival rate

$$L_2=\sum_{n=1}^{N}n\,P_{1n}(0)+\sum_{n=N+1}^{r}n\,P_{1n}(0)+\sum_{n=r+1}^{R-1}n\,P_{1n}(0)$$

$$=\sum_{n=1}^{N}\frac{n\,s\left(1-s^n\right)}{1-s}P_{00}(0)+\sum_{n=N+1}^{r}n\,s^{n-N+1}\frac{\left(1-s^N\right)}{1-s}P_{00}(0)+\sum_{n=r+1}^{R-1}n\left(\frac{1-s^N}{1-s}\right)s^{n-N+1}\left(\frac{s^n-s^r}{s^r-s^R}\right)P_{00}(0)$$

After simplifying

$$L_2=\left[\frac{N(N+1)s}{2}+\frac{Ns^2}{1-s}+\frac{AB(rs-R)}{1-s}-\frac{AB(R+r)(R-r-1)}{2}\right]\frac{P_{00}(0)}{(1-s)}$$

Where $A=\left(1-s^N\right)s^{-N+1}$, $B=\frac{s^{r+R}}{s^r-s^R}$, and $s\neq 1$

Eq. 29

And $P_{00}(0)$ is given by Eq. 27
L_3 = the expected number of customers in the system when the server is busy with slower rate of arrivals

$$=\sum_{n=r+1}^{R-1}n\,P_{1n}(1)+\sum_{n=R}^{K}n\ P_{1n}(1)$$

$$=\sum_{n=r+1}^{R-1}n\left(\frac{1-t^{n-r}}{1-t}\right)P_{1r+1}(1)+\sum_{n=R}^{K}n\left(\frac{t^{n-R}-t^{n-r}}{1-t}\right)P_{1r+1}(1)$$

After simplifying

$$L_3=\left[1/2(R+r)(R-r-1)+\frac{R-rt}{(1-t)}+\frac{(K-Kt+1)t^{K+1}\left(t^{-r}-t^{-R}\right)}{(1-t)}\right]\frac{P_{1r+1}(1)}{(1-t)},\quad s\neq 1\ t\neq 1$$

Eq. 30

Where $P_{1r+1}(1)$ is given by Eq. 20 and $P_{00}(0)$ is given by Eq. 27
Hence the expected number of customers in the system

$$L_N=L_1+L_2+L_3$$

$$=\frac{P_{00}(0)}{1-s}\left[\frac{N(N-1)}{2}+\frac{sN}{1-s}+\frac{AB(rs-R)}{(1-s)}-\frac{AB(r+R)(R-r-1)}{2}\right]$$

$$+\frac{AB\,P_{00}(0)}{(1-t)}\left[\frac{1}{2}(R+r)(R-r-1)+\frac{R-rt}{(1-t)}+\frac{(K-Kt+1)t^{K+1}\left(t^{-r}-t^{-R}\right)}{(1-t)}\right]$$

Where $s\neq 1\ t\neq 1$

Eq. 31

Where $P_{00}(0)$ is given by Eq. 27

V. EXPECTED WAITING TIME

Using the Little's formula the expected waiting time of the customer, who arrive while the server is busy, is calculated as

$$W_s=\frac{L_s}{\bar{\lambda}} \qquad \text{Eq. 32}$$

Where L_s= the number of customers in the system when the server is busy.
$=L_2+L_3$ where L_2 and L_3 are given by Eq. 29 and Eq. 30 respectively
And $\bar{\lambda}$ = the actual mean arrival of the system

$= \lambda_0 P_1(0) + \lambda_1 P(1)$ where $P_1(0)$ and $P(1)$ are given by Eq. 24 and Eq. 26 respectively.

Eq. 33

VI. MEAN ADDITIONAL DELAY DUE TO BUILD UP PERIOD

Now we want to derive the waiting time of the customer who arrives while the server is idle

Let I_R= Residual build up period (i.e. the duration from the arrival of test customer to the instant when N-th customer arrives)

Now the test customer can be any of the 1,2, …….N customers that arrive during build up period. We get-

$$E(I_R) = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{N-i}{\lambda}\right) = \frac{N-1}{2\lambda}$$

Hence E (D) = Mean of the additional delay due to the build-up period.

$$= \frac{N(N-1)}{2\lambda} P_{00}(0) \qquad \text{Eq. 34}$$

Where $P_{00}(0)$ is given by Eq. 27

VII. SOME OTHER SYSTEM CHARACTERISTICS

Let E(I), E(F) and E(S) denote the expected lengths of the idle period, busy period with faster arrival rate and busy period with slower arrival rate respectively. Then the expected length of a cycle is:

$$E(C) = E(I) + E(F) + E(S) \qquad \text{Eq. 35}$$

The long run fractions of time the service is idle, busy with faster arrival rate and busy with slower arrival rate and given by

$$\frac{E(I)}{E(C)} = P_0(0) \qquad \text{Eq. 36}$$

$$\frac{E(F)}{E(C)} = P_1(0) \qquad \text{Eq. 37}$$

$$\frac{E(S)}{E(C)} = P(1) \qquad \text{Eq. 38}$$

Clearly $E(I) = \frac{N}{\lambda}$, Hence from Eq. 34

$E(C) = \frac{N}{\lambda}\left[P_{00}(0)\right]^{-1}$, Where $P_{00}(0)$ is given by Eq. 26

Hence for Eq. 37 and Eq. 38

$$E(F) = P_1(0)\,E(C) = \left[\frac{NS}{(1-S)} - \frac{AB(R-r)}{(1-S)}\right]\frac{N}{\lambda} \qquad \text{Eq. 39}$$

$$E(S) = P(I)\ E(C)$$

$$= \left[\frac{(R-r)}{(1-t)} - \frac{\left(t^{K-R+1} - t^{K-r+1}\right)}{(1-t)^2}\right]\frac{N}{\lambda}AB \qquad \text{Eq. 40}$$

VIII. NUMERICAL ANALYSIS

For various values of λ_0, λ_1, μ, ε, r, R, K, N the values of $P_0(0)$, P(0), P(I), L_N and W_N are computed and tabulated in the tables. It is observed that when the mean dependence rate increases and the other parameters are kept fixed, L_N and W_N decrease and $P_{00}(0)$ (the probability that the system is empty) increases. When the arrival rate increases (the other parameters being fixed), $P_{00}(0)$ decreases and L_N, W_N increase. However, with increase in service rate, both L_N, and W_N decrease. Moreover, increase in K results an increment in Ls and Ws while other parameters are kept unchanged.

S. No.	r	R	λ_0	λ_1	μ	ε	N	K	$P_{00}(0)$	P(0)	P(1)
1	7	12	8	2	10	0.5	4	25	0.058383	0.963573	0.036427
2	8	12	8	2	10	0.5	4	25	0.057796	0.967294	0.032706
3	9	12	8	2	10	0.5	4	25	0.057251	0.970746	0.029254
4	10	12	8	2	10	0.5	4	25	0.056747	0.973936	0.026064
5	9	13	8	2	10	0.5	4	25	0.056626	0.974702	0.025298
6	9	14	8	2	10	0.5	4	25	0.056075	0.978194	0.006
7	9	15	8	2	10	0.5	4	25	0.05559	0.981265	0.018735
8	9	16	8	2	10	0.5	4	25	0.055165	0.983955	0.016045
9	9	11	6	5	9	0.5	5	30	0.071103	0.978113	0.021887
10	9	11	6.2	5	9	0.5	5	30	0.066648	0.972869	0.027131
11	9	11	6.3	5	9	0.5	5	30	0.06445	0.969919	0.030081
12	9	11	6.4	5	9	0.5	5	30	0.062272	0.966741	0.033259
13	9	11	7	5	9	0.5	5	30	0.049766	0.942476	0.057524
14	9	11	7	5.3	9	0.5	5	30	0.049535	0.938101	0.061899
15	9	11	7	5.6	9	0.5	5	30	0.049266	0.933007	0.066993
16	9	11	7	5.9	9	0.5	5	30	0.048949	0.927001	0.072999
17	9	13	8	5	8.2	0.5	5	30	0.019262	0.819479	0.180521
18	9	13	8	5	8.4	0.5	5	30	0.021974	0.844018	0.155982
19	9	13	8	5	8.6	0.5	5	30	0.024775	0.865538	0.134462
20	9	13	8	5	8.8	0.5	5	30	0.027641	0.884304	0.115696
21	9	13	8	5	8.5	0.5	5	30	0.023365	0.855139	0.144861
22	9	13	8	5	8.5	0.6	5	30	0.023498	0.857276	0.142724
23	9	13	8	5	8.5	0.7	5	30	0.023634	0.859422	0.140578
24	9	13	8	5	8.5	0.8	5	30	0.023773	0.861577	0.138423
25	9	13	6	5	9.5	0.5	4	25	0.097379	0.994341	0.005659
26	9	13	6	5	9.5	0.5	5	25	0.077953	0.99212	0.00788
27	9	13	6	5	9.5	0.5	6	25	0.065021	0.988855	0.011145
28	9	13	6	5	9.5	0.5	7	25	0.055809	0.984012	0.015988

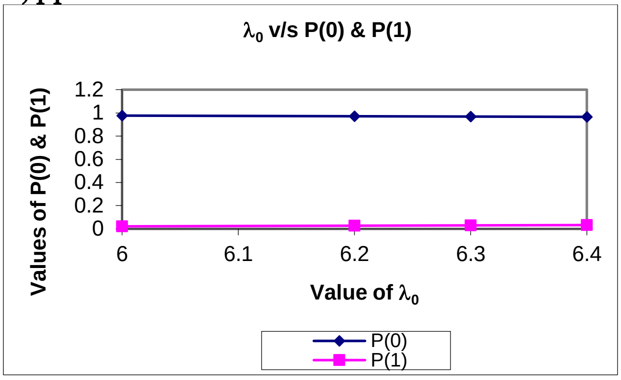
XI. CONCLUSION

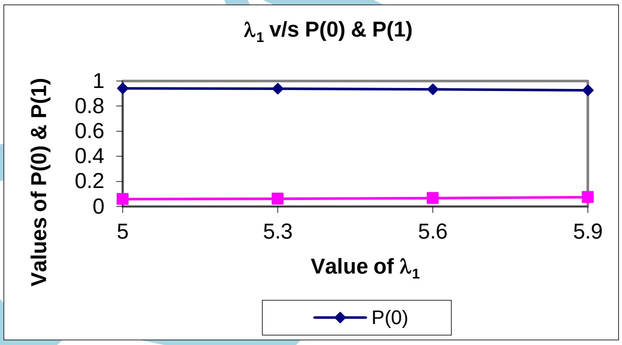
Many queueing systems occurring in message and packet switching applications have finite waiting room (capacity) for the customers. The incorporation of varying input rates make the model closer to real life congestion situations. Our study gives an insight to improve the service quality of the system. .

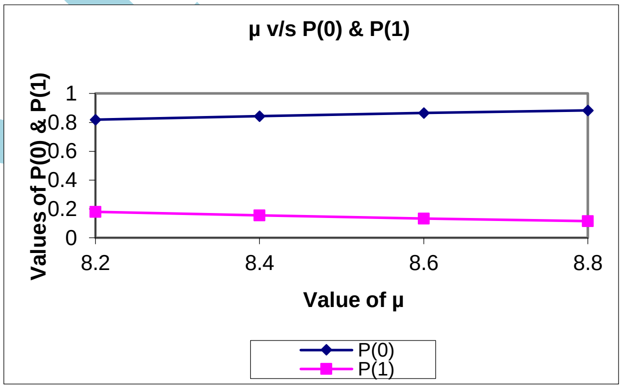
Table-1

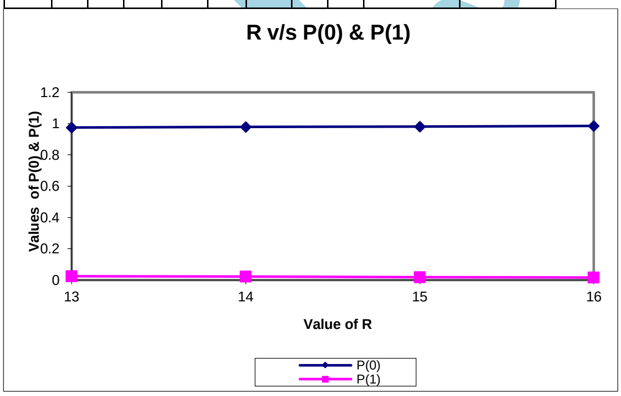
Table-2

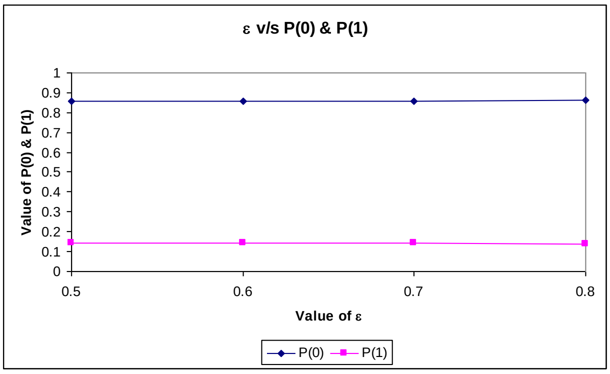
S. No.	r	R	λ_0	λ_1	μ	ε	N	K	L_N	W_N
1	7	12	8	2	10	0.5	4	25	4.191349	0.538634
2	8	12	8	2	10	0.5	4	25	4.250422	0.544663
3	9	12	8	2	10	0.5	4	25	4.312059	0.551099
4	10	12	8	2	10	0.5	4	25	4.375239	0.557809
5	9	13	8	2	10	0.5	4	25	4.400936	0.560757
6	9	14	8	2	10	0.5	4	25	4.485431	0.570001
7	9	15	8	2	10	0.5	4	25	4.565005	0.578758
8	9	16	8	2	10	0.5	4	25	4.639292	0.586975
9	9	11	6	5	9	0.5	5	30	3.755816	0.628261
10	9	11	6.2	5	9	0.5	5	30	3.912168	0.634326
11	9	11	6.3	5	9	0.5	5	30	3.994263	0.63797
12	9	11	6.4	5	9	0.5	5	30	4.078949	0.642007
13	9	11	7	5	9	0.5	5	30	4.638807	0.67376
14	9	11	7	5.3	9	0.5	5	30	4.681902	0.679051
15	9	11	7	5.6	9	0.5	5	30	4.734122	0.685488
16	9	11	7	5.9	9	0.5	5	30	4.798556	0.693463
17	9	13	8	5	8.2	0.5	5	30	7.259152	0.973281
18	9	13	8	5	8.4	0.5	5	30	6.917821	0.918451
19	9	13	8	5	8.6	0.5	5	30	6.600254	0.868842
20	9	13	8	5	8.8	0.5	5	30	6.305646	0.823954
21	9	13	8	5	8.5	0.5	5	30	6.756114	0.893026
22	9	13	8	5	8.5	0.6	5	30	6.731716	0.889048
23	9	13	8	5	8.5	0.7	5	30	6.707214	0.885059
24	9	13	8	5	8.5	0.8	5	30	6.682602	0.88106
25	9	13	6	5	9.5	0.5	4	25	3.052022	0.509151
26	9	13	6	5	9.5	0.5	5	25	3.545531	0.591699
27	9	13	6	5	9.5	0.5	6	25	4.036393	0.673984
28	9	13	6	5	9.5	0.5	7	25	4.523453	0.755923

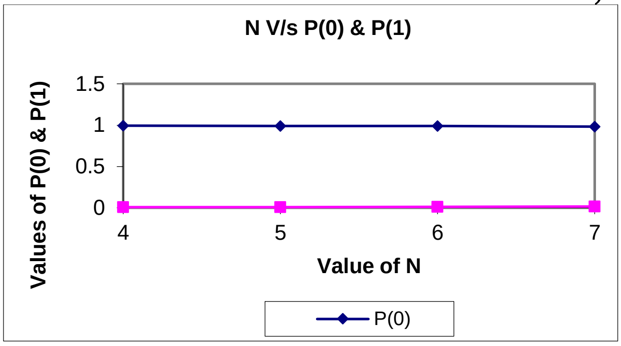


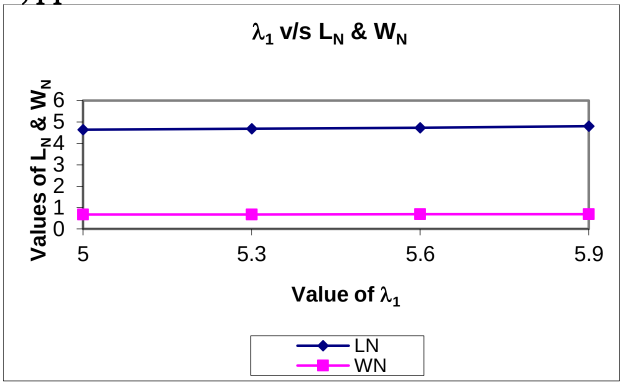


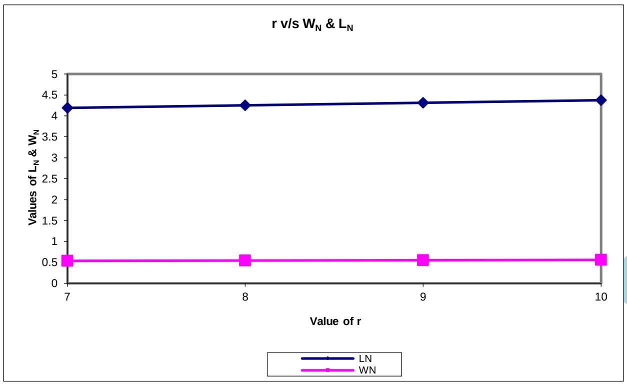


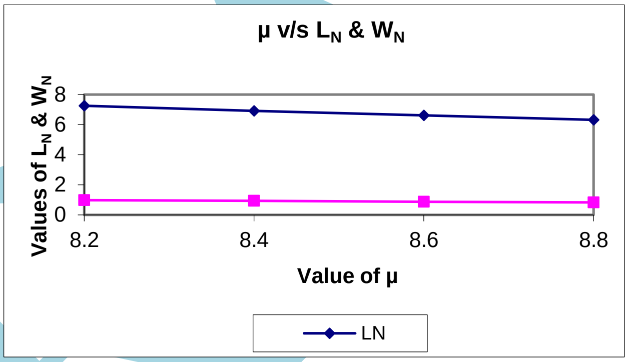


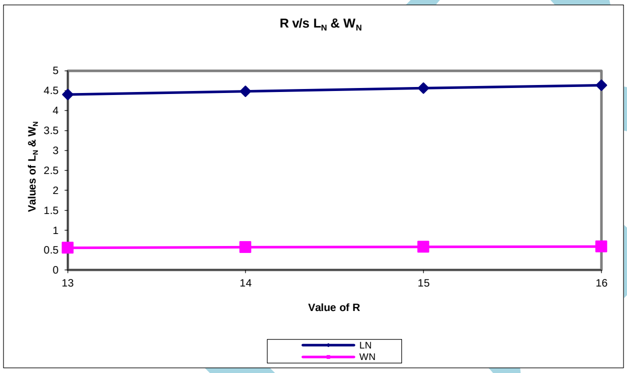


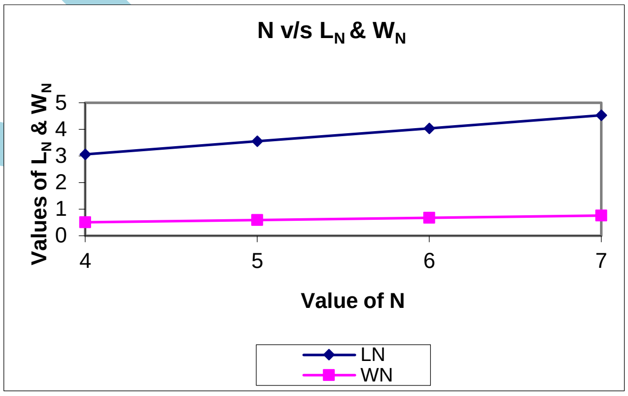


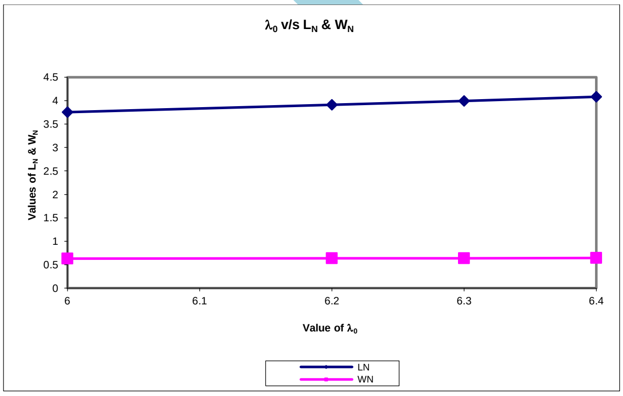


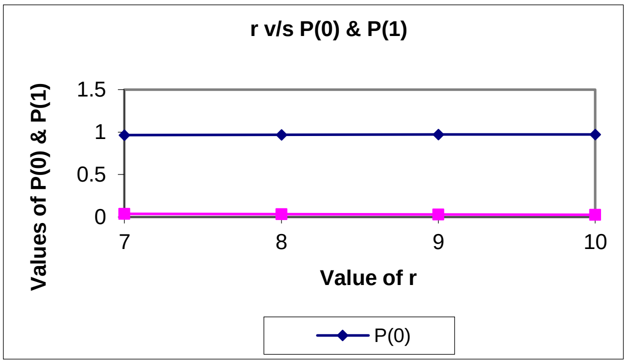












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