Designer nuclei and some of their properties

Hezekiah K. Cherop[1], Kenneth K. Sirma[2], Kapil M. Khanna[1] and Paul Seurey[1].

[1]Department of Physics, University of Eldoret, P.O Box 1125-30100, Eldoret, Kenya.
[2]Department of Renewable energy and Technology, Turkana University College, P.O Box 69-30500, Lodwar, Kenya.
Author for correspondence: khannak700@gmail.com

Abstract- **Designer nuclei are those nuclei that rarely exist in nature as stable nuclei. They are a consequence of the fabrication process at the scale of atomic nuclei. Designer atomic nuclei are new, rare isotopes, with unusual numbers of protons or neutrons with unusual decay modes. For instance, super heavy isotopes of light elements such as $^{11}_{3}$Li, have a very high ratio of neutrons to protons $\left(\frac{N}{Z}=\frac{8}{3}\right)$ such that the neutrons in $^{11}_{3}$Li have very low binding energy. The size of $^{11}_{3}$Li is roughly $10fm = 10^{-14}$m (it stretches to a longer size) which corresponds to roughly ten times the volume of the stable $^{6}_{3}$Li nucleus, and it has the size of $^{220}_{86}$Ra nucleus. The $^{11}_{3}$Li nucleus has a diffuse surface of neutron matter, and this is due the fact that quantum mechanically the wave function of the neutrons can extend far beyond the normal range of the nucleus. The existence of such nuclei allows the study of interaction of neutrons in pure neutron matter, such as neutrons stars, and also in the case of finite nuclei $^{N}_{Z}X$ where $N \gg Z$. In such a system it can be assumed that the core of the nucleus is made up of Z neutron-proton pairs (deuterons) which are surrounded by the unpaired neutrons (N-Z). As the neutron number becomes very large compared to the proton number, variation of the charge radius of the nucleus can also be studied. Such calculations can assist in the development of new nuclear theory.**

Keywords: Designer nuclei, Neutron excess, Bogoliubov technique, Isotopes, Finite nuclei.

I. Introduction

A designer nucleus can be defined as the nucleus that is produced in the laboratory by the collision of two nuclei. It has an abnormal number of protons or neutrons when compared to the parent nucleus like $^{6}_{3}$Li (parent) and $^{11}_{3}$Li (designer). The creation of designer nuclei is an experimental effort to decide or to observe as to how many neutrons and protons can constitute a bound atomic nucleus. There are around 3200 known isotopes [1] out of which 286 primordial nuclides have existed in their current form since the earth was formed. On the nuclear landscape, they form the valley of stability. The designer nuclei are created by arranging nuclear reactions in the laboratory, i.e., a stable nucleus acts as a target and a beam of projectiles bombard the target with the aim of transferring more neutrons or protons to the target nucleus. This results in the creation of new isotopes (or nuclei) by adding neutrons or protons. When the number of neutrons or protons added to the stable isotope is comparatively large, then short-lived radioactive nuclei or beta unstable nuclei are created. If we go on adding neutrons or protons to a nucleus, we reach a drip line stage, whereby strong internucleon interaction does not exist; nucleons are no longer attached to the nucleus, and the neutron or proton simply drips off. With reference to the present theoretical estimates, the number of bound nuclides with atomic number Z between 2 and 120 are around 7000 [2, 3].

It is important to note that nucleonic pairing and or nuclear superfluidity strongly affect the separation energies and drip line positions [4]. The knowledge of these two quantities is important in the creation of designer nuclei since breaking nucleonic pair costs energy, and the nuclei with even numbers of nucleons are more bound than their odd-nucleon-number neighbours. Consequently, one-nucleon drip line is reached earlier than the two nucleon drip line. This can help in the creation of designer nucleus with odd number of nucleons when compared with its neighbour that will have even number of nucleons. The lowest atomic number (Z) super heavy nucleus that was experimentally created is $^{11}_{3}$Li in which the neutron (N) to proton (Z) ratio is very large compared to the naturally existing isotopes ($^{6}_{3}$Li and $^{7}_{3}$Li) [2, 5]. The nucleus $^{11}_{3}$Li stretches to the extent that its volume is roughly 10 times the volume of the normal $^{6}_{3}$Li nucleus [5, 6] and designer nuclei with neutron excess will have a neutron skin of finite size. Now as the mass number (A) increases, the number of neutrons (N) increases faster than the proton number (Z), and in the case of heavy and super heavy nuclei (SHN or SHE) there is finite neutron skin of the nucleus. It is thus likely that designer nuclei with large neutron excess may have properties similar to the SHN with large neutron excess.

A number of neutron-rich nuclei have been discovered from time to time [7-10] for the production of the so-called designer nuclei or the super heavy nuclei. A suitable projectile-target combination is to be chosen such that the target and projectile do not get excited very much and are able to cross the potential barrier so as to collide with each other, and result in a new stable nucleus [11-13]. The challenge is to predict a correct projectile and target-combinations to produce desired nuclei [14].

Thus, in an effort to produce designer stable nuclei, whether low mass or super heavy nuclei, chemists and physicists have been trying a fabrication process to create stable nuclei with abnormal neutron number or proton number [15]. Such nuclei are rare isotopes with unusual decay modes [15]. The demand for the unusual type of new isotopes is high since the new isotopes may lead to the understanding of the characteristic properties of large finite nuclei with abnormally large neutron excess. The properties could be the pairing among the nucleons, constitution of the core region of the nucleus, the neutron skin of the nucleus, the size of the nucleus, the role of Coulomb interaction in the decay process from the nucleus, the role of the neutron excess in studying the creation of nuclear energy and nuclear fission, and the existence of the island of stability of super heavy nuclei. There could be a host of other still unknown nuclear properties and phenomena that may be studied in future depending upon what kind of isotopes or nuclei we are able to produce, and what kind of experimental methods are used to produce them.

The techniques to produce super heavy isotopes of light nuclei with large neutron excess can affect the properties of nuclei. For instance, in super heavy Lithium(${}^{11}_{3}Li$), the ratio $\frac{N}{Z}$ is large when compared to ${}^{6}_{3}Li$. The inter neutron distance will become large and this can result in decrease of binding energy of the neutrons. The nucleus ${}^{11}_{3}Li$ has diffused surface of neutron matter, and its size is of the order of a very heavy nucleus ${}^{220}_{88}Ra$ [16].

It is being argued that the region of neutron-rich nuclei is the best range of nuclei to open the way for understanding nuclear structure and do more research in this area [17]. The region of great interest is the Calcium region in which the heaviest isotope discovered so far is ${}^{60}_{20}Ca$, such that Z=20 and N=40. The heaviest stable Calcium isotope is ${}^{48}_{20}Ca$, and hence ${}^{60}_{20}Ca$ has 12 more neutrons when compared to the heaviest stable calcium isotope. Another set of seven recently found neutron-rich nuclei (isotopes) are ${}^{47}_{15}P({}^{31}_{15}P)$, ${}^{49}_{16}S({}^{32}_{16}S)$, ${}^{52}_{17}Cl({}^{35}_{17}Cl)$, ${}^{54}_{18}Ar({}^{36\text{-}40}_{18}Ar)$, ${}^{57}_{19}K$ and ${}^{59}_{19}K$ (${}^{39}_{19}K$), and ${}^{62}_{21}Sc({}^{45}_{21}Sc)$. This discovery extends the range of known neutron-rich nuclei.

Now quantum mechanically, the wave-function of the neutron (neutron halo) can extend far beyond the normal range of nucleus, just like the case of neutrons and the core in a square well potential. The interaction of neutrons with protons in nuclei with abnormal neutron excess is an important area of research in the sense that the charge radius of nuclei such as ${}^{11}_{3}Li$ can be estimated. Such studies may assist in the development of new concepts in nuclear theory [18].

The discovery of new isotopes (nuclei) with abnormal values of neutrons (N) and protons (Z) has shown that the quantum magic numbers in nuclei are not generally the same. They are different from what is observed for electrons in atomic physics. For example, a nucleus with 28 neutrons (28 is a magic number for nuclei) is sometimes but not always magic [19]. On the other hand, a change in the magic numbers in rare isotopes is not always found. This is clear from the results of the direct mass measurement of the rare isotope ${}^{132}_{50}Sn$ (proton number Z=50 and the neutron number N=82) that revealed a very large shell gap, which corresponds to the energy difference between the filled and unfilled shell model orbits [20].

It is well known that the atomic nuclei are used as fuel for the fission and fusion processes that are responsible for the creation of energy on the earth and in the sun and stars. Transuranic elements, like ${}^{235}_{92}U$ and ${}^{240}_{94}Pu$ are used as fissile materials in fission reactions that fuel the nuclear reactors on the earth. Element ${}^{2}_{1}H$ and ${}^{3}_{1}H$ are used as fuel for fusion reactors (known as stars on the Earth). These nuclei have sufficient neutron excess. For ${}^{235}_{92}U$, it is of the order of $\frac{N}{Z} = \frac{143}{92} = 1.554$, and for ${}^{3}_{1}H$, it is 2. Thus, if we can artificially produce nuclei with this ratio $\frac{N}{Z} = 1.554$ or more, such nuclei can be used as fuel in the fission and fusion reactors. Another alternative method could be to create designer nuclei with reduced Z such that $\frac{N}{Z} > 1.554$.

From the above, we conclude that the key to the development of nuclear fission reactors, fusion reactors or new nuclear theory could be to create nuclei with large neutron excess, or reduced number of protons, or increased number of protons. In the process, theoretical and experimental studies will have to be conducted to decide as to how to select the target nucleus and the bombarding nucleus. The target nucleus and the bombarding nucleus must be able to come close to each other, form a compound nucleus, and then disintegrate to appear as a designer nucleus with abnormal number of protons and neutrons.

We now have to decide which properties of the nuclei could have significant influence on the process of creating designer nuclei, and how the properties of designer nuclei could differ from the normal counterpart nucleus. One such essential property which is common to all kinds of nuclei is the ground state binding energy of nuclei that determines nuclear synthesis and existence. Another set of microscopic quantities are Coulomb repulsion and surface tension. The strongest effect on the stability of nuclei is the shell closure [21]. Some of the other macroscopic properties could be specific heat, entropy etc.

In an earlier attempt [16] it was assumed that a nucleus may be composed of a core of neutron-proton pairs (np-pairs or deuterons which are bosons since the spin is of deuterons is 1 which is an integer) surrounded by an envelop of unpaired neutrons which are called neutron skin of a nucleus with neutron excess (N > Z). In this manuscript, properties of some isotopes for which $\frac{N}{Z} = 1.554$ or more are studied. Some of such nuclei are ${}^{11}_{3}Li$, ${}^{125}_{49}In$, ${}^{173}_{66}Dy$, ${}^{220}_{88}Ra$ and ${}^{235}_{92}U$. Some nuclei that could be produced in the laboratories in future could be ${}^{15}_{6}C$, ${}^{18}_{6}C$, ${}^{20}_{8}O$, ${}^{76}_{26}Fe$, etc. Nuclei with Z= 114, 120 or 126 and N=172, 184 could also be produced [21]. We have chosen some of the low mass number (A) stable nuclei. How their stability will be affected by the excess neutrons is still to be investigated, possibly both theoretically and experimentally. The newly created heavy nuclei that have been added to the periodic table are Flerovium with Z=114, Livermorium with Z=116, Tennessine with Z=117 and Oganesson with Z=118 [22].

II. Theory

In the core of the nucleus which is composed of neutron-proton pairs, neutron and proton pairs interact with each other harmonically. The neutron-proton pair can be treated as a deuteron [23, 24]. The Hamiltonian, H, of such an assembly can be written as [25],

$$H = H_0 + H' = Z(n + \tfrac{1}{2})\hbar\omega + H' \qquad (1)$$

where H_0 corresponds to the energy of the core of the nucleus in which the neutron and proton pairs interact with each other harmonically, such that the core has Z pairs or deuterons. H' is the perturbation of the core by the neutrons in the neutron skin, and thus neutron number = N-Z. The form of H' is assumed to be,

$$H' = \beta x^3 + \gamma x^4 \qquad (2)$$

where β and γ are parameters for the perturbation such that,

$$\beta = \frac{\hbar\omega}{a_0^3} \text{ and } \gamma = \frac{\hbar\omega}{a_0^4} \qquad (3)$$

The numerical values of the physical quantities in Eq.(3) and their associated terms are,

$\hbar$ = reduced Planck's constant = $1.054 \times 10^{-27} erg.S$

μ = reduced mass of the np − pair = $8.369 \times 10^{-25} gm$

a_0 = radius constant = $1.3 \times 10^{-13} A^{\frac{1}{3}} cm$

K = Boltzmann's constant = $1.3807 \times 10^{-16} erg/K$

ω = Angular radian frequency = $6 \times 10^{22} S^{-1}$

We have now to write the trial wave function that will correspond to the interaction of an unpaired neutron in the core region of a nucleus with large mass number such that N > Z. The trial wave function can be written as, ψ, such that,

$$\psi = a_l^+(U_k + V_k a_k^+ a_k^+)|0\rangle \qquad (4)$$

Now, the expectation value of the perturbation is,

$$\langle\psi|H'|\psi\rangle = \langle 0|a_l(U_k + V_k a_k a_k)H'(U_k + V_k a_k^+ a_k^+)a_l^+|0\rangle \qquad (5)$$

Since we are dealing with protons and neutrons in the nucleus and these nucleons are fermions, then,

$$U_k^2 + V_k^2 = 1 \qquad (6a)$$

The value of the trial wave function ψ depends on the values of U_k and V_k that have the following possible values [26, 27],

$$U_k = 0 \text{ and } V_k = 1 \qquad (6)$$

$$U_k = 1 \text{ and } V_k = 0 \qquad (7)$$

$$U_k = \frac{1}{\sqrt{2}} \text{ and } V_k = \frac{1}{\sqrt{2}} \qquad (8)$$

Consequently, the trial wave function can account for the following possibilities:

i. If $V_k = 1$ and $U_k = 0$, this means that ψ contains the term $a_l^+ a_k^+ a_k^+$, and this means that the neutron-proton pair and the interacting neutron in the neutron skin (surface region of the nucleus) must co-exist always. However, this may not be true at all the times, and hence we need not to accept this possibility.

ii. If $V_k = 0$ and $U_k = 1$, this will mean that ψ contains the term a_l^+only, and that will mean that the neutron-proton pairs play no role in ψ. This situation cannot be accepted since the model is based on the existence of neutron-proton pairs in the wave function, as per the nuclear model.

iii. If $V_k = \frac{1}{\sqrt{2}}$ and $U_k = \frac{1}{\sqrt{2}}$, then ψ will contain terms like $\frac{1}{\sqrt{2}} a_l^+|0\rangle$ and $\frac{1}{\sqrt{2}} a_l^+ a_k^+ a_k^+|0\rangle$. The term $\frac{1}{\sqrt{2}} a_l^+|0\rangle$ will mean that the neutron in the surface region exists as an independent entity (which is true), and the neutron-proton pair in the core exists as an independent entity and this is represented by $\frac{1}{\sqrt{2}} a_k^+ a_k^+|0\rangle$. The term $\frac{1}{\sqrt{2}} a_l^+ a_k^+ a_k^+$ asserts that the neutron-proton pair in the core region interacts with the neutron in the surface region. Hence, the trial wave function, ψ, that will be used to calculate the expectation value of the perturbation H' will be $V_k = \frac{1}{\sqrt{2}}$ and $U_k = \frac{1}{\sqrt{2}}$.

Now neutron-proton pairs in the core interact with the neutrons in the surface region and the resulting perturbation $H' = \beta x^3 + \gamma x^4$ is an anharmonic perturbation such that the displacement x will be written in terms of creation operator, a^+, and destruction operator, a , in the form,

$$x = \frac{1}{\alpha\sqrt{2}}(a^+ + a) \qquad (9)$$

$$\text{where } \alpha = \sqrt{\frac{\mu\omega}{\hbar}} \qquad (10)$$

μ =reduced mass of the neutron-proton pair and neutron and it is given by,

$$\mu = \frac{(m_n + m_p)m_n}{(m_n + m_p + m_n)} \qquad (11)$$

ω = Natural frequency of oscillation of the oscillator which is a neutron-proton pair.

The total energy (E_n), which will be the binding energy of the nucleus can be written as,

$$E_n = E_0 + E' = Z\left(n + \frac{1}{2}\right)\hbar\omega + (N - Z)\langle\psi|H'|\psi\rangle \qquad (12)$$

where n=0,1,2,……

Here, E_0 is the energy of the neutron-proton core (np-pairs) of the nucleus, and E' is the interaction energy of the unpaired neutrons (N-Z) in the surface region of the nucleus with the Z np-pairs of the core. We have now to evaluate the value of $\langle\psi|H'|\psi\rangle$. We can write it as,

$$\langle\psi|H'|\psi\rangle = \langle\psi|\beta x^3|\psi\rangle + \langle\psi|\gamma x^4|\psi\rangle \qquad (13)$$

Substituting for x from Eq. (9) in Eq. (13) gives,

$$\langle\psi|H'|\psi\rangle = \frac{\beta}{\alpha^3\sqrt{8}}\langle n, 0|a_l(U_k + V_k a_k a_k)(a + a^+)^3 * (U_k + V_k a_k^+ a_k^+)a_l^+|n, 0\rangle + \cdots \qquad (14)$$

After lengthy calculations using anti-commutation laws for the creation and annihilation operators gives,

$$\langle\psi|H'|\psi\rangle = \frac{\gamma\frac{1}{2}}{4\alpha^4}\{6n^5 + 86n^4 + 467n^3 + 1180n^2 + 1378n + 585\} = E_n \qquad (15)$$

Hence, $E' = (N - Z)E_n$ (16)

and $E_n = E_0 + E' = Z\left(n + \frac{1}{2}\right)\hbar\omega + (N - Z)E_n$

(n = 0, 1, 2…) (17)

Substituting $U_k = V_k = \frac{1}{\sqrt{2}}$ in Eq. (17) gives,

$$E_n = Z\left(n + \frac{1}{2}\right)\hbar\omega + (N - Z)\frac{\gamma}{4\alpha^4}\frac{1}{2}(6n^5 + 86n^4 + 467n^3 + 1180n^2 + 1378n + 585) \qquad (18)$$

Binding Fraction (f)

Binding fraction (f) is the binding energy per nucleon when the nucleus is in the ground state, i.e., it is the value of $\frac{E_n}{A}$

when $n = 0$. Eq. (18) for $n = 0$ will lead to the value of binding fraction, i.e.,

$$f = \frac{E_0}{A} = \frac{Z}{A}\left(\frac{\hbar\omega}{2}\right) + \left(\frac{N-Z}{A}\right)\left(\frac{\gamma}{4\alpha^4}\right)\frac{1}{2}.(585) \qquad (19)$$

In Eq. (19), the quantity $\frac{N-Z}{A} = \eta$ is called the neutron excess parameter, i.e,

$$\eta = \frac{N-Z}{A} \qquad (20)$$

Eq. (19) emphatically establishes that the binding energy E_0 of the nucleus, and also the binding fraction explicitly depends on the neutron excess parameter (η) as it should be for N > Z nuclei. Clearly, for N = Z, the perturbation energy in this model is $E' = 0$, since for such nuclei there will be no such a thing as neutron skin or neutron surface of the nucleus.

Specific heat (C)

To calculate the specific heat C, it is necessary to include the probability amplitude Green's function factor. The corresponding thermal activation factor is $e^{-\frac{\hbar\omega}{KT}}$. Using this in Eq. (18), we can write,

$$C = \frac{\partial E_n}{\partial T} = (N-Z)\frac{\gamma}{8\alpha^4}\frac{\hbar\omega}{KT^2}(6n^5 + 86n^4 + 467n^3 + 1180n^2 + 1378n + 585)e^{-\frac{\hbar\omega}{KT}} \qquad (21)$$

Eq. (21) gives the value of C as a function of temperature T.

Transition temperature (T_C)

The transition temperature (T_C) of the nucleus is given by,

$$\left(\frac{\partial C}{\partial T}\right)_{T=T_C} = 0 \qquad (22)$$

Substituting for C from Eq. (21) in Eq. (24) we get,

$$T_C = \frac{\hbar\omega}{2K} \qquad (23)$$

Entropy (S)

The expression for entropy S is,

$$S = \int \frac{MCdT}{T} \qquad (24)$$

where M = mass of the nucleus.

Substituting Eq. (21) in Eq. (22) and integrating leads to,

$$S = (N-Z)\frac{\gamma}{8\alpha^4}\frac{\hbar\omega}{K}(6n^5 + 86n^4 + 467n^3 + 1180n^2 + 1378n + 585)\left(\frac{K}{\hbar\omega T}e^{-\frac{\hbar\omega}{KT}} + \frac{K^2}{\hbar^2\omega^2}e^{-\frac{\hbar\omega}{KT}}\right) \qquad (25)$$

III. RESULTS AND DISCUSSION

The calculations of the binding fractions using Eq. (19) are shown in Table 1. It is noted that the values of the binding fractions with A, Z and N is very large in the case of $^{11}_{3}$Li. This is caused by the large value of the neutron excess parameter the yields greater value of the neutron-proton ratio, that is, $\frac{N}{Z} = 2.67$. As the values of A increase, the binding fractions decrease since the binding fraction depends on the neutron excess parameter.

Table 1: Variation of binding fraction (f) with A, Z, N and η for n=0 using Eq. (19)

A	Z	N	η	f (MeV/nucleon)
11	3	8	0.4545	88.232
125	49	76	0.2160	9.288
173	66	107	0.2370	8.636
220	88	132	0.2000	8.576
235	92	143	0.2170	8.404

The graphical illustration of the excitation energies (E_n) with mass number (A) is shown in Fig. 1. The values of E_n were obtained from Eq. (18) for the six levels of the excitations namely n = 0, 1, 2, 3, 4 and 5. At the ground level excitation (n = 0), the excitation energies among all the selected designer nuclei are relatively small compared to the other levels (they range between 900MeV to 2000MeV). As the values of n increase the E_n also increase with greater values noted in the level n = 5, with $^{11}_{3}$Li recording the highest value of over 260000MeV. This implies that, there is a linear relationship between E_n, the levels of excitations (n) and the mass numbers (A).

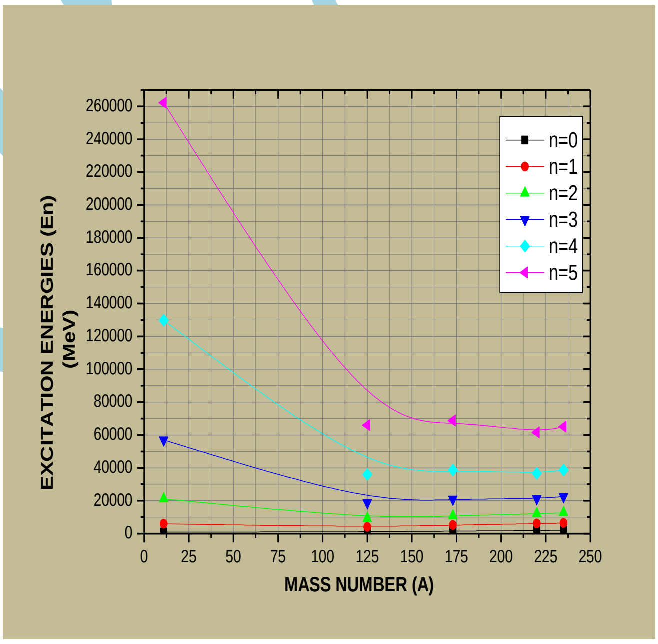

Figure 1: Variation of excitation energies with mass number (A) using Eq. (18)

On the variation of specific heat capacities (C) and entropy (S) with the mass number (A), it was noted that, the specific heat capacities and entropies of the designer nuclei are directly proportional to the excitation energies and the mass number as shown in Fig. 2 and Fig. 3. The transition temperature(T_C) was calculated using Eq. (23) and the numerical values for the physical quantities in Eq. (3). It was found that $T_C = 2.29 \times 10^{11}K$ and this value was used to calculate the specific heat capacities and entropies of the selected nuclei in Eq. (21) and Eq. (25) respectively.

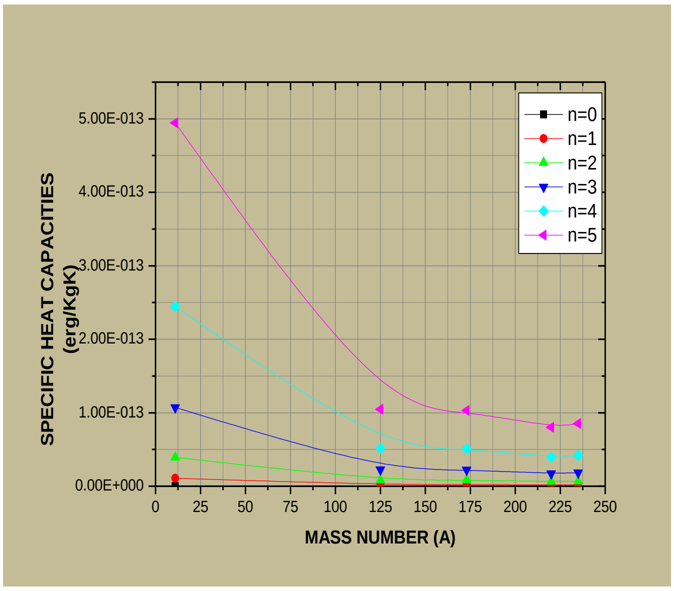

Figure 2: Variation of specific heat capacities (C) with mass number (A) using Eq. (21)

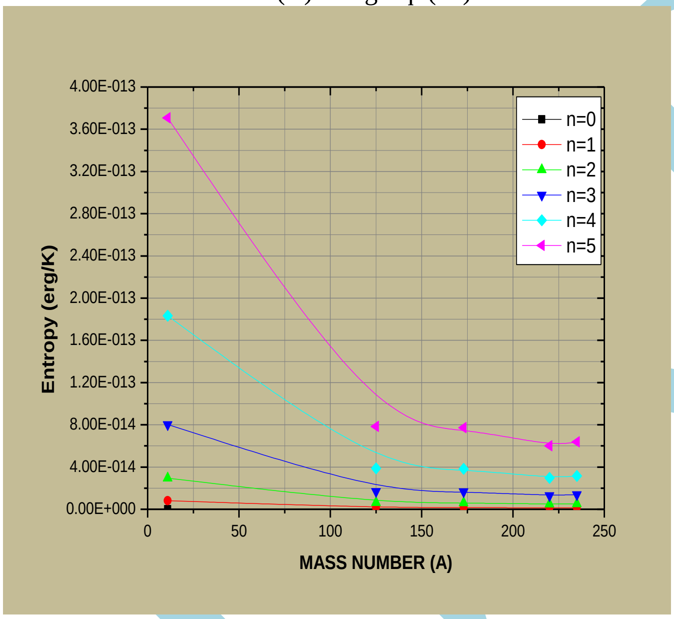

Figure 3: Variation of Entropy (S) with mass number (A) using Eq. (25)

IV. CONCLUSION

In this research, it is assumed that the interaction of np-pairs in the core region of the nucleus is harmonic. Similarly, the interaction of the un-paired neutrons in the surface region with the np-pairs in the core region leads to anharmonic interactions that favour the formation of fissile materials. Such nuclei have abnormally large number of neutrons or protons. Consequently, they yield very large values of specific heat capacities, entropies and excitation energies just like the case of a pure neutron matter.

The exact form of the nuclear reactions for such designer nuclei is not exactly known. Thus, it is proposed that the following nuclear reactions can be tried on the ${}^{11}_{3}Li$ designer nucleus, and such nuclear reactions can possibly be used to produce nuclear energy and new nuclear reactors can be designed.

$$ {}^{11}_{3}Li + {}^{1}_{0}n \rightarrow {}^{4}_{2}He + {}^{1}_{1}H + 7\,{}^{1}_{0}n $$

$$ {}^{11}_{3}Li + {}^{1}_{1}H \rightarrow 2\,{}^{4}_{2}He + 4\,{}^{1}_{0}n $$

$$ {}^{11}_{3}Li + {}^{3}_{2}He \rightarrow 2\,{}^{4}_{2}He + {}^{1}_{1}H + 5\,{}^{1}_{0}n $$

$$ {}^{11}_{3}Li + {}^{2}_{1}H \rightarrow 2\,{}^{4}_{2}He + 5\,{}^{1}_{0}n $$

However, the exact value of the energy released in the nuclear reaction can not be calculated since the exact mass of ${}^{11}_{3}Li$ is not yet measured.

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