Persistence and Long-Range Dependence in Indian Stock Market Returns using Fractal Market Hypothesis

Rajnish Jindal[1], Dr. Deepak Kumar Agarwal[2]

[1]Ph.D. Research Scholar in Department of Commerce
Multanimal Modi College, Modinagar
Chaudahry Charan Singh university, Meerut, Uttar Pradesh, India
[2]Associate Professor, Multanimal Modi College, Modinagar
Chaudahry Charan Singh university, Meerut, Uttar Pradesh, India

Abstract: **The Indian Stock Market is made up of participants from tick traders to long-term investors. Each participant having different investment horizons trade simultaneously in the market. This paper finds evidences of long memory in the returns from the data collected from Indian Bombay Stock Exchange. Fractal Market Hypothesis (FMH) explores the application of chaos theory and fractals to finance. The main idea of the fractal model is that the investors may not directly react to information. Instead, the investors may normally react with delay. The standard complement of statistical techniques used to identify predictable market structure, is capable of identifying regular periodic cycles. The Indian stock market has a high degree of persistence.**

Keywords: Persistent, Long Range, Dependence, Rescaled Range Analysis, Hurst Exponent, Non-Periodic cycles, Bombay Stock Exchange

I. Introduction

The nonparametric statistical techniques provide a viable alternative testing for nonlinear structure and are ideal for modelling financial data as they make prior assumption about the distribution of data. One such nonparametric technique is Rescaled Range Analysis (Hurst H.E -1951). The Rescaled Range value iscalled the rescaling range. In a time series, the range increases with increments of time. Rescaled Range Analysis is a useful tool for revealing some well disguised properties of stochastic time series such as persistence and anti-persistence characterized by non-periodic cycles (Jonathan M Blackledge-2008).

The Fractal Model accounts for markets that consist of many irrational investors, trading at different investment horizons. As long as the market maintains a fractal structure with no characteristic time scale, the market should remain stable. If all investors' ime horizon were to become unique, the market would become unstable because everyone would respond according to the same information at the same time. The standard complement of statistical techniques used to identify predictable market structure is capable of identifying regular periodic cycles (McKenzie Michael D-2001).

II. Literature Reviews

The previous studies analyzed the long memory and market cycles in stock markets are reviewed below. [4], Edgar E.Peters (1989) tested the behaviour of stock and bond returns of Standard & Poor 500 using Rescaled Range Analysis for computing Hurst Exponent and found that the returns showed stronger influence of investors' sentiment. [7] Hsing Fang, et al., (1994) estimated the fractal structure in Currency Futures prices of British Pound, German Mark, Japanese Yen and Swiss Franc. A time series having fractal structure was characterized by long-term dependence and non-periodic cycles. [3] Muller U.A., et al., (1995) demonstrated the Intra-day Foreign Exchange prices of USD- DEM of monthly and weekly seasonal aspect. The study found that a time value and Fractal properties lead to the hypotheses of a heterogeneous market. [9] John T Barkoulas, et al., (1999) found the Fractional dynamics characterized by irregular cyclical fluctuations with long-term dependence and the fractal structure with long memory in several series highlighting the similarities and differences in dynamic behaviour among future return series. [3] Dr. Singh J P and Parikshit Dey (2002) examined the efficacy of the statistical measures of risk of stock market using the Rescaledrange analysis and Hurst's exponent, Fractal dimensions and the Lyapunov exponents. The results found that the stock market has nonlinearities. [16] Tran Van Quang (2005) analysed the Czech equity price index using the Fractal Market Hypothesis. The result reveals that the price changes on Czech equity market did not follow a random walk and the market was far behind efficient. [1] Assaf A (2006) tested the stock market of Kuwait having long memory in the monthly returns, absolute returns, squared returns and modified long squared returns using the analysis of rescaled range statistic and rescale variance statistic. [17]Wen-Cheong Chin, et al., (2007) investigated the volatility measurement of Kuala Lumpur stock exchange index prices. Autoregressive Fractionally Integrated Moving Average model and Heterogeneous Autoregressive was used to find out the long memory and volatility in the market. The results showed that the long memory exhibited in the market due to significant relation between news and volatility. [5] Gayathri M. and Selvam M. (2011a) studied the efficiency of Fractal Market Hypothesis in the Indian Stock Market. According to this study, any new information would be immediately and fully reflected in prices and stock returns of equity and also the short term and long term trade follow technical information and fundamental information respectively. [6] Gayathri M. and Selvam M. (2011b) analyzed the Fractal Structure in the National Stock Exchange of India and examined the long range dependence of daily returns of Nifty in the stock market and the stock prices did not reflect the information in the past series of stock prices. [14] Murugesan Selvam, et al (2011) tested the Fractal Structure Analysis in the Indian Stock Market. The study analyzed the Fractal Structure in Sensex returns, and found that the trend occasionally followed the random walk initially. Thus the study found that the fractal structure existed in the BSE Sensex.

There is no comprehensive study analyzing the recent scenario in the Indian stock markets using the Rescaled Range Analysis. Hence, the present study aims to investigate the monthly returns of long memory in Indian Stock Market.

III. Statement of the Problem

A stock market crash is often defined as a sharp dip in share prices of equities listed on the stock exchanges. In parallel with various economic factors, a reason for stock market crashes is also due to panic and investing publics' loss of confidence. Often, the stock market crashes end speculative economic bubbles. The new information regarding securities comes to the market in a random fashion and the timing of one announcement is generally independent of others [18]. The competing investors attempt to adjust the security prices rapidly to reflect the effect of new information [19]. If all investors' time horizon were to become unique, the market would become unstable because everyone would respond according to the same information.

It is to be noted that there was the lack of quantitative evidences about the potential bias of equity returns in Indian Stock Market and the failure of standard statistical analysis to uncover any long-run trends or cycles in financial market data [20]. As of today, there is no empirical evidence that supports or rejects the randomwalk model. Hence an attempt is made to study the long range dependence in the Indian Stock Market.

IV. Need of the Study

This study using the Rescaled Range Analysis, a useful tool for revealing stock price returns of stochastic time series such as persistence and long range dependence characteristics by non- periodic cycles. The purpose of R/S analysis is to provide an assessment of how the apparent variability of a series changes with the length of the time-period being considered and to determine the fractal dimension of a time series. In assessing the fractal dimension of asset returns, the investors may better understand the systematic pattern of price returns and consequently may adjust their asset pricing strategies accordingly.

V. Objectives of the Study

The objective of the study is to analyze the stationary and to examine the long range dependence of monthly index returns of Sensex.

VI. Hypotheses of the Study

The following null hypotheses are tested.

NH1 - The monthly index returns of Sensex are non stationary and

NH2 - The monthly index returns of Sensex have no long range dependence.

VII. Methodology of the Study

A. Sources of Data and Computational Details

The monthly index returns of BSE Sensex were used to examine the evidence of non-periodic cycles and long range dependence. BSE is the oldest stock exchange in Asia. The equity market capitalization of the companies listed on the BSE was US$1 trillion as of December

2011, making it the 6th largest stock exchange in Asia and the 14th largest in the world. The BSE has the largest number of listed companies in the world. The required data were collected from the PROWESS, a corporate database. The other required data were collected from Books, Journals and various Websites.

B. Period of the Study

The study period covered a period of ten years from 1st January, 2010 to 31st December, 2021.

C. Tools Used in this Study

The following tools were used for this study.

1. Descriptive Statistics

The descriptive statistics include Mean, Median and Standard Deviation.

2. Augmented Dickey Fuller Test

In statistics and econometrics, an Augmented Dickey–Fuller test(ADF) is a test for a unit root in a time series sample. The more negative it is, the stronger the rejection of the hypothesis that there is a unit roots at some level of confidence. The testing procedure

for the ADF test is the same as for the Dickey–Fuller test but it is applied to the model

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \delta_1 \Delta y_{t-1} + \cdots + \delta_p \Delta y_{t-p} + \varepsilon_t,$$

where α is a constant, β the coefficient on a time trend and p the lag order of the autoregressive process. By including lags of the order p the ADF formulation allows for higher-order autoregressive processes. This means that the lag length p has to be determined when applying the test. The unit root test is carried out under the null hypothesis $\gamma = 0$ against the alternative hypothesis of $\gamma < 0$.

3. Rescaled Range Analysis

Hurst (1965) developed the rescaled range analysis, a statistical method to analyze long records of natural phenomena. There are two factors used in this analysis: firstly the range R, which is the difference between the minimum and maximum 'accumulated' values or cumulative sum of X (t,tau)of the natural phenomenon at discrete integer-valued time t over a time span tau, and secondly the standard deviation S, which is estimated from the observed values Xi(t). Hurst found that the ratio R/S is very well described for a large number of natural phenomena by the following empirical relation

$$\frac{R}{S} = (C.T)^H$$

where tau is the time span, and H the Hurst exponent. The coefficient c was taken equal to 0.5 by Hurst. R and S are defined as:

$$R(\tau) = \max_{1\le t\le \tau} X(t,\tau) - \min_{1\le t\le \tau} X(t,\tau)$$ and

$$S = \left(\frac{1}{\tau} \sum_{t=1}^{\tau} \{\xi(t) - \langle\xi\rangle_\tau\}^2 \right)^{\frac{1}{2}}$$

Where, $\langle\xi\rangle_\tau = \frac{1}{\tau}\sum_{t=1}^{\tau} \xi(t)$ and

$$X(t,\tau) = \sum_{u=1}^{t} \{\xi(u) - \langle\xi\rangle\tau\}$$

VIII. Limitations of the Study

The following are the major limitations of the study

1. The study was confined only to the Sensex returns.
2. As the study was based mainly on secondary data, it was beset with certain limitations which were bound to arise while dealing exclusively with secondary data, and
3. All limitations associated with various tools like Descriptive Statistics, Augmented Dickey Fuller Test and Rescaled Range Analysis, are also applicable to this study.

IX. Analysis of Stationary and Long Range Dependencein the BSE Sensex Returns

For the purpose of this study, the analysis is made as follows

1. Analysis of Descriptive Statistics of BSE Sensex returns
2. Analysis of Augmented Dickey-Fuller test (ADF) of BSE Sensex returns
3. Estimating the Hurst Exponent using Rescaled Range Analysis of BSE Sensex returns
4. Estimating the Hurst Exponent for non-periodic cycles of BSE Sensex returns, and
5. R/S Chart for 20 months, 30 months, 50 months, 80 months and 120 months period of BSE Sensex returns

A. Analysis of Descriptive Statistics of BSE Sensex Returns

Table-1 reveals the results of Descriptive Statistics for BSE Sensex Returns from January 2002 to December 2011. The perusal of the above Table clearly shows that Sensex had earned positive average returns of 0.0164 and it implies that the price series have slowly increased over the period of time. The Sensex returns earned the standard deviation of 0.0768 which reveals the risk in the market. The negative skewness i.e., -

2.06832 indicates a distribution with an asymmetrical extending towards more negative values. It is clearly understood from the analysis that the market was negative skewness characterized the degree of asymmetry of a distribution around its mean. The Kurtosis measure of positive return distribution was characterized the relative peakedness, whose value was 4.150352. Thus the possibility of a distribution was leptokurtic as the value of kurtosis was positive. It indicates that the investors can mean a greater chance of extremely negative outcomes in BSE during the study period.

B. Analysis of Augmented Dickey-Fuller (ADF) of BSE Sensex

The results of ADF Test with regard to the BSE Sensex returns from January 2002 to December 2011 are presented in Table-2. According to above Table, the statistical value for the Augmented Dickey-Fuller (ADF) was -9.557375 and its probability was 0.00000. The test critical values were -3.486064 at 1% level,
-2.885863 at 5% level and -2.579818 at 10% level. It is to be noted that the calculated statistical value (-9.557375) was lesser than the critical values (-3.486064) at 1% level, (-2.885863) at 5% level and at 10% level (-2.579818). It is clear that the ADF Test provided the evidence of stationary in respect of monthly Sensex returns. The overall analysis of ADF test proved that the null hypothesis (NH1) - The monthly index returns are non stationary, is rejected. It was not necessary to analyse the data at the further level as it was stationary at the unit root test itself.

C. Estimating the Hurst Exponent using Rescaled Range Analysis on BSE Sensex

It is to be noted that if Hurst Exponent value H is close to 0.5, it indicates a random walk of the return. If Hurst Exponent value H lies between 0 and 0.5, it means that time series exhibits "anti- persistent behaviour". It indicates that the prices increasing in the past imply the prices decreasing in the future and vice versa. If Hurst Exponent value H lies between 0.5 and 1, it means that the time series has "persistent behaviour". It implies the fact that the prices increasing trend in the past clearly implies the increasing trend in the future prices too and vice versa.

Table 3 shows the analysis of Hurst Exponent estimation of a time series from January 2002 to December 2011. During the study period, the value of R/S analysis for the Sensex returns was 0.5690. It provides the evidence of persistence in the monthly index returns as the Hurst value is greater than 0.5. Besides, the overall analysis of the above Table indicates that the monthly index returns of Sensex were discovered to be long memory during the study period. Hence the null hypothesis, (NH2) - "The monthly index returns have no long range dependence" is rejected. The fractal dimension of time series of stock returns for Sensex was 1.4289 (calculated according to formula D=2-H). It is clearly understood that the stock prices did not reflect the information in the past series of stock prices. Therefore, the investors should make their investment decision based on important information available in the stock market.

D. Estimating the Hurst Exponent for Non-Periodic Cycles on BSE Sensex

An added advantage of rescaled range analysis lay in its ability to discern cycles within data. This not only includes regular periodic cycles but also non-periodic cycles as well. Table-4 shows the Hurst Exponent values from January 2002 to December 2011. The analysis of Hurst Exponent for Non Periodic Cycles of Sensex reveals the Hurst Exponent value in the long range dependence. It is to be noted that none of the values for the time lags was equal to 0.50, indicating that the Indian Stock Market did not follow random walk in so far as the monthly returns were concerned. This shows that there was a definite possibility for persistence in the Sensex returns but the values for time lags for 80 months were close to 0.50, indicating that the trend was not perfectly established. However, for the longer periods, the values were reasonably higher than 0.5, indicating a definite possibility for persistence. Therefore, the investors may share the same risk level with different investment horizon, which would make the market stable.

E. R/S Chart for 20 Month, 30 Month, 50 Month, 80 Month and 120 Month Period of BSE Sensex Returns

The movements of monthly index returns of Sensex using Rescaled
Range Analysis were given in figs. 1 to 5.

Fig. 1 shows the monthly index returns of Sensex movements for 20 months. The plots of log(R/S) and log(N) represents the empirical rescaled ranges of the data set for 20 months. It is to be noted that the log (R/S) values slowly increases while the log
(N) period (i.e., the returns of Sensex values) increasing during the period of 20 months. The above Figure clearly reveals the persistence of time series. Therefore the movement of stock market (BSE) was predictable during the study period.

The monthly index returns of Sensex movements for 30 months are depicted in Figure-2. The above Figure reveals that the price trend moves upward direction during the period of 30 months. It shows the downward direction of price trend during the beginning of a study period, but not for the entire period of 30 months. Therefore the analysis of Hurst exponent value for 30 months detects the presence of increasing trend in time series. Thus the investors may easily determine the price distribution of a time series.

The monthly index returns movements for 50 months are presented in Figure-3. According to the above Figure, the series of plots move towards the upward direction through the period of 50 months. It implies that the series of data clearly reveals persistence, as the time series reflect the increasing trend. Thus the current market prices impact the future price trend forever. This may be a result of longer returns periods picking up trends in the stock prices in the BSE Sensex returns.

Fig. 4, portrays the monthly index returns movements for 80 months. The plots, as shown in the above Figure, clearly indicate that the trend was not perfectly established. Because, the plots shows upward and slightly downward trend for the period of 80 months, as the Hurst value is nearby 0.50 (i.e.,0.5271). However, for the longer period, the series were reasonably increasing through the period of 80 months, indicating a definite possibility for persistence of Sensex returns. It reveals the fluctuations in Sensex returns of 80 months covering more distance as the tendency of the price to continue its current direction. Hence the investors can easily predict the stock prices in the market during the study period.

Fig. 5 shows the monthly index returns movements for 120 months. The stock market was persistence, as the price trend moves the upward direction throughout the period. It demonstrates that stock market participants habitually overreact to past news and information. Thus, the series of data reveals the long memory of Sensex returns, when the numbers of observations are increasing.

X. Conclusion and Discussion

The present study examined the long range dependence in the BSE Sensex returns using the Rescaled Range Analysis. It was found that there was persistence in stock market during 20 months, 30 months, 50 month s, 80 months and 120 months as all measures of Hurst range from 0.5 to 1. It indicates the index returns of Sensex confirmed persistence during the study period. It is clear that the investors reacted to the information being received from past news, reveals high degree of persistence in BSE Sensex returns. Besides, the rescaled range analysis confirmed persistent behaviour of Sensex returns.

Edgar E.Peters (1989) tested the behaviour of stock and bond returns of Standard & Poor 500 and suggested that the prices are reflected on the basis of stronger influence of investors' sentiment. Muller U.A, et.al., (1995) examined the foreign exchange prices and found that the time value and fractal properties lead to Heterogeneous market. The other studies by Hsing Fang et.al., (1994), John T Barkolas et.al., (1999) and Gayathri M and Selvam M (2011 b) also confirmed the time series having fractal dynamics characterised by long term dependence and non periodic or irregular cycles in the market by using the Rescaled Range Analysis. A similar study was conducted by Assaf .A (2006) who investigated the presence of fractional dynamics in the returns and volatility of the emerging stock market of Kuwait. The Variance Rescaled statistic for the returns which indicated that there was persistence and long range dependence in the Emerging Stock market of Kuwait. The findings of Hsing Fang et.al., John T Barkolas et.al., Gayathri M and Selvam M and Assaf .A confirmed the findings of the present study.

Murugesan Selvam et.al., (2011) by using fractal structure analysis in Indian Stock Market found that the market trends occasionally followed the random walk. The findings of present study did not confirm the findings of Murugesan Selvam et.al.,

Thus the study described the fractal structure of a series at long lags. There is persistent temporal dependence even between distant observations as a series exhibited long memory. Hence, the identification of persistent and long range dependence in the market may give rise to profitable trading opportunities to the investors and technical analysts.

References

[1] Assaf, A.,"Persistence and Long-range Dependence in the Emerging Stock Market of Kuwait", The Middle East Business and Economic Review, Vol. 18, No. 1, 2006, pp.1-17.

[2] Clifford Brown Larry Liebovitch.,"Fractal Analysis", Sage Publications India Pvt.Ltd. New Delhi, 2010.

[3] Dr.Singh, J.P., Parikshit Dey.,"Risk Measurement, Nonlinearities, and chaos", Singapore Management Review, Vol. 24, No. 2, 2002, pp. 47-55.

[4] Edgar E.Peters.,"Fractal Structure in the Capital Markets",Financial Analysts Journal, Vol. 45, No.

4, 1989, pp. 32- 37.

[5] Gayathri M., Selvam M. (2011a),"Efficiency of Fractal Market Hypothesis", National Conference on Management in the age of innovation, Arun Vasam Publishers, pp. 542

[6] Gayathri M., Selvam M. (2011b),"Analyzing the Fractal Structure in the Indian Stock Market with special reference to NSE index", International Conference on Changing Perspectives of Management, HSI publications. pp. 191.

[7] Hsing Fang, Kon S.Lai, Michael Lai.,"Fractal Structure in Currency Futures Price Dynamics: Introduction", The Journal of Futures Market, Vol. 14, No. 2, 1994, pp. 169- 181.

[8] Hurst.H.E.,"The Long Term Storage Capacity of Reservior,Transactions of the American Society of Civil Engineers",pp. 770-799.

[9] John T Barkoulas, Walter C.Labys, Joseph I. Onochie., "LongMemory In Futures Prices", The Financial Review, Vol. 34,No. 4, 1999, pp. 91-100.

[10] Jonathan M Blackledge.,"Application of the Fractal Market Hypothesis for Macroeconomic Time Series Analysis", ISAST Transactions on Electronics and Signal Processing,Vol. 1, No. 2, 2008, pp. 1-22.

[11] Mandelbrot, B., Fisher, A., Calvet, L.,"A Multifractal Model of Asset Returns", Yale University, Cowles Foundation Discussion Paper, 1997, pp. 1164.

[12] McKenzie Michael D.,"Non-Periodic Australian Stock Market Cycles: Evidence from Rescaled Range Analysis", Economic Record, Vol. 77, No. 239, 2001, pp. 393-406.

[13] Muller U.A, Dacorogna M.M, Dav'e R.D, Pictet O.V, Olsen R.B, Ward J.R,"Fractals and Intrinsic Time - A Challenge to Econometricians", International Conference of the Applied Econometrics Association, Luxembourg, 1995.

[14] Murugesan Selvam, Gayathri Jayapal, Saranya G. (2011), "Fractal Structure Analysis in the Indian Stock Market". [Online] Available: http://www.ssrn.com/abstract 1885030.

[15] Pravir Malik,"Redesigning the Stock Market - A Fractal Approach", Sage Publications India Pvt.Ltd. New Delhi, 2012.

[16] Tran Van Quang.,"The Fractal Market Analysis and its Application on Czech Conditions", Acta Oeconomica Pragensi, Vol. 13, No. 1, 2005, pp. 101-111.

[17] Wen-Cheong Chin, Zaidi Isa, Abu Hassan Shaari Mohd Nor.,"A Simple Generalized Long Persistence Realized Volatility Model", International Journal of Business and Society, Vol. 8, No. 1, 2007, pp. 1-14.

[18] Jaglan, V., Dalal, S., & Srinivasan, S. (2011). Improving performance of business intelligence through case based reasoning. International Journal of Engineering Science and Technology, 3(4), 2880–2886.

[19] Dalal, S., Jaglan, V., & Sharma, K. (2014). Designing architecture of demand forecasting tool using multi-agent system. International Journal of Advanced Research in Engineering and Applied Sciences, 3(1), 11–20.

[20] Dalal, S., & Athavale, V. (2012). Analysing supply chain strategy using case-based reasoning. Journal of Supply Chain Management Systems, 1(3), 40.

Table 1: Descriptive Statistics for Sensex Returns from 2012 to 2021

Descriptive Variable	Sensex
Mean	0.0164
Maximum	0.2826
Minimum	-0.2389
Standard Deviation	0.0768
Skewness	-0.206832
Kurtosis	4.150352
Number of observations	120

Source: Computed from PROWESS Corporate Database Using
E-Views 5.1

Table 2: Augmented Dickey-Fuller Test for Sensex Returns from 2012 to 2021

		t-Statistic	Probability
Augmented Dickey-Fuller test statistic		-9.557375	0.0000
Test critical values:	1% level	-3.486064	
	5% level	-2.885863	
	10% level	-2.579818	

Source: Computed from PROWESS Corporate Database Using E-Views 5.1

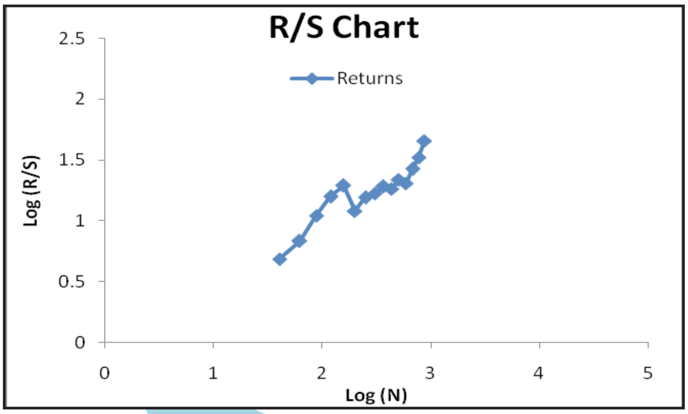

Fig. 1: Chart Showing R/S for 20 Months

Table 3: Fractal Dimension for Sensex Returns from 2012 to 2021

Hurst Exponent	Statistic Value
Rescaled range analysis	0.5690
Standard Deviation	0.0768
Fractal dimension	1.4289
Number of Observations	120
Degrees of Freedom	2

Source: Computed from PROWESS Corporate Database Using
KaotiXL 1.1 Software

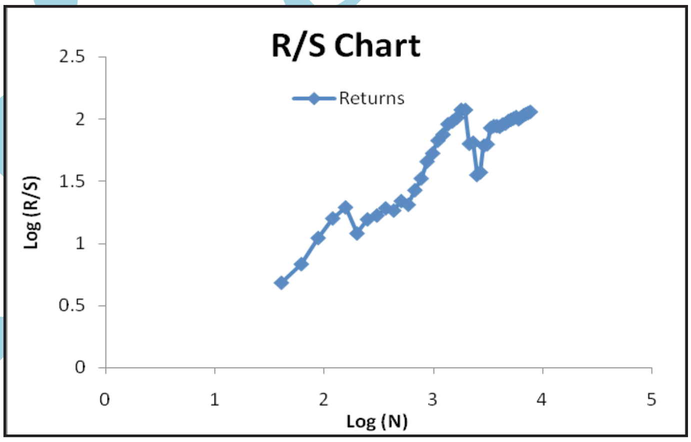

Fig. 2: Chart Showing R/S for 30 Months

Table 4: Hurst Exponent for Non Periodic Cycles of Sensex from 2012 to 2021

Period N	20 months	30 months	50 months	80 months	120 months
Hurst Exponent	0.5457	0.7393	0.5710	0.5271	0.5711

Source: Computed from PROWESS corporate database using
KaotiXL 1.1 software

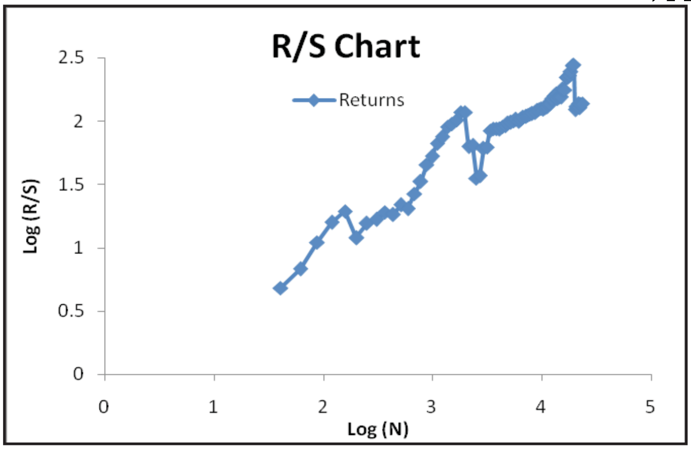

Fig. 3: Chart Showing R/S for 50 Months

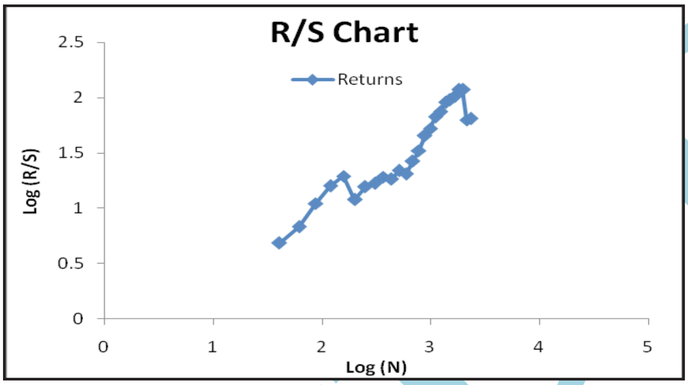

Fig. 4: Chart Showing R/S for 80 Months

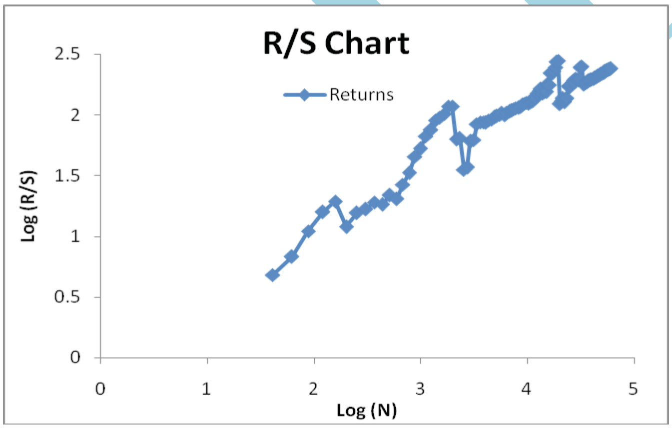

Fig. 5: Chart Showing R/S for 120 Months

